12th KIAS Workshop on Particle Physics and Cosmology

And 2024 Korea-France STAR workshop

Axion kinetic misalignment from PQ inflation

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HML, A. Menkara, M-J. Seong, J-H. Song, JHEP 05 (2024) 295

& arXiv: 2408.17013 [hep-ph]

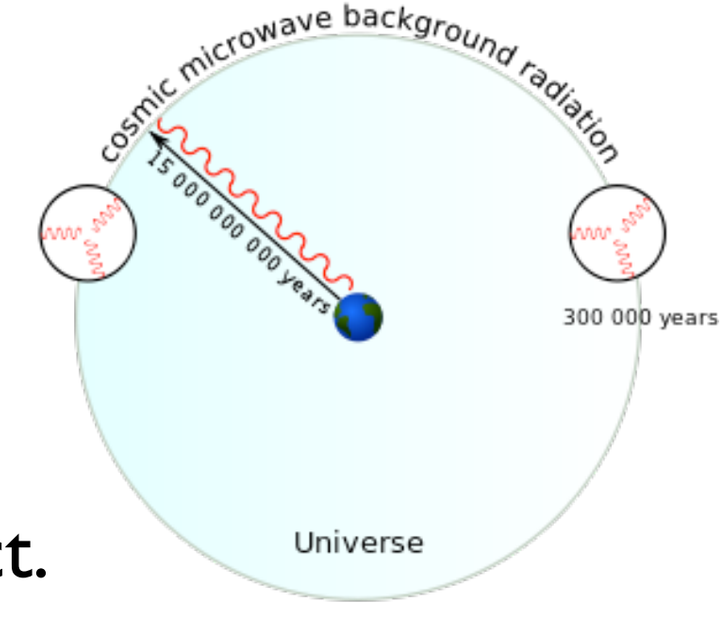
Outline

- Introduction
- Axion models and PQ inflation
- Reheating and axion relic density
- Conclusions

Introduction

Cosmic inflation

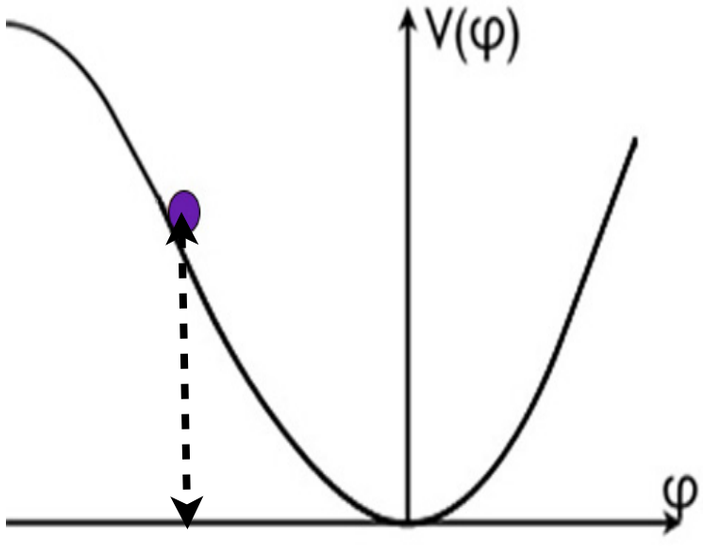
- Cosmic Inflation solves various problems in Standard Big Bang: horizon, homogeneity, flatness, relic problems, etc.



e.g. CMB would have been not in causal contact.

- "Slowly-rolling scalar (inflaton)" derives inflation and reheats the universe.

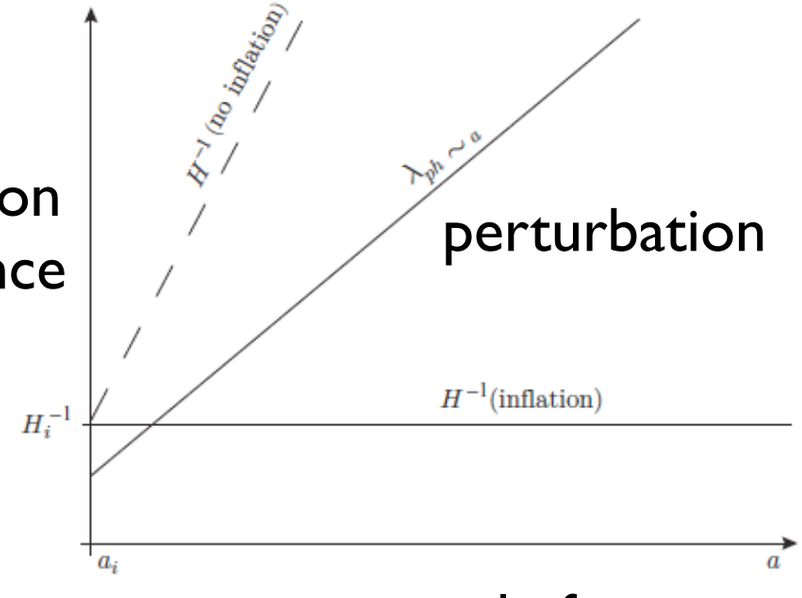
→ CMB anisotropies, galaxies, clusters.



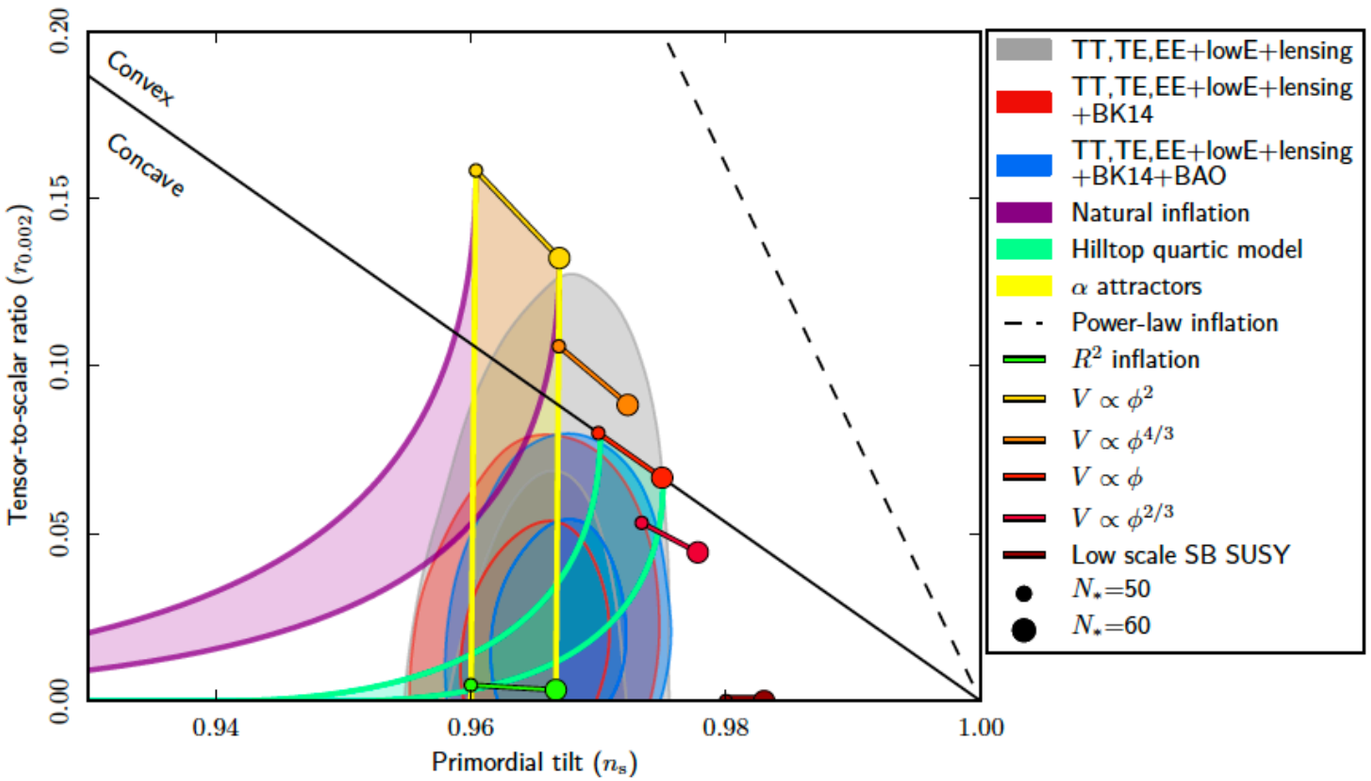
$$ds^2 = -dt^2 + a^2(t)d\vec{x}^2$$

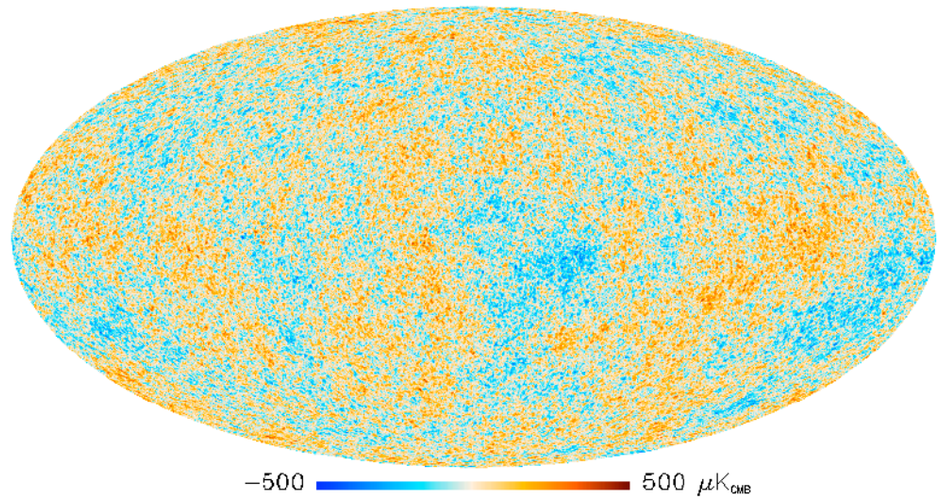
$$e^N = \frac{a_{\rm end}}{a_*},$$

$$N \approx 60.$$



Planck and inflation





n_s=0.9665±0.0038
[Planck+BAO]

- Almost scale-invariant CMB ➡ slow-roll inflations

$$\langle \mathcal{R}(\vec{k})\mathcal{R}(\vec{k}')\rangle = \delta^3(\vec{k}-\vec{k}')\,\frac{2\pi^2}{k^3}\mathcal{P}_{\mathcal{R}}(k), \qquad \mathcal{P}_{\mathcal{R}}(k) \propto k^{n_s-1},$$

$$n_s = 1 - 6\epsilon + 2\eta, \qquad \epsilon = \frac{M_P^2}{2}\left(\frac{V'}{V}\right)^2 \ll 1,\ \eta = M_P^2\frac{V''}{V} \ll 1.$$

- Tensor perturbation: $r = 16\epsilon$, r<0.035 [Planck18+BK18+BAO]

  small r accessible by Bicep3, Simon, CMB-S4, LiteBird.

- Almost Gaussian: $f_{NL}^{\rm local} = 2.7 \pm 5.8$ ➡ canonical single field

Starobinsky model

- Starobinsky-like models are a best fit to Planck data.

$n_s = 1 - \frac{2}{N},\ r = \frac{12}{N^2}$: successful for N=40-80 at 95% C.L.

- Starobinsky model based on "pure gravity" is dual to a scalar-tensor theory. [Starobinsky (1984)]

$$\mathcal{L} = \sqrt{-g}\left(\frac{R}{2} + \xi^2 R^2\right) \Longleftrightarrow \mathcal{L} = \sqrt{-g}\left(\frac{R}{2} + 2\xi\phi R - \phi^2\right)$$

$$\Rightarrow \quad \mathcal{L}_E = \sqrt{-g_E}\left(\frac{R}{2} - \frac{1}{2}(\partial_\mu \chi)^2 - \frac{1}{16\xi^2}\left(1 - e^{-\sqrt{\frac{2}{3}}|\chi|}\right)^2\right)$$

$g_{\mu\nu} \to g_{\mu\nu}/(1+4\xi\phi)$

$1 + 4\xi\phi = e^{\sqrt{\frac{2}{3}}|\chi|}$ in Einstein frame

$\xi \sim 10^4$ needed for CMB.

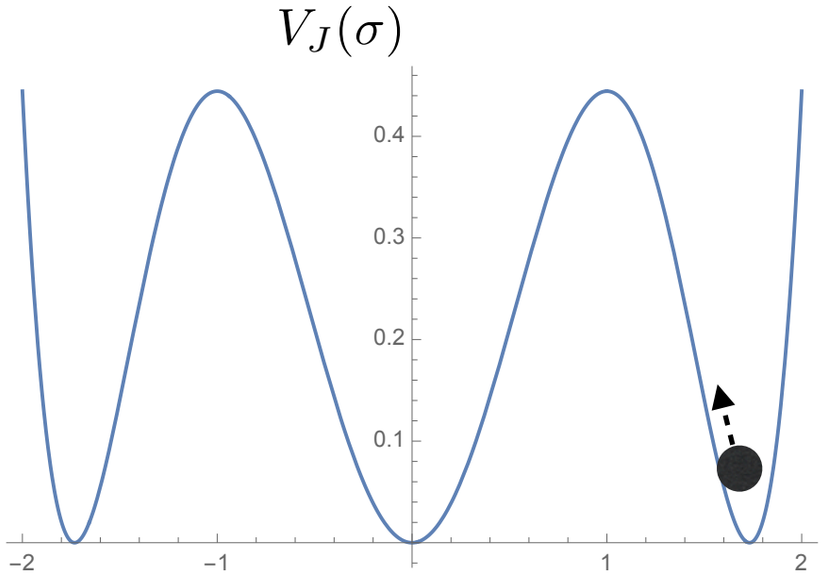
Pole inflation

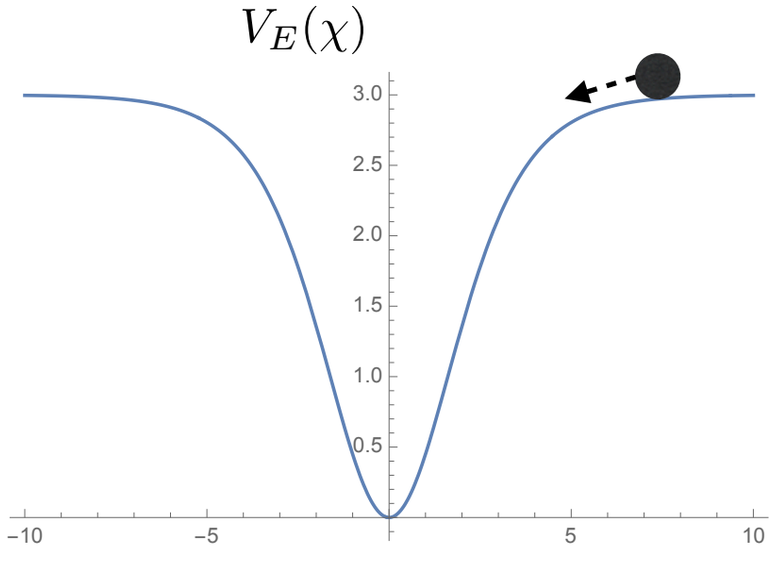
- Singlet scalar field σ with a conformal coupling:

$$\frac{\mathcal{L}_J}{\sqrt{-g_J}} = -\frac{1}{2}\left(1-\frac{1}{6}\sigma^2\right)R + \frac{1}{2}(\partial_\mu\sigma)^2 - V_J(\sigma)\ , \quad V_J = \frac{1}{2}M^2\sigma^2\left(1-\frac{1}{6}\sigma^2\right)^2$$

$$\Rightarrow \quad \frac{\mathcal{L}_E}{\sqrt{-g_E}} = -\frac{1}{2}R + \frac{1}{2}\frac{(\partial_\mu\sigma)^2}{\left(1-\frac{1}{6}\sigma^2\right)^2} - \frac{1}{2}M^2\sigma^2$$

$$= -\frac{1}{2}R + \frac{1}{2}(\partial_\mu\chi)^2 - 3M^2\tanh^2\left(\frac{\chi}{\sqrt{6}}\right)$$





Inflation occurs near the pole.

Complex scalar = inflaton

e.g. Kallosh, Linde, Roest (2013)

PQ symmetry & axion

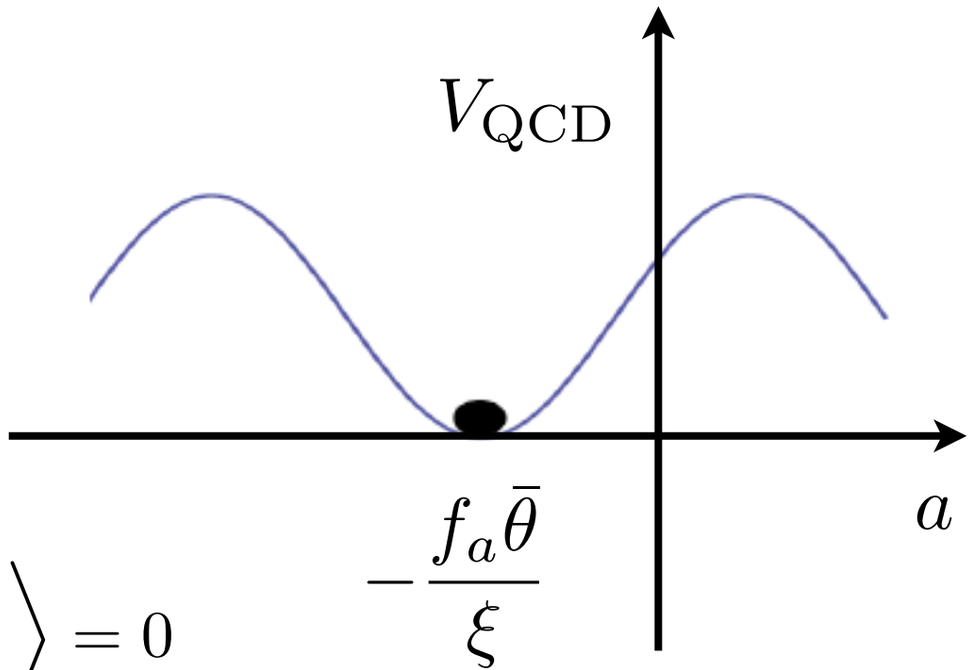
- Axion is a pseudo-Nambu-Goldstone coming from the spontaneous breakdown of U(1) Peccei-Quinn symmetry.

$$\Phi(x) = \frac{1}{\sqrt{2}} f_a \, e^{ia(x)/f_a}, \qquad a(x) \to a(x) + 2\pi f_a$$

- U(1) PQ symmetry becomes anomalous due to QCD anomalies (in KSVZ or DFSZ): axion-gluon coupling leads to the axion potential via QCD instanton.

$$\mathcal{L}_\theta = \frac{g_s^2}{32\pi^2}\left(\bar{\theta} + \xi\,\frac{a}{f_a}\right) G^a_{\mu\nu}\tilde{G}^{a\mu\nu}$$

$$\longrightarrow \quad V_{\rm QCD} = -\Lambda^4_{\rm QCD}\cos\left(\bar{\theta} + \xi\,\frac{a}{f_a}\right)$$



Dynamically relaxed to CP-conserving vacuum

$$\left\langle \bar{\theta} + \xi\,\frac{a}{f_a} \right\rangle = 0$$

Axion quality

- Bounds from neutron electric dipole moment:

$$d_n = \frac{e}{\Lambda_{\rm QCD}^2}\frac{m_u m_d}{m_u+m_d}\,\theta_{\rm eff} < 1.8\times 10^{-26} e\,{\rm cm} \longrightarrow \theta_{\rm eff} = \bar{\theta} + \xi\frac{\langle a\rangle}{f_a} < 10^{-10}$$

- Quantum gravity breaks continuous global symmetries explicitly, so its effect on effective θ must be small enough.

"Axion quality problem"

Explicit PQ breaking at Planck scale: $V_{\rm PQV} = \frac{c_n}{M_P^{n-4}}\,\Phi^n + {\rm h.c.}$

$$\Phi(x) = \frac{1}{\sqrt{2}}\,f_a\,e^{ia(x)/f_a} \longrightarrow V_{\rm PQV} = \frac{c_n}{2^{n/2-1}}\left(\frac{f_a}{M_P}\right)^n M_P^4\cos\left(n\,\frac{a}{f_a}+\alpha\right)$$

$$\theta_{\rm eff} < 10^{-10} \longrightarrow n \gtrsim 12 \text{ for } f_a = 10^{12}\,{\rm GeV}$$

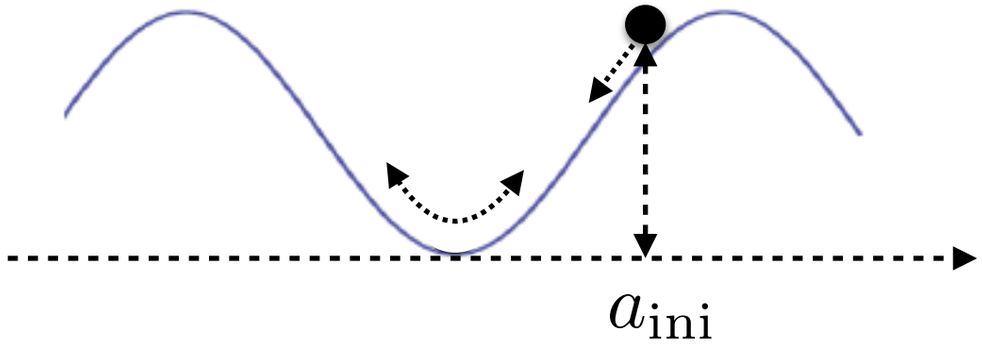
e.g. higher order discrete R symmetries

[HML et al (2011)]

Axion dark matter

- Axion abundance for dark matter is determined by the initial misalignment before QCD phase transition.

[Preskill et al; Abbott et al; Dine et al (1983)]



"Axion misalignment mechanism"

$m_a \simeq 3H$: coherent oscillation

⟶ Axion cold dark matter

Axion abundance:

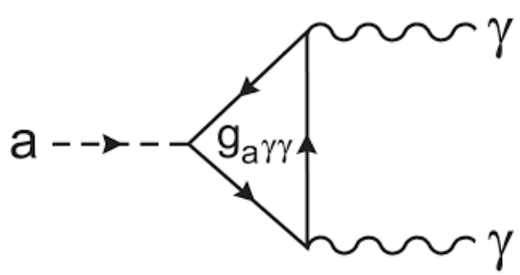
$$\Omega_a h^2 = \frac{\rho_a(a_{\rm ini})}{\rho_c/h^2}\,\frac{m_a(0)}{m_a(T_{\rm osc})}\left(\frac{g_{s*}(T_0)}{g_{s*}(T_{\rm osc})}\right)^{1/3}\left(\frac{T_0}{T_{\rm osc}}\right)^3$$

$$\simeq 0.12\left(\frac{f_a}{9\times 10^{11}\,{\rm GeV}}\right)^{1.165}\left(\frac{a_{\rm ini}}{f_a}\right)^2$$

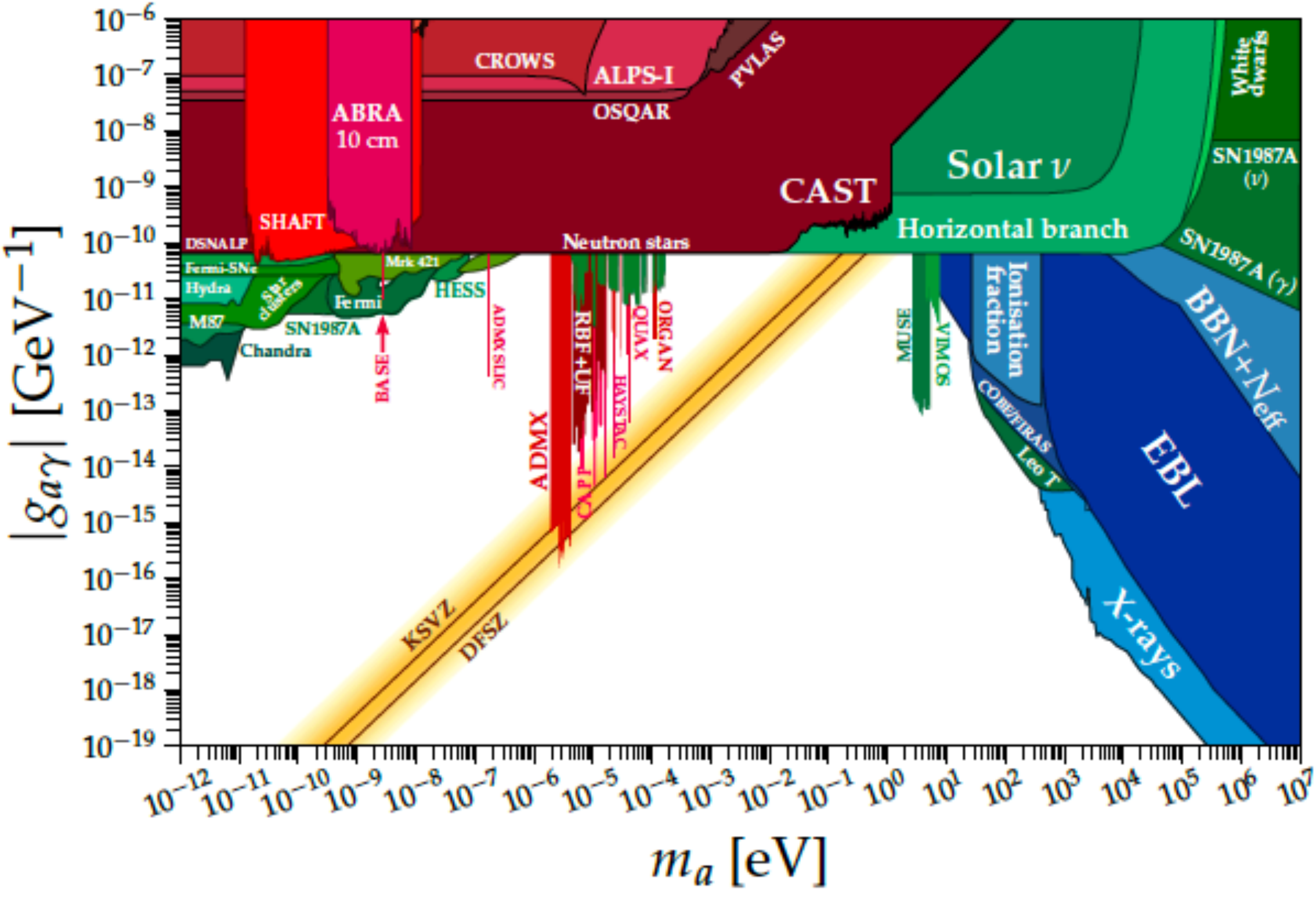
Axion window:

$$10^8\,{\rm GeV} < f_a < 10^{12}\,{\rm GeV}$$

Supernova — Relic density



Bounds on axions



Initial velocity for axion

- When the PQ radial mode is far away from the minimum, the PQ violating term generates a large potential for axion.

$$\Phi = \frac{1}{\sqrt{2}}\rho\, e^{i\theta} \ : \ V_{\rm PQV} = \frac{c_n}{2^{n/2-1}}\left(\frac{\rho}{M_P}\right)^n M_P^4 \cos(n\,\theta + \alpha), \quad \rho \gg f_a$$

⟶ Velocity kick for axion before QCD phase transition.

[Co, Hall, Harigaya (2019)] "Axion kinetic misalignment"

- PQ radial mode settles down to minimum before QCD PT.

$$\rho = f_a,\ \theta = \frac{a}{f_a} : \ \text{PQ charge,} \quad n_\theta = f_a^2\dot{\theta}, \quad a^3 n_\theta = \text{const}$$

Axion oscillation delayed: $\rho_{a,\rm rot} \simeq m_a n_\theta \sim \rho_{a,\rm pot}$

⟶ "smaller" axion decay constant: $f_a < 1.5 \times 10^{11}\,\text{GeV}$

PQ inflation => unification of axion and inflation!

Axion models and PQ inflation

Models for QCD axion

- Model content and PQ charges

Content/models	KSVZ	DFSZ
SM fermions	0	nonzero
Higgs	H : 0	H_1, H_2: $q_1 - q_2 = p\, q_\Phi$
PQ field	$\Phi : q_\Phi$	$\Phi : q_\Phi$
Extra quarks	Q_L , $Q_R : -q_\Phi/2$	not present
QCD anomaly	$\xi = q_\Phi$	$\xi = 3p\, q_\Phi$
Axion decay constant	$f_a = q_\Phi\, v_\Phi$	$f_a \simeq q_\Phi\, v_\Phi$

$H_1^\dagger H_2 \Phi^p$

DFSZ models

- DFSZ: Z_2 parities and PQ charges (in detail)

Z_2	Φ	H_1	H_2	q_L	u_R	d_R	l_L	e_R
Type I	$+$	$-$	$+$	$+$	$+$	$+$	$+$	$+$
Type II	$+$	$-$	$+$	$+$	$+$	$-$	$+$	$-$
Type X	$+$	$-$	$+$	$+$	$+$	$+$	$+$	$-$
Type Y	$+$	$-$	$+$	$+$	$+$	$-$	$+$	$+$

No FCNC

PQ	Φ	H_1	H_2	q_L	u_R	d_R	l_L	e_R
Type I	q_Φ	q_1	$q_1 - pq_\Phi$	0	$q_1 - pq_\Phi$	$-q_1 + pq_\Phi$	0	0
Type II	q_Φ	q_1	$q_1 - pq_\Phi$	0	$q_1 - pq_\Phi$	$-q_1$	0	$-q_1$
Type X	q_Φ	q_1	$q_1 - pq_\Phi$	0	$q_1 - pq_\Phi$	$-q_1 + pq_\Phi$	0	$-q_1$
Type Y	q_Φ	q_1	$q_1 - pq_\Phi$	0	$q_1 - pq_\Phi$	$-q_1$	0	$-q_1 + pq_\Phi$

e.g. Type II: $\mathcal{L}_Y = -Y_u \bar{q}_L \tilde{H}_2 u_R - Y_d \bar{q}_L H_1 d_R - Y_e \bar{l}_L H_1 e_R$

Only Type II, Y leads to QCD anomalies.

Axion-photon couplings

$$S_{\rm anom} = \frac{k_G}{8\pi^2}\int \theta\,{\rm tr}(G\wedge G) + \frac{k_F}{8\pi^2}\int \theta\, F\wedge F$$

$$\rightarrow\quad \mathcal{L}_{\rm photon} = -\frac{1}{4} g_{a\gamma\gamma}\, a\, F_{\mu\nu}\tilde{F}^{\mu\nu}, \qquad g_{a\gamma\gamma} = \frac{\alpha}{2\pi f_a/\xi}\left(\frac{E}{N} - 1.92\right).$$

		k_G	k_F	E/N	
DFSZ	Type I	0	$3(pq_\Phi - q_1)$	$-$	→ gluons X
	Type II	$3pq_\Phi$	$4pq_\Phi$	$\frac{8}{3}$	⇢ unification
	Type X	0	$3pq_\Phi$	$-$	→ gluons X
	Type Y	$3pq_\Phi$	pq_Φ	$\frac{2}{3}$	
KSVZ		q_Φ	0	0	

PQ pole inflation

- Consider a complex singlet scalar field with PQ charge:
  "radial mode" => inflaton, "angular mode" => axion

$$\Phi = \frac{1}{\sqrt{2}}\rho\, e^{i\theta}$$

[HML, A. Menkara, M-J, Seung, J-H, Song (2023)]

$$\frac{\mathcal{L}_J}{\sqrt{-g_J}} = -\frac{1}{2}M_P^2\,\Omega(\Phi)R(g_J) + |\partial_\mu\Phi|^2 - \Omega^2(\Phi)V_E(\Phi)$$

$$\Omega(\Phi) = 1 - \frac{1}{3M_P^2}|\Phi|^2,$$

Conformal coupling to gravity

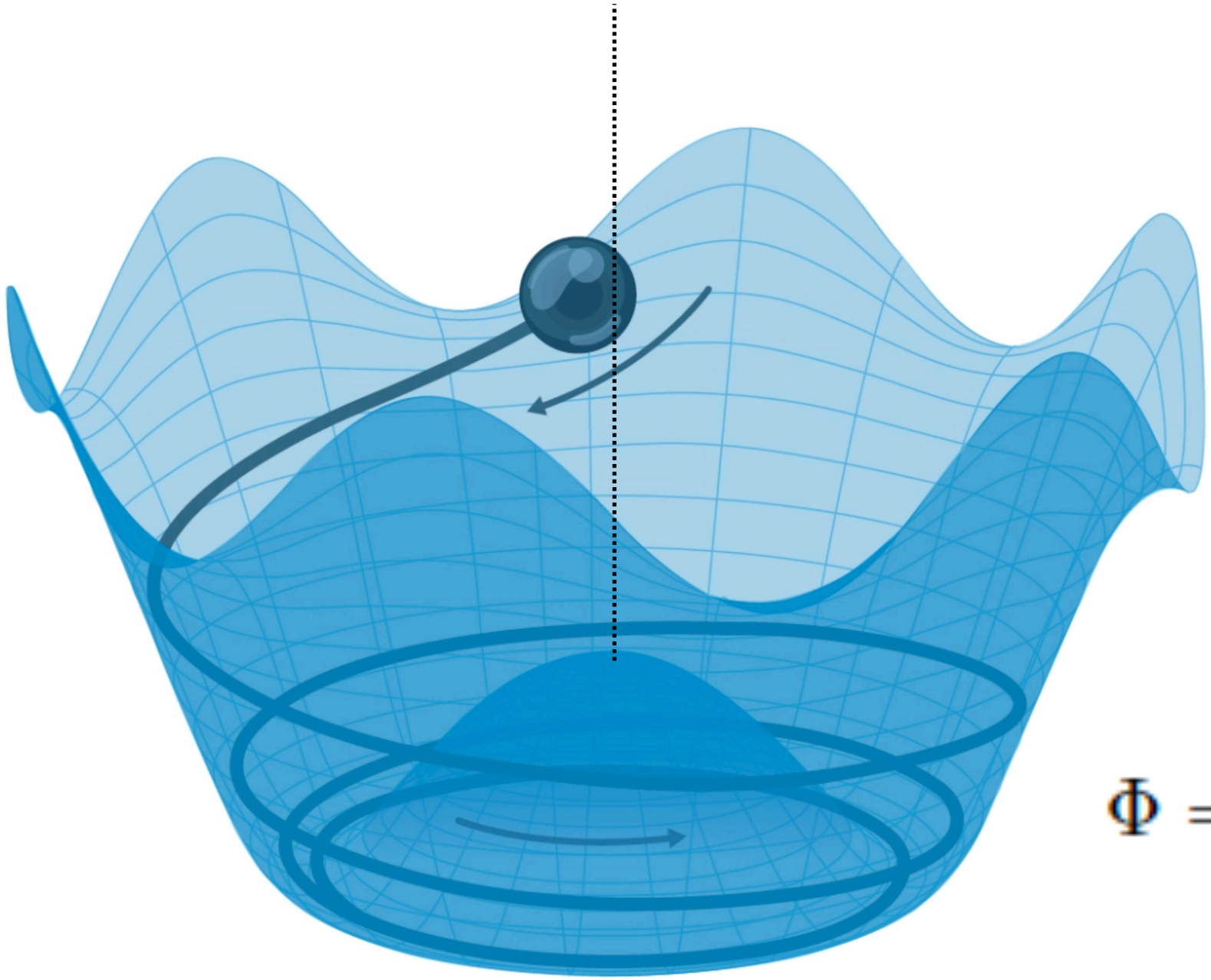
$$V_E(\Phi) = V_0' + \frac{\beta_m}{M_P^{2m-4}}|\Phi|^{2m} - m_\Phi^2|\Phi|^2 + \left(\sum_{k=0}^{[n/2]}\frac{c_k}{2M_P^{n-4}}|\Phi|^{2k}\Phi^{n-2k} + \text{h.c.}\right)$$

PQ conserving

PQ violating

PQ inflation, SSB of U(1) PQ	Axion kinetic misalignment

Motion of PQ field



$$\Phi = \frac{1}{\sqrt{2}} \rho \, e^{i\theta}$$

[R. Co et al, PRL (2020)]

PQ conserving potential

- Bounds on PQ conserving potential.

$$m = 2: \quad V_{\rm PQ} = V_0 + \lambda_\Phi \left(|\Phi|^2 - \frac{f_a^2}{2} \right)^2 , \qquad f_a = \sqrt{-m_\Phi^2/\lambda_\Phi}$$

$$\Phi = \frac{1}{\sqrt{2}} \rho\, e^{i\theta}, \qquad \rho = \sqrt{6} M_P \tanh\left(\frac{\phi}{\sqrt{6} M_P} \right),$$

$$\longrightarrow \quad \frac{\mathcal{L}_E}{\sqrt{-g_E}} = -\frac{1}{2} M_P^2 R + \frac{1}{2} (\partial_\mu \phi)^2 + 3 M_P^2 \sinh^2\left(\frac{\phi}{\sqrt{6} M_P} \right) (\partial_\mu \theta)^2 - V_E(\phi, \theta),$$

Inflaton potential: $V_E(\phi,\theta) = V_{\rm PQ}(\phi) + V_{\rm PQV}(\rho,\theta),$

$$V_{\rm PQ}(\phi) = V_0 + \frac{1}{4}\lambda_\Phi \left(6 M_P^2 \tanh^2\left(\frac{\phi}{\sqrt{6} M_P} \right) - f_a^2 \right)^2, \qquad \lambda_\Phi = 1.1 \times 10^{-11}$$

- General PQ conserving potential.

$$V_{\rm PQ} = \frac{\beta_m}{M_P^{2m-4}} |\Phi|^{2m} - m_\Phi^2 |\Phi|^2 , \qquad \langle \rho \rangle \sim \langle \phi \rangle \sim f_a = (m_\Phi^2 M_P^{2m-4})^{1/(2m-2)}$$

$$\longrightarrow \quad V_{\rm PQ}(\phi) = V_0' + 3^m \beta_m M_P^4 \left[\tanh\left(\frac{\phi}{\sqrt{6} M_P} \right) \right]^{2m} - 3 m_\Phi^2 M_P^2 \tanh^2\left(\frac{\phi}{\sqrt{6} M_P} \right), \qquad 3^m \beta_m = 1.0 \times 10^{-10}$$

PQ violating potential

- Bounds on PQ violating potential.

$$V_{\rm PQV}(\rho,\theta) \;=\; 3^{n/2} M_P^4 \tanh^n\left(\frac{\phi}{\sqrt{6}M_P}\right) \sum_{k=0}^{[n/2]} |c_k| \cos\Big((n-2k)\theta + A_k\Big)$$

Axion quality: $$|\theta_{\rm eff}| = \left|\bar\theta + \xi\frac{\langle a\rangle}{f_a}\right| < 10^{-10}$$

$$V_{\rm eff}(a) = V_0 - \Lambda_{\rm QCD}^4 \cos\left(\bar\theta + \xi\frac{a}{f_a}\right) + V_{\rm PQV} \longrightarrow \left(\frac{f_a}{M_P}\right)^n \lesssim \frac{2^{n/2}\xi}{(n-2k)|c_k|}\left(\frac{\Lambda_{\rm QCD}}{M_P}\right)^4$$

$f_a = 10^{12}\,\text{GeV},\ \xi = 1,\ \ |c_k| = \mathcal{O}(1),$ $n \gtrsim 12$ In general, $n \propto 1/\ln(M_P/f_a)$

PQ inflation leads to stringent bounds on PQ violation:

$$V_n(\theta_i)/M_P^4 = 3^{n/2}\sum_{k=0}^{[n/2]} |c_k| \cos\Big((n-2k)\theta_i + A_k\Big) < 1.0\times 10^{-10}, \qquad 3^{n/2}|c_k| \lesssim 10^{-10}$$ each

$\longrightarrow$ $n \gtrsim 10(5)$ for $f_a = 10^{12}(10^6)\,\text{GeV}, \xi = 1$ and $|c_k| \lesssim 10^{-12}$

Axion velocity from inflation

- Nonzero axion velocity is set at the end of inflation.

PQ inflaton: $$\ddot{\phi} + 3H\dot{\phi} - \sqrt{6}M_P \sinh\left(\frac{\phi}{\sqrt{6}M_P}\right)\cosh\left(\frac{\phi}{\sqrt{6}M_P}\right)\dot{\theta}^2 = -\frac{\partial V_E}{\partial \phi}$$

$$\phi \sim \sqrt{6}M_P, \qquad \dot{\phi} \simeq -\frac{1}{3H}\frac{\partial V_E}{\partial \phi} = -\sqrt{2\epsilon_\phi}\, M_P H$$ Slow-roll inflation

Axion: $$6M_P^2 \sinh^2\left(\frac{\phi}{\sqrt{6}M_P}\right)\left[\ddot{\theta} + 3H\dot{\theta} + \frac{2}{\sqrt{6}M_P}\coth\left(\frac{\phi}{\sqrt{6}M_P}\right)\dot{\phi}\,\dot{\theta}\right] = -\frac{\partial V_E}{\partial \theta}$$

$$\dot{\theta} \simeq -\frac{1}{3H}\frac{\frac{\partial V_E}{\partial \theta}}{6M_P^2 \sinh^2\left(\frac{\phi}{\sqrt{6}M_P}\right)} = -\frac{\sqrt{2\epsilon_\theta}\, H}{6\sinh^2\left(\frac{\phi}{\sqrt{6}M_P}\right)} \ll H$$

slow-rolling or sub-dominant dynamics for inflation

Bound on axion velocity during inflation:

$$3^{n/2}|c_k| \lesssim 10^{-10} \longrightarrow |\dot{\theta}_{\rm end}| \lesssim \frac{M_P \tanh^n\left(\frac{\phi_{\rm end}}{\sqrt{6}M_P}\right)}{18 H_I \sinh^2\left(\frac{\phi_{\rm end}}{\sqrt{6}M_P}\right)} \cdot 10^{-10} \simeq 0.9^n \times 6 \times 10^{-7} M_P$$

Reheating and axion relic density

Post-inflation dynamics

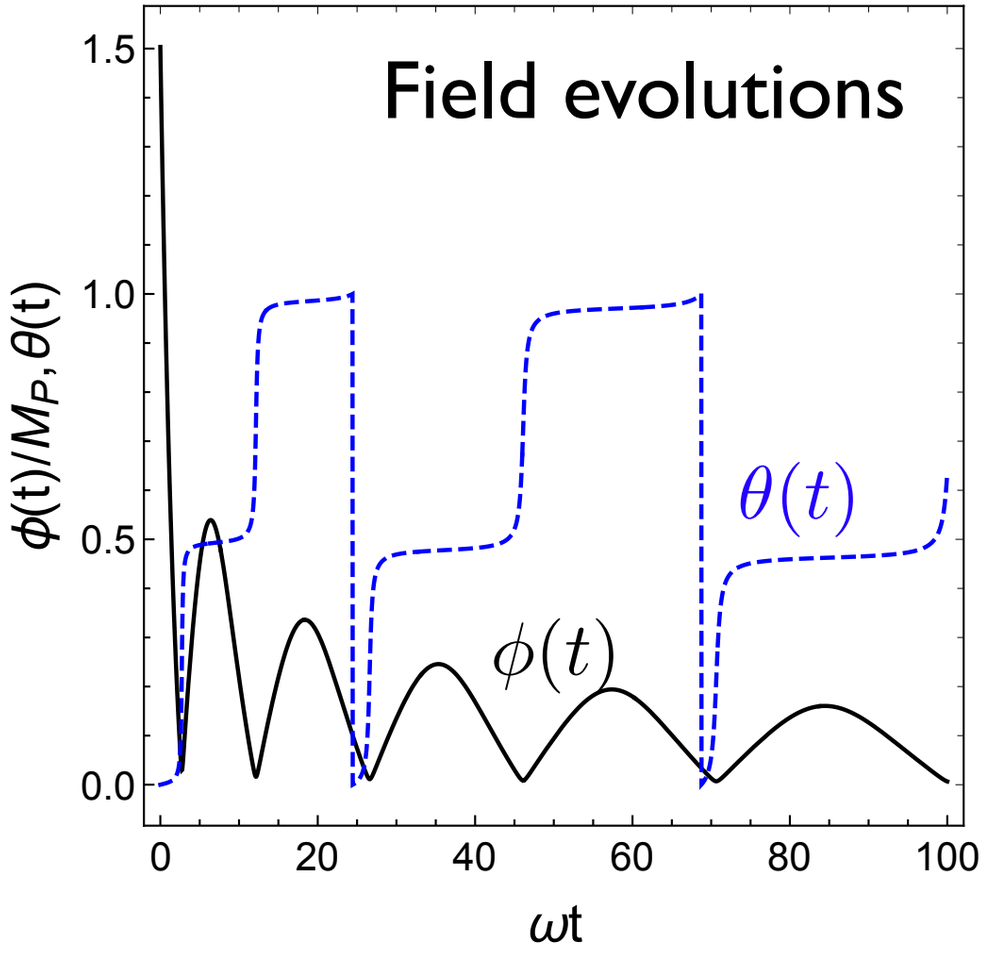
PQ radial mode: damps out due to Hubble friction.

$$\phi \ll \sqrt{6} M_P\,, \quad \ddot{\phi} + 3H\dot{\phi} - \phi\,\dot{\theta}^2 \;\simeq\; -\frac{\partial V_E}{\partial \phi}, \qquad \text{elliptic => circular motion}$$

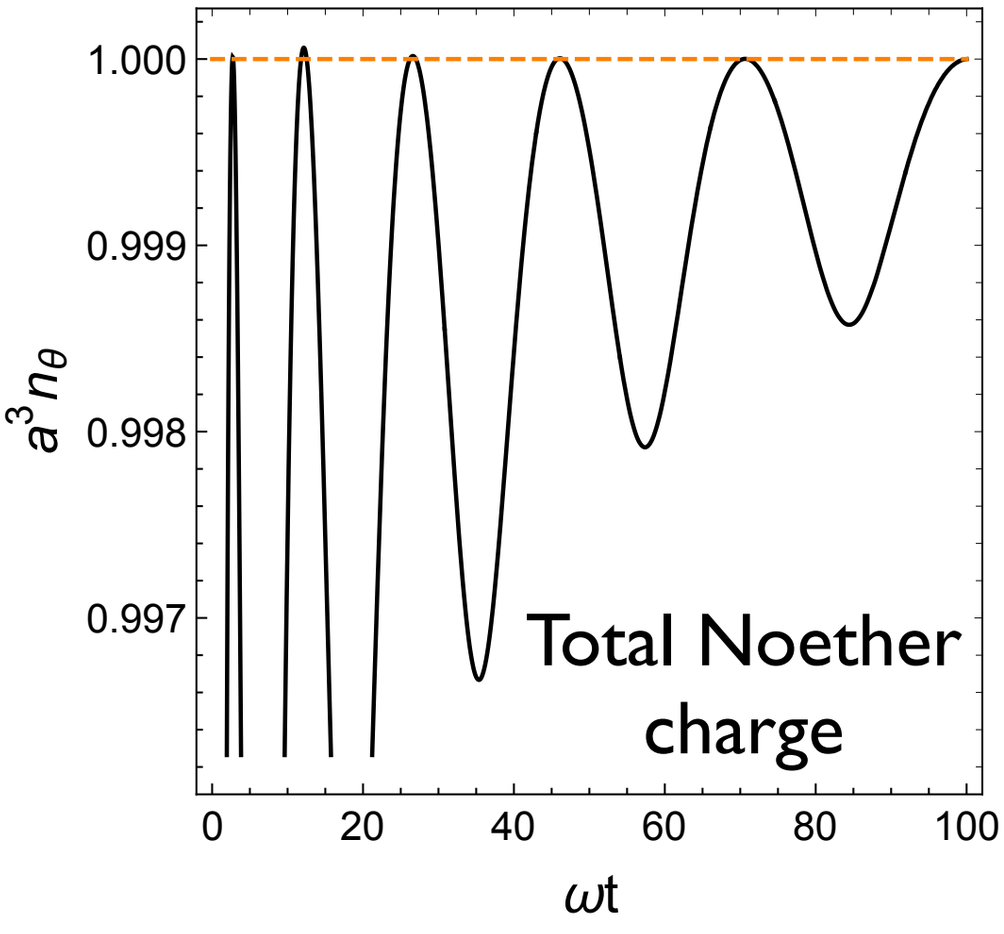
Axion: PQ Noether charge conserved approximately.

$$\phi^2(\ddot{\theta} + 3H\dot{\theta}) + 2\phi\dot{\phi}\,\dot{\theta} \;\simeq\; -\frac{\partial V_E}{\partial \theta} \simeq 0 \longrightarrow\quad n_\theta = \phi^2\dot{\theta}\,, \quad \frac{d}{dt}(a^3 n_\theta) = 0$$

f_a=10^{11} GeV, n=10, c_0=10^{-13}, A_0=0.5



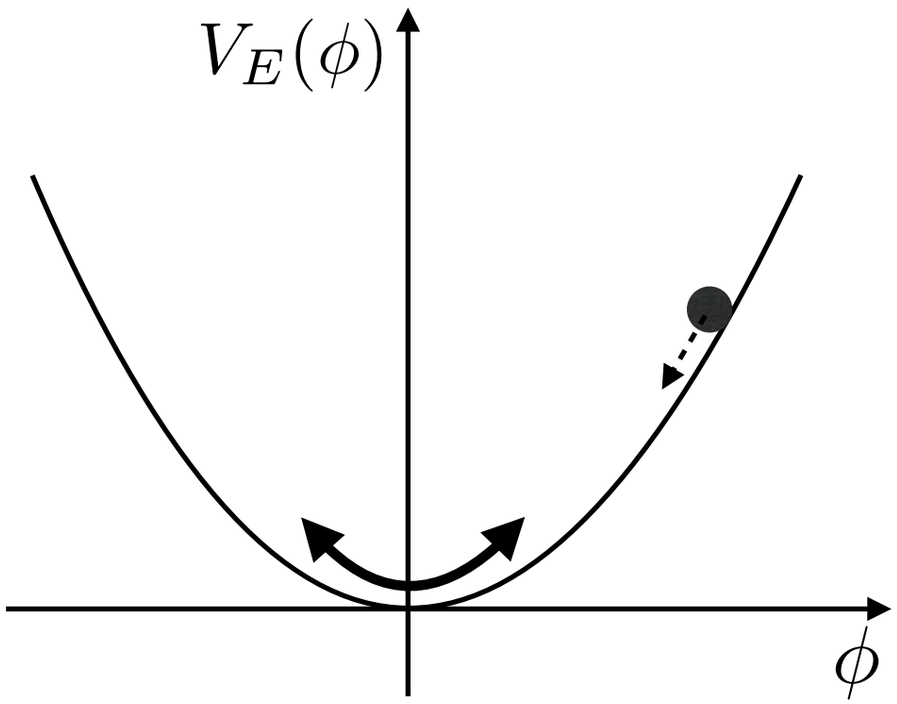
f_a=10^{11} GeV, n=10, c_0=10^{-13}, A_0=0.5



Inflaton condensate

- PQ field oscillates around zero after inflation.

$$V_E(\phi) \simeq \frac{c_m}{2^m}\Lambda^{4-2m}\phi^{2m} \equiv \alpha_m \phi^{2m}$$



Virial theorem: $\left\langle \frac{dS}{dt} \right\rangle = 0, \quad S \equiv \phi\dot{\phi}$

$$\longrightarrow \left\langle \frac{1}{2}\dot{\phi}^2 \right\rangle = -\left\langle \frac{1}{2}\phi\ddot{\phi} \right\rangle = m\langle V_E(\phi)\rangle$$

(Hubble friction ignored)

$$\rho_\phi = \left\langle \frac{1}{2}\dot{\phi}^2 \right\rangle + \langle V_E(\phi)\rangle = (m+1)\langle V_E(\phi)\rangle,$$
$$p_\phi = \left\langle \frac{1}{2}\dot{\phi}^2 \right\rangle - \langle V_E(\phi)\rangle = (m-1)\langle V_E(\phi)\rangle.$$

$$\longrightarrow \langle w_\phi \rangle = \frac{p_\phi}{\rho_\phi} = \frac{m-1}{m+1}$$

"general equation of state(e.o.s)"

With Hubble friction, Inflaton = "damped" harmonic oscillator.

$$\boxed{\phi = \phi_0(t)\mathcal{P}(t)}$$

$\phi_0(t)$: inflaton amplitude,

$$\mathcal{P}(t) = \sum_{n=-\infty}^{\infty} \mathcal{P}_n\, e^{-in\omega t}$$: oscillating part, $\omega = m_\phi \sqrt{\frac{\pi m}{2m-1}} \frac{\Gamma\left(\frac{1}{2}+\frac{1}{2m}\right)}{\Gamma\left(\frac{1}{2m}\right)}, \quad m_\phi^2 = V_E''(\phi_0)$

Couplings of PQ field

- KSVZ model:

$$\mathcal{L}_{\rm int} = -y_Q \Phi \bar{Q}_R Q_L + \text{h.c.} - \lambda_{H\Phi}|\Phi|^2|H|^2 \ -\frac{1}{4}\lambda_\Phi|\Phi|^4$$

- DFSZ model:

$$\mathcal{L}_{\rm int} = -\lambda_{1\Phi}|H_1|^2|\Phi|^2 - \lambda_{2\Phi}|H_2|^2|\Phi|^2 - (2^{p/2}\kappa_p H_1^\dagger H_2 \Phi^p + \text{h.c.}) - \frac{1}{4}\lambda_\Phi|\Phi|^4$$

- Right-handed neutrinos for seesaw mechanism.

$$\mathcal{L}_N = -\frac{1}{2}\lambda_N \Phi \overline{N_L^c} N_R - Y_\nu \bar{l}\tilde{H}_{1,2} N_R + \text{h.c.}$$

KSVZ: $H_{1,2} \longrightarrow H,$

DFSZ: $Z_2-\text{odd}\ N_R \to H_1, \quad Z_2-\text{even}\ N_R \to H_2$

Perturbative reheating

- Reheating due to decays of PQ inflaton.

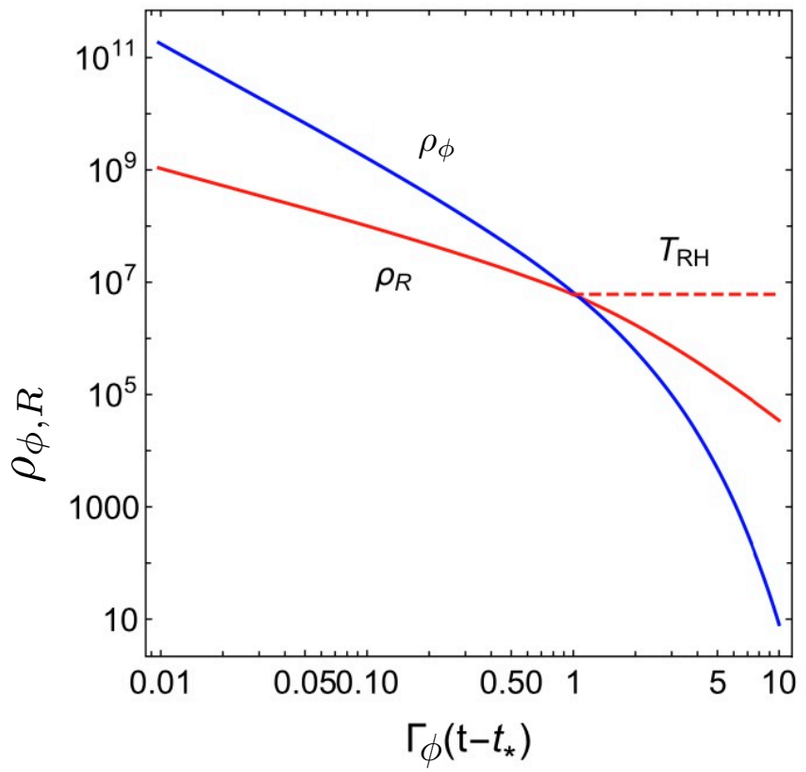
$$\ddot{\phi} + (3H + \Gamma_\phi)\dot{\phi} + V'_E = 0 \longrightarrow \dot{\rho}_\phi + 3(1+w_\phi)H\rho_\phi = -\Gamma_\phi(1+w_\phi)\rho_\phi,$$

$$\dot{\rho}_R + 4H\rho_R = \Gamma_\phi(1+w_\phi)\rho_\phi$$

KSVZ: $\phi \to \bar{Q}Q,\ \phi\phi \to H^\dagger H, \bar{Q}Q$

DFSZ: $\phi \to H_1^\dagger H_2, H_2^\dagger H_1,\ \phi\phi \to H_i^\dagger H_j$

$+\ \phi \to NN, \phi\phi \to NN$



- Reheating temperature:

Small Higgs-portal couplings are efficient!

$$T_{\rm RH}|_{\rm decay} \simeq 5.2\times10^5\,{\rm GeV}\left(\frac{100}{g_*(T_{\rm reh})}\right)^{1/4}\left(\frac{\max[y_f\,,\kappa_1/\omega]}{10^{-4}}\right)^2\left(\frac{\lambda_\Phi}{10^{-11}}\right)^{1/4},$$

$$T_{\rm RH}|_{\rm scattering} \simeq 3.0\times10^{11}\,{\rm GeV}\left(\frac{100}{g_*(T_{\rm reh})}\right)^{1/4}\left(\frac{\max[\lambda_{H_i\Phi},\kappa_2,\sqrt{2}y_f^2\,m_N/\omega]}{10^{-7}}\right)^2\left(\frac{10^{-11}}{\lambda_\Phi}\right)^{3/4}$$

Axion dark radiation

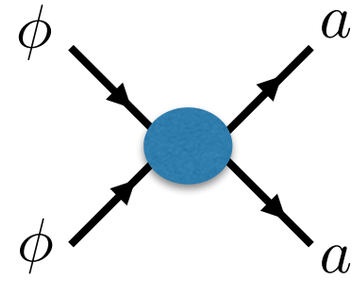
- Hot axions produced from inflaton scattering.

$$\frac{\Gamma_{\phi\phi\to aa}}{\Gamma_{\phi\phi\to HH}} \simeq \frac{\lambda_\Phi^2}{2\lambda_{H\Phi}^2} \lesssim 1 \quad \text{for sub-dominant dark radiation}$$

$$\longrightarrow \quad \Delta N_{\rm eff} = 0.02678 \left(\frac{Y_a}{Y_a^{\rm eq}}\right)\left(\frac{106.75}{g_{*s}(T_{\rm reh})}\right)^{4/3}, \qquad \frac{Y_a}{Y_a^{\rm eq}} = \frac{3\lambda_\Phi^{3/2} g_{*s}(T_{\rm reh}) T_{\rm reh}}{2.2\lambda_{H\Phi}^2 \phi_0}$$

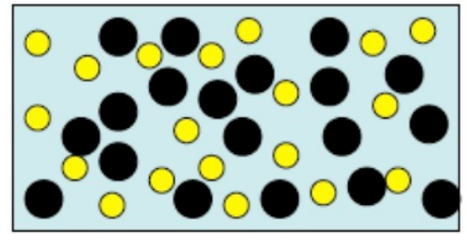
Low reheating temperature: suppressed axion DR!

$$T_{\rm reh} < T_{a,{\rm eq}}\,, \qquad \frac{Y_a}{Y_a^{\rm eq}} = 0.025\left(\frac{T_{\rm reh}}{T_{a,{\rm eq}}}\right)\left(\frac{10^{-11}}{\lambda_{H\Phi}}\right)^2\left(\frac{f_a}{10^{11}\,{\rm GeV}}\right)^{2.246}$$



- High reheating temperature: thermalized axions!

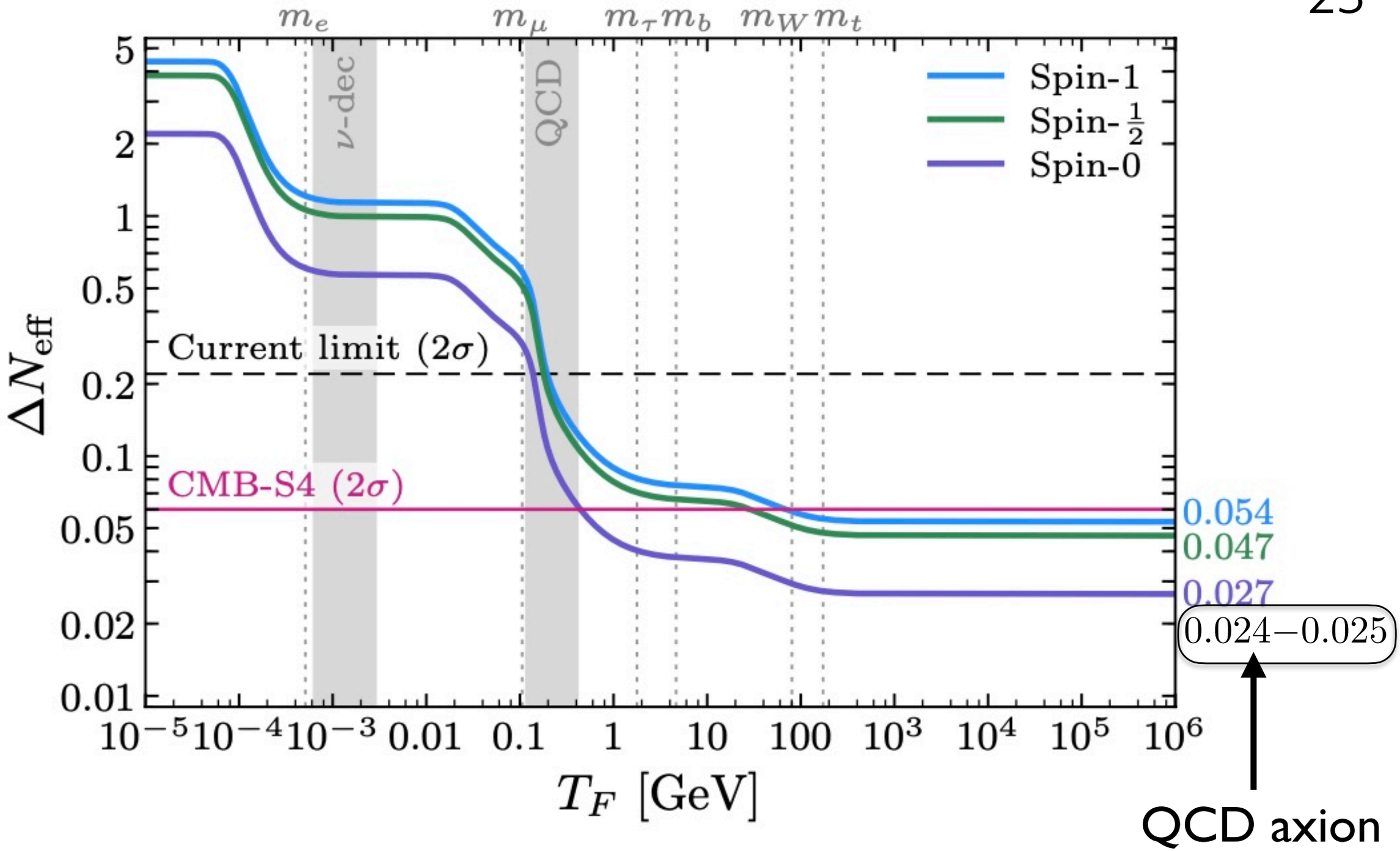
$$T_{\rm reh} \gtrsim 1.7\times 10^9\,{\rm GeV}\left(\frac{f_a}{10^{11}\,{\rm GeV}}\right)^{2.246} \equiv T_{a,{\rm eq}}$$



After axion decoupling: $\Delta N_{\rm eff} = \frac{4}{7}\left(\frac{T_{a,0}}{T_{\nu,0}}\right)^4 = \frac{4}{7}\left(\frac{11}{4}\right)^{4/3}\left(\frac{g_{*s}(T_0)}{g_{*s}(T_{\rm reh})}\right)^{4/3}$

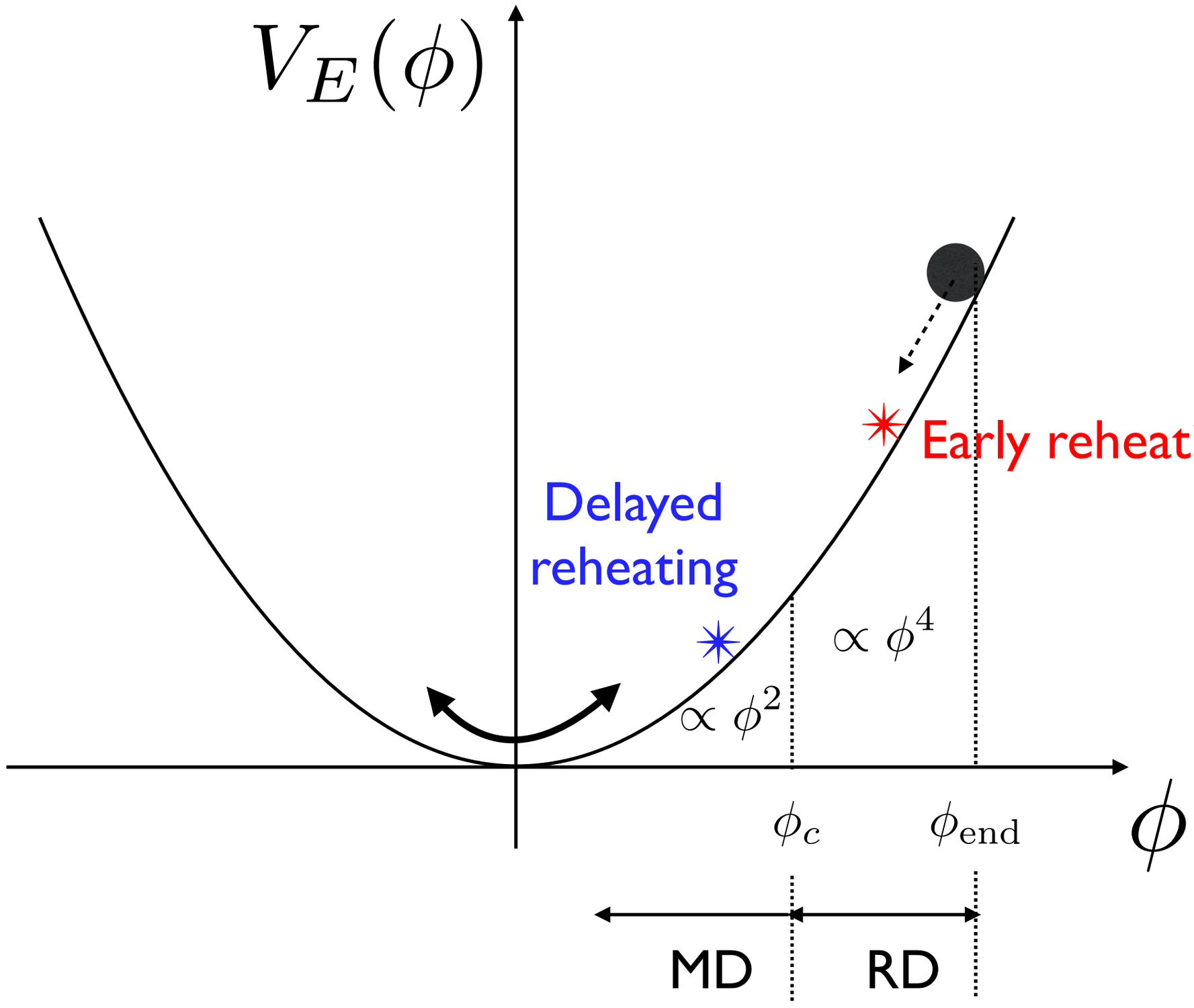
$$\longrightarrow \Delta N_{\rm eff} = 0.02363 \text{ (KSVZ)}\,,\ 0.02550 \text{ (DFSZ)}$$

CMB and ΔN_{eff}



[Planck 2018; CMB-S4 (2019)]

Reheating complete?



PQ charge and axion density

PQ inflaton energy during reheating: $T_{\rm RH}^c = \left(\frac{100}{g_*}\right)^{1/4}\left(\frac{f_a}{10^{11}\,{\rm GeV}}\right)(1.2\times 10^8\,{\rm GeV})$

$$\longrightarrow \rho_\phi = \begin{cases} \rho_{\rm end}\left(\frac{a_{\rm end}}{a_{\rm RH}}\right)^4, & T_{\rm RH} > T_{\rm RH}^c, \\ \rho_{\rm end}\left(\frac{a_{\rm end}}{a_c}\right)^4\left(\frac{a_c}{a_{\rm RH}}\right)^3, & T_{\rm RH} < T_{\rm RH}^c \end{cases}$$

early reheating during RD

delayed reheating until MD

PQ charge density at reheating: $a^3 n_\theta = a^3\phi^2\dot\theta \simeq {\rm const}$

$$n_\theta(T_{\rm RH}) = \begin{cases} n_{\theta,{\rm end}}\left(\frac{a_{\rm end}}{a_{\rm RH}}\right)^3 = n_{\theta,{\rm end}}\left(\frac{\pi^2 g_*(T_{\rm RH})T_{\rm RH}^4}{45V_E(\phi_{\rm end})}\right)^{3/4}, & T_{\rm RH} > T_{\rm RH}^c, \\ n_{\theta,{\rm end}}\left(\frac{a_{\rm end}}{a_c}\right)^3\underbrace{\left(\frac{a_c}{a_{\rm RH}}\right)^3} = n_{\theta,{\rm end}}\left(\frac{\pi^2 g_*(T_{\rm RH})T_{\rm RH}^4}{45V_E(\phi_{\rm end})}\right)^{3/4}\left(\frac{T_{\rm RH}}{T_{\rm RH}^c}\right), & T_{\rm RH} < T_{\rm RH}^c \end{cases}$$

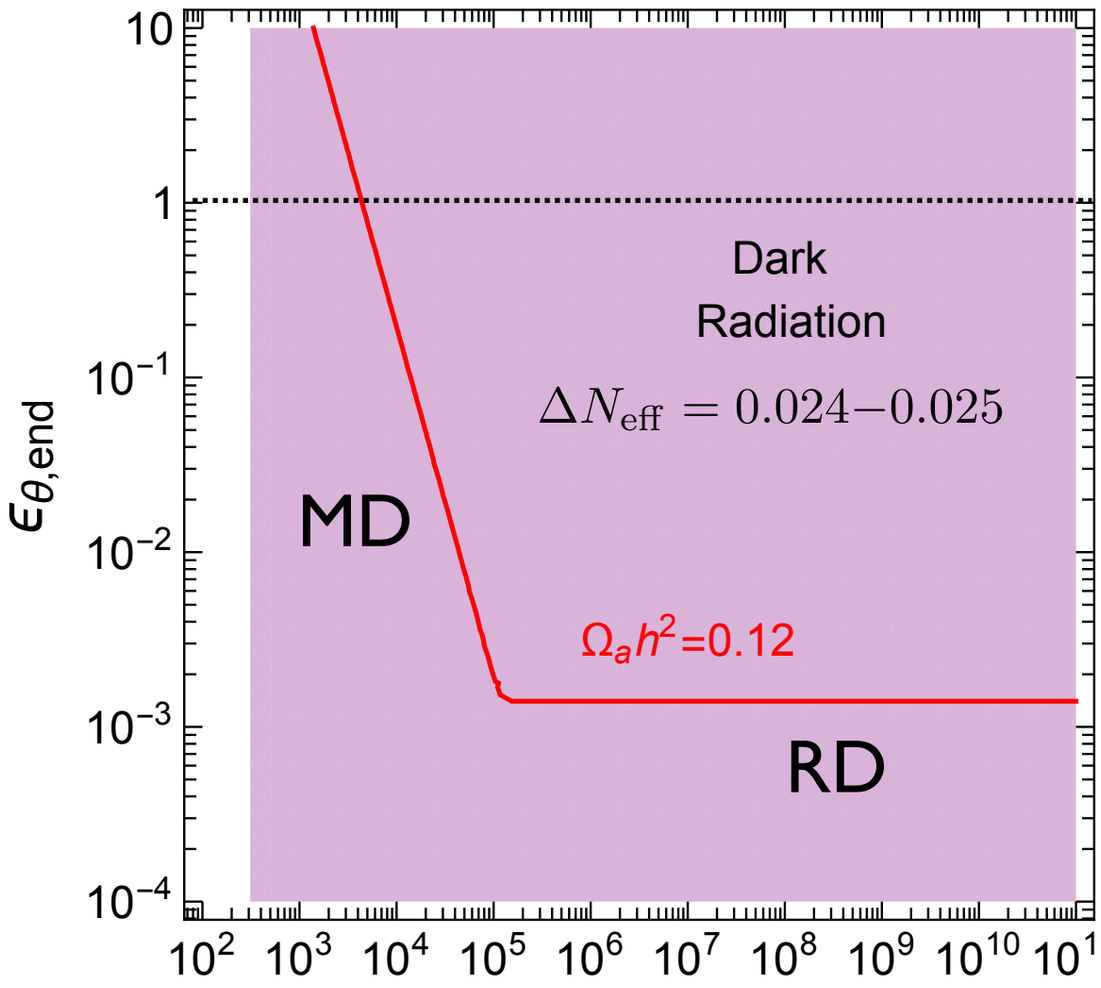
faster expansion: more diluted PQ charge

PQ charge to axion relic density:

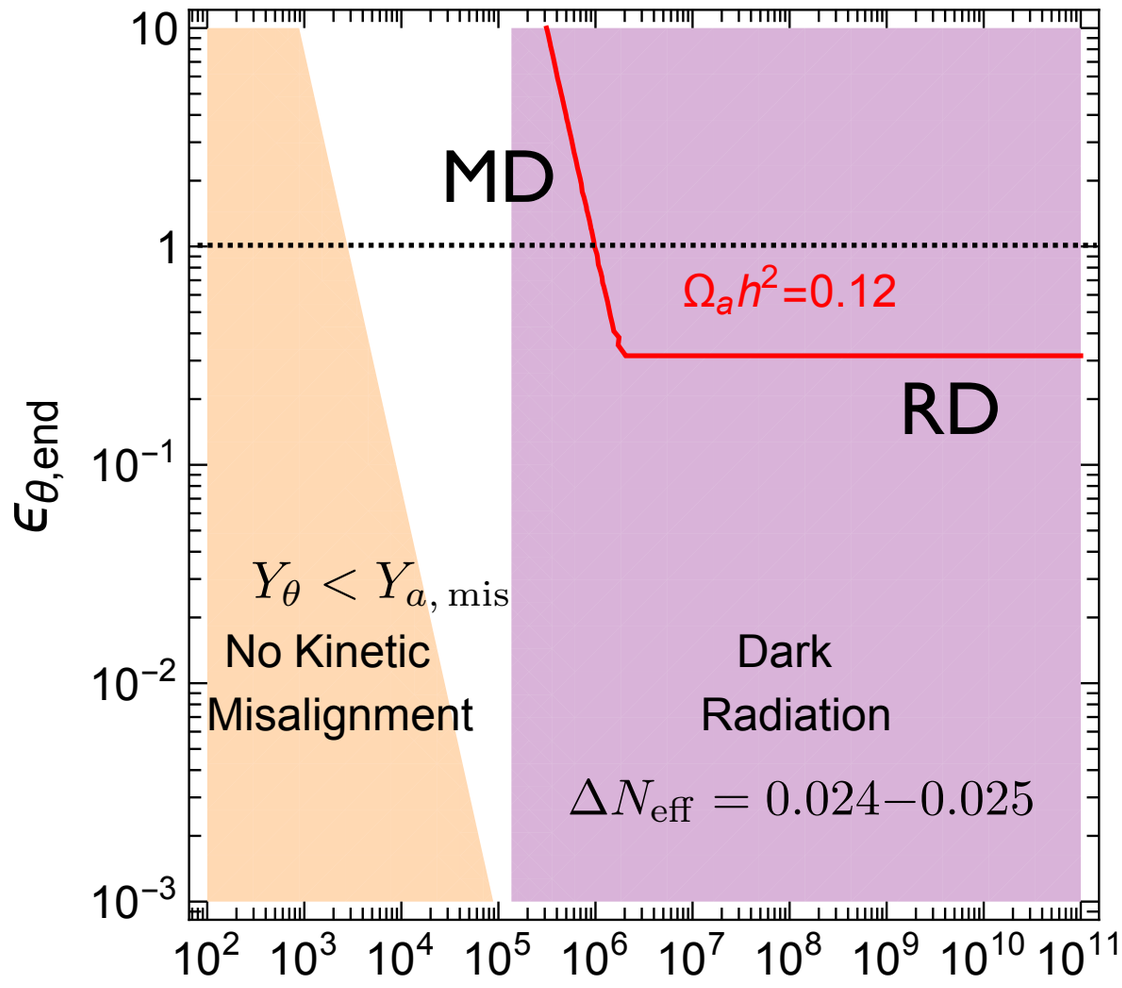
$$\frac{\rho_a}{s} = C m_a(0) Y_\theta \text{ with } C\simeq 2\,,\quad Y_\theta = \frac{n_\theta(T_{\rm RH})}{s(T_{\rm RH})} \longrightarrow \boxed{\Omega_a h^2 = 0.12\left(\frac{10^9\,{\rm GeV}}{f_a}\right)\left(\frac{Y_\theta}{40}\right)}$$

Axion velocity vs reheating

f_a=10^8GeV



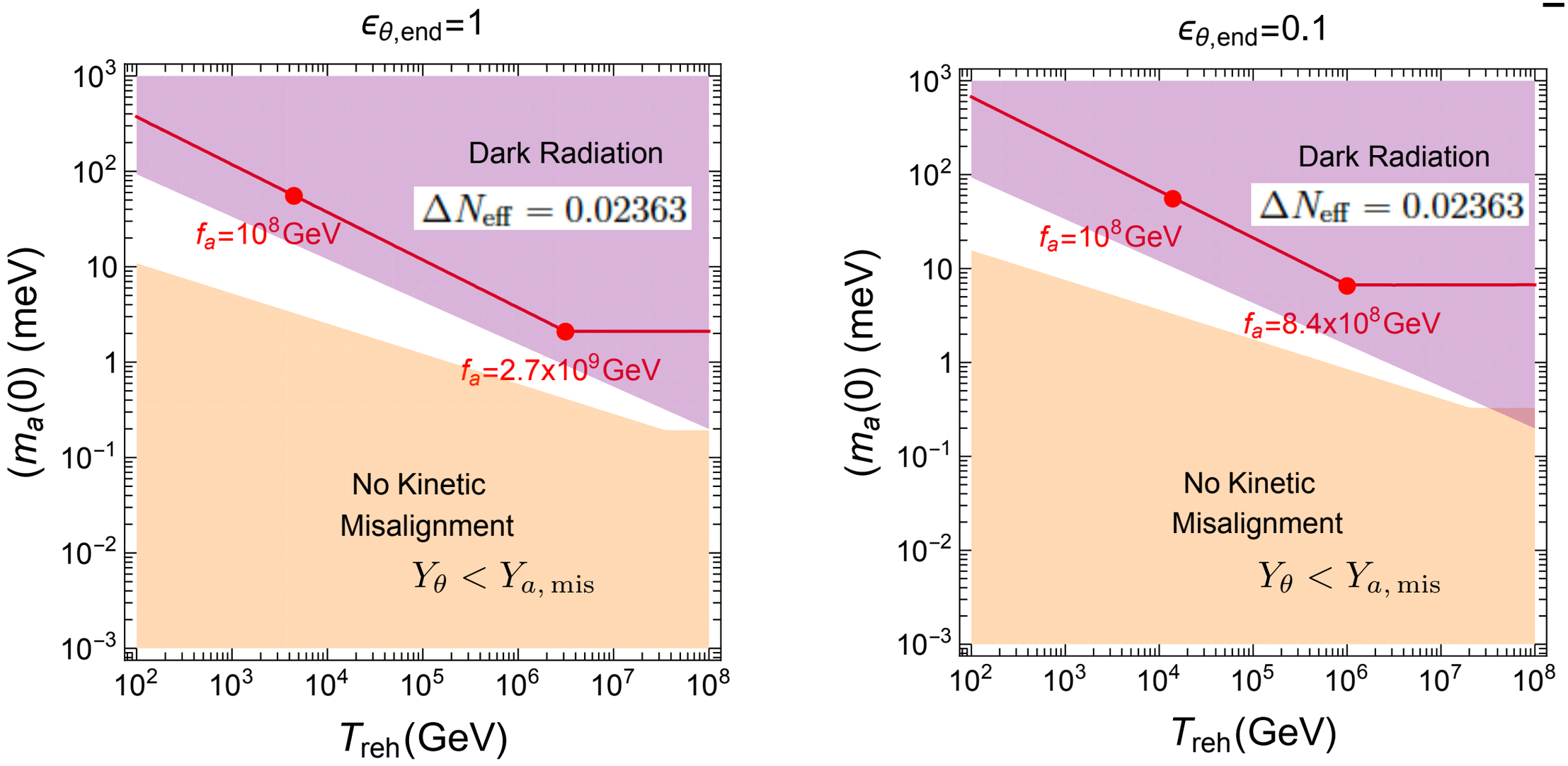
f_a=1.5x10^9GeV



$$\epsilon_\theta \equiv \frac{M_P^2}{2V_E^2}\left(\frac{\partial V_E}{\partial\theta}\right)^2 \longleftrightarrow \dot{\theta} = -\frac{\sqrt{2\epsilon_\theta}\,H}{6\sinh^2\left(\frac{\phi}{\sqrt{6}M_P}\right)}$$

axion slow-roll parameter during inflation

Axion mass vs reheating

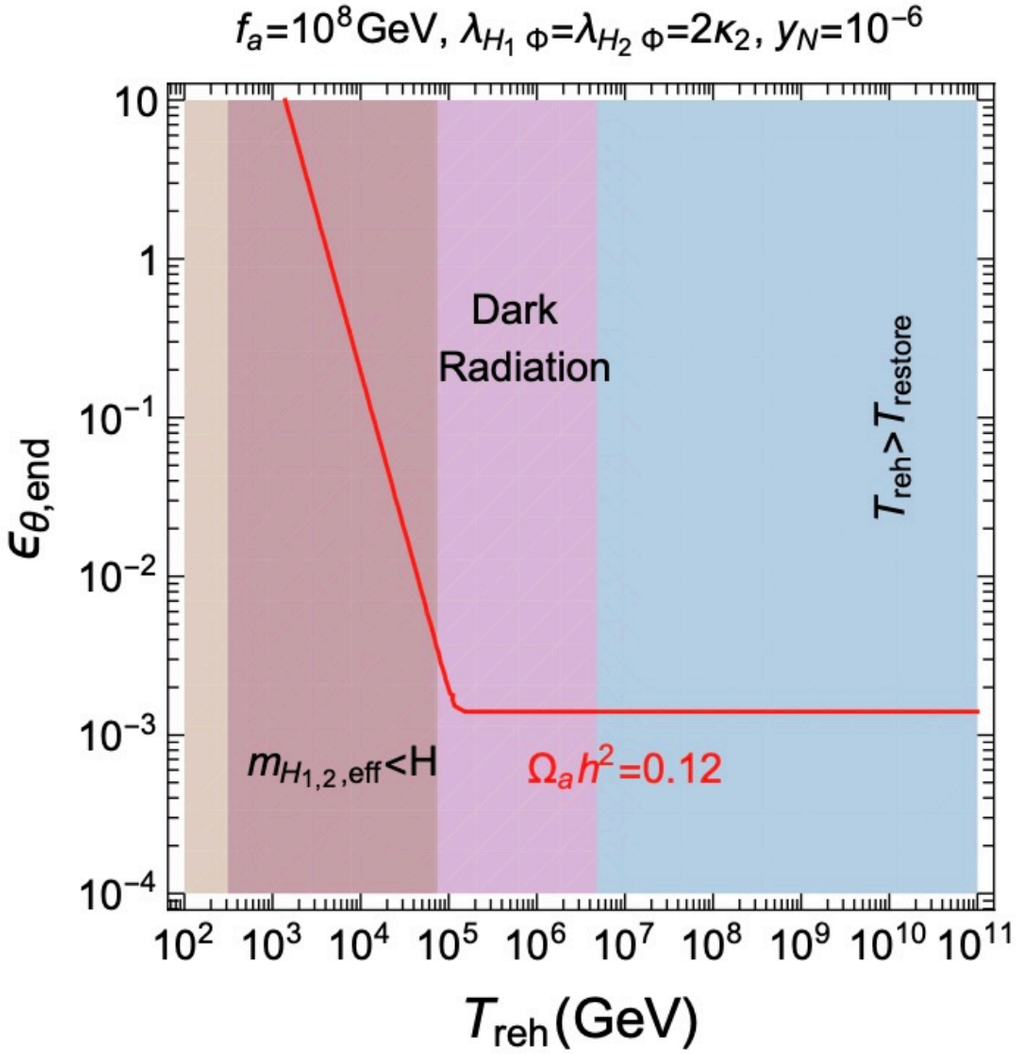


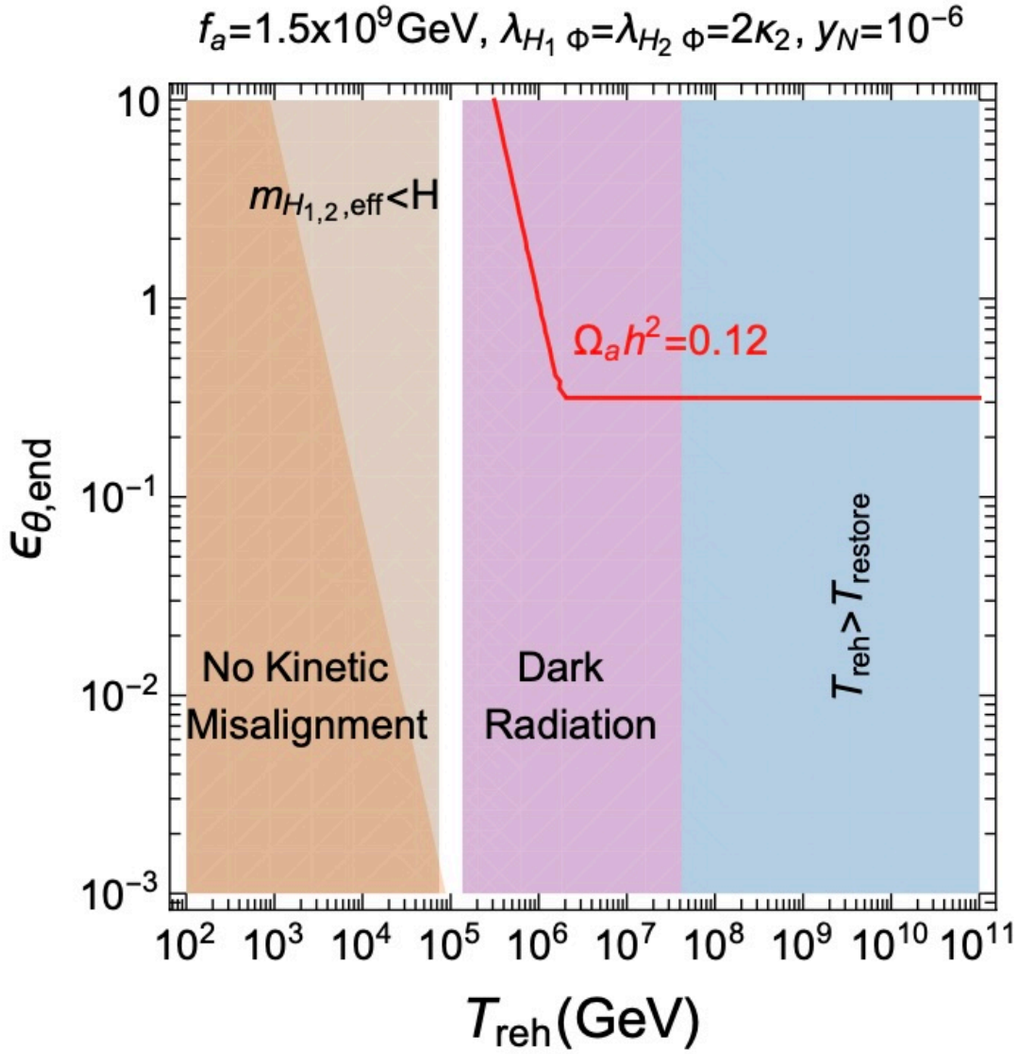
[HML, A. Menkara, M-J, Seong, J-H, Song (2023)]

Axion kinetic misalignment explains axion DM production.

Axions produced from inflaton scattering become DR.

More constraints





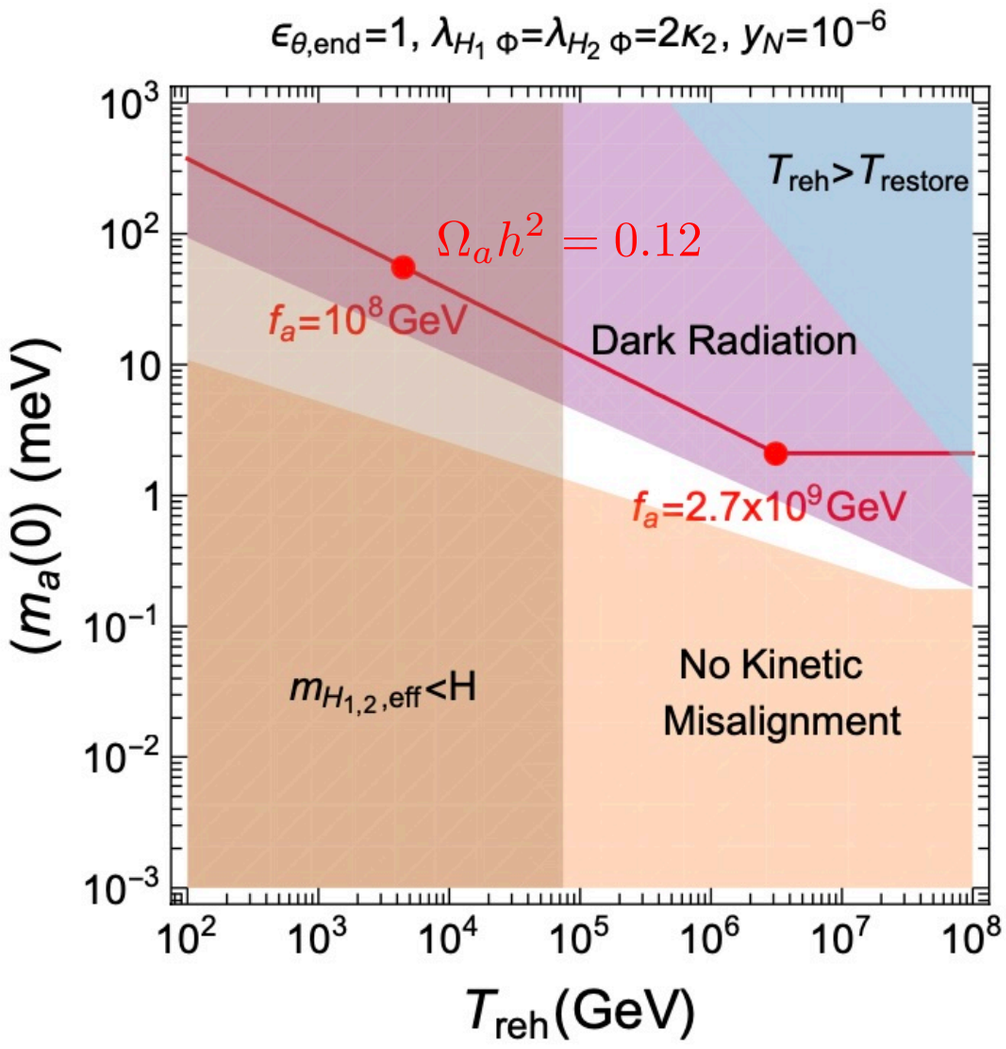
[HML, A. Menkara, M-J, Seong, J-H, Song (2024)]

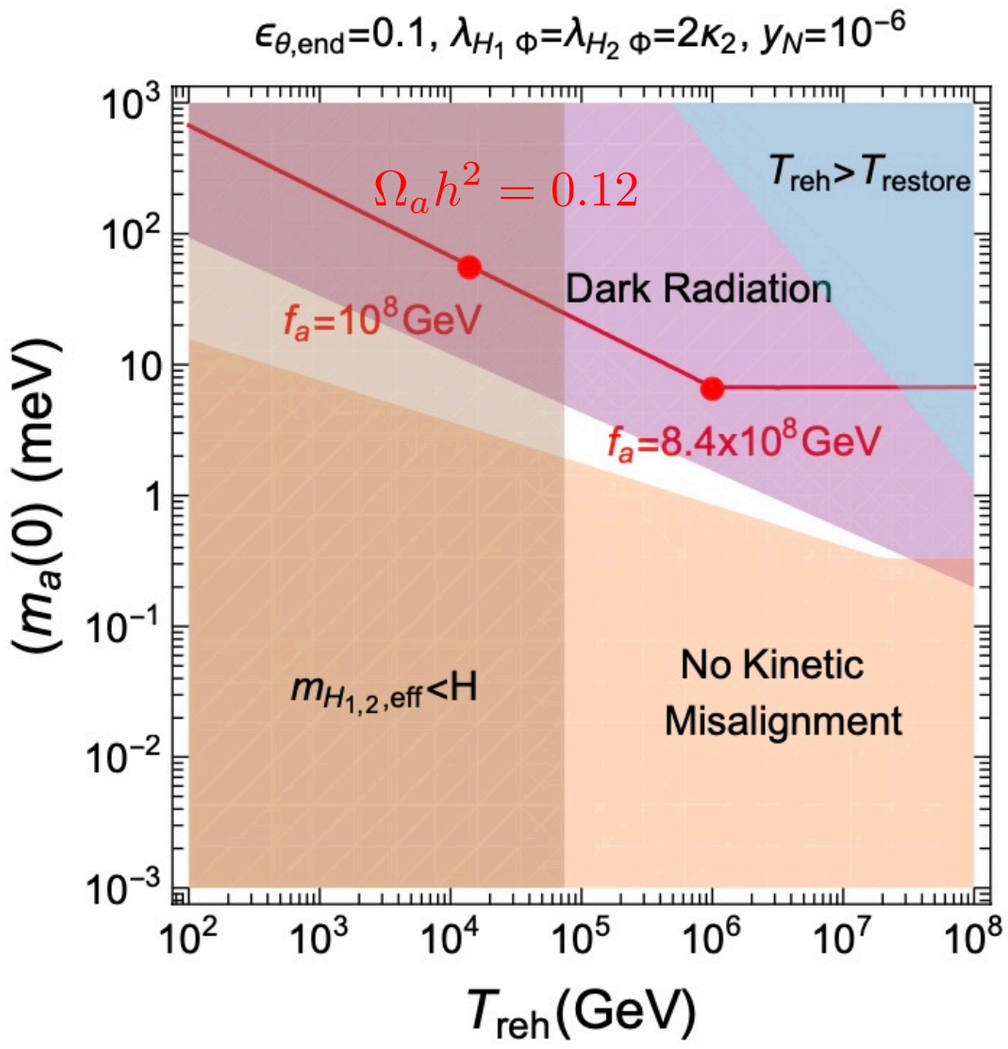
Brown: Higgs fields are lighter than Hubble scale during inflation.

Blue: Restoration of PQ symmetry after reheating.

(But, no axion domain wall problem due to PQ violating term.)

More constraints





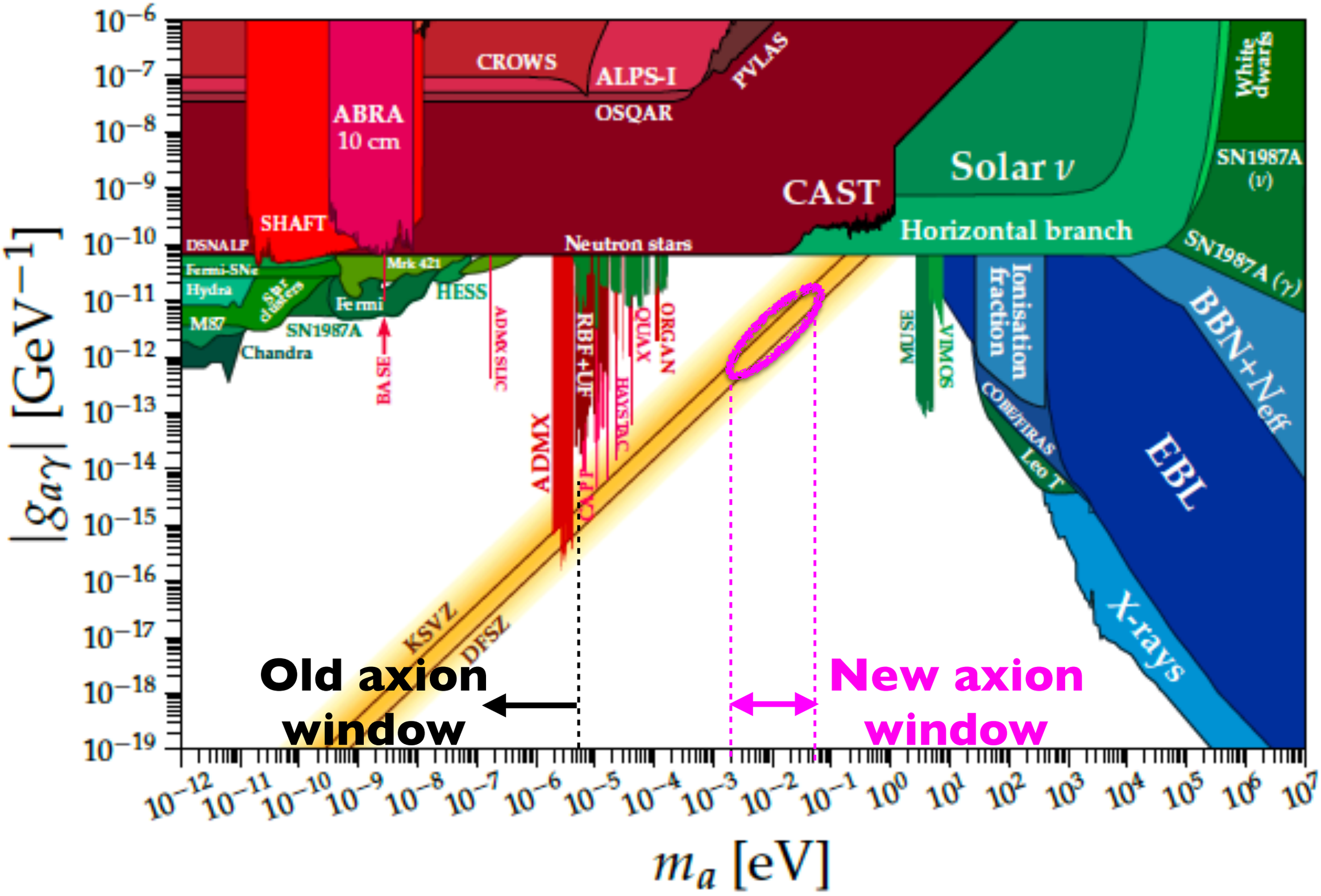
[HML, A. Menkara, M-J, Seong, J-H, Song (2024)]

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Bounds on axions



Isocurvature perturbations

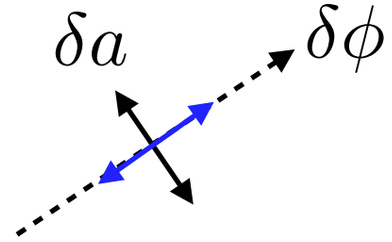
- Bounds on isocurvature perturbations.

$$P_{\rm iso}(k_*) = \left(\frac{1}{Y_a}\frac{\partial Y_a}{\partial \theta_*}\right)^2 \langle \delta\theta_*^2 \rangle ,$$

$\langle \delta\theta_*^2 \rangle$: fluctuations during inflation.

$P_\zeta(k_*) = 2.1 \times 10^{-9}$ at $k_* = 0.05$/Mpc.

$$\beta_{\rm iso} \equiv \frac{P_{\rm iso}(k_*)}{P_\zeta(k_*) + P_{\rm iso}(k_*)} < 0.038,$$ at 95% C.L. (Planck)



- The case with axion (kinetic) misalignment.

$$Y_a = Y_{a,\rm mis} + Y_{a,\rm kin}, \quad sY_{a,\rm kin} = 2m_a n_\theta \text{ and } sY_{a,\rm mis} = \tfrac{1}{2} m_a^2 f_a^2 \theta_*^2$$

$$P_{\rm iso}(k_*) = \left[\frac{4}{\theta_*^2}\left(\frac{Y_{a,\rm mis}}{Y_a}\right)^2 + \frac{1}{4}\left(\frac{1}{\epsilon_{\theta,*}}\frac{\partial \epsilon_{\theta,*}}{\partial \theta_*}\right)^2 \left(\frac{Y_{a,\rm kin}}{Y_a}\right)^2\right] \langle \delta\theta_*^2 \rangle ,$$

$$\langle \delta\theta_*^2 \rangle = \frac{1}{f_{a,\rm eff}^2}\left(\frac{H_I}{2\pi}\right)^2, \quad f_{a,\rm eff} \equiv \sqrt{6}\left|\sinh\left(\tfrac{\phi_*}{\sqrt{6}M_P}\right)\right| \simeq 22.$$

"Large effective $f_{a,\rm eff}$ "
=> suppression of angle fluc.

Safe from Planck bound: $\frac{4}{\theta_*^2}\left(\frac{Y_{a,\rm mis}}{Y_{a,\rm kin}}\right)^2 + (l-2k)^2 \cot^2\left((l-2k)\theta_* + A_{0,l,k}\right) \lesssim 10$

Axion domain walls

- Axion domain walls after QCD phase transitions.

PQ symmetry can be restored during reheating.

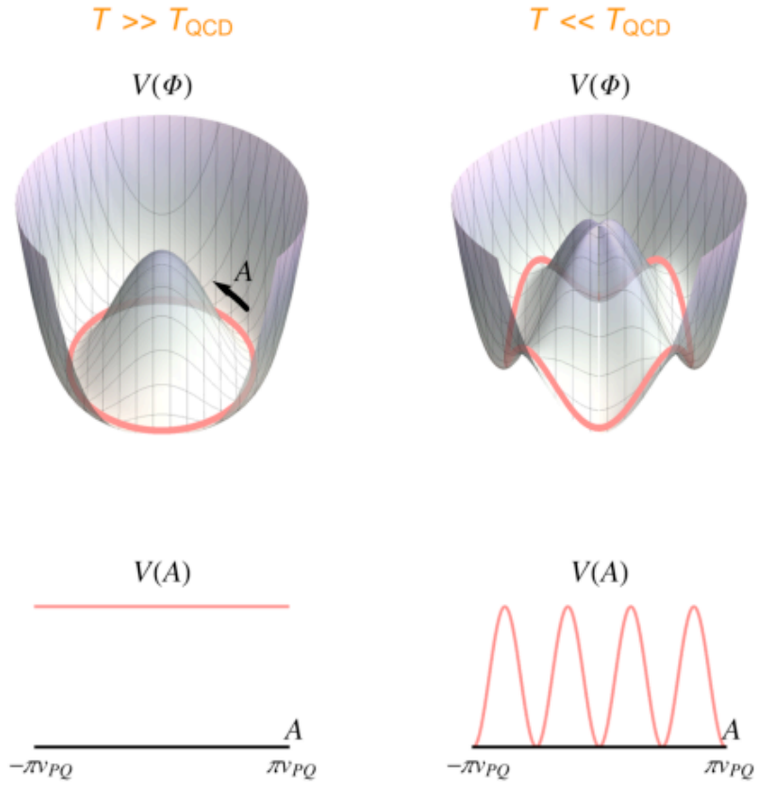
PQ symmetry is broken below a critical temperature.

=> Z_3 (Z_6) domain walls for p=1(2) in DFSZ

Domain wall energy : $\rho_{\rm walls} \sim \frac{\sigma}{R}$

Domain wall energy can be dominant.

=> Axion domain wall problem.



[1904.090155]

Axion domain walls

- PQ violation and domain wall problem.

PQ violation: $\Delta V = c\,\Lambda_{\rm QCD}^4 \times 10^{-10}, \quad c = |c_{0,l,k}| \left(\frac{f_a}{\sqrt{2} q_\Phi M_P}\right)^l \left(\frac{M_P}{\Lambda_{\rm QCD}}\right)^4 \times 10^{10}$

Axion quality => $c \lesssim 1$

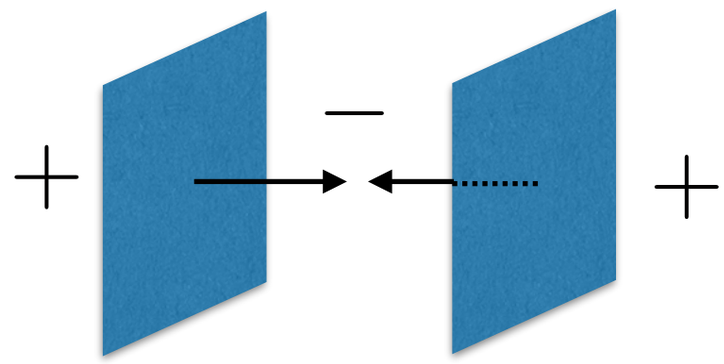
DW energy density dominates:

$$\rho_{\rm walls} \sim \rho_R \longrightarrow t_* \sim \frac{M_P^2}{\sigma}$$

[S. Abel et al, hep-th/9506359]

Pressure > energy density before BBN.

$$\Delta V \gtrsim \rho_{\rm walls} \sim \frac{\sigma}{t} \gtrsim \frac{\sigma}{0.1\,{\rm sec}}$$



=> annihilating domain walls.

Axion DW tension: $\sigma \sim \Lambda_{\rm QCD}^2 f_a \longrightarrow c \gtrsim 10^{-3} \left(\frac{f_a}{10^9\,{\rm GeV}}\right)$

$$|c_{0,l,k}| \lesssim 10^{-11} \longrightarrow l \lesssim 9 \ , \ f_a \sim 10^9\,{\rm GeV}.$$

Conclusions

- PQ pole inflation is the perturbative expansion about conformal coupling, being distinguishable from inflation models with large non-minimal coupling due to the general equation of state during reheating.
- Small couplings of the PQ inflaton in both KSVZ and DFSZ models are needed for the PQ pole inflation, but they allow for efficient reheating by decays/ annihilations of PQ inflaton.
- Initial axion velocity is set by the PQ violating potential during PQ inflation, accounting for the correct relic density by axion kinetic misalignment for a small axion decay constant.

Backup slides

Bosonic Lagrangian

- Jordan-frame Lagrangian in DFSZ model.

$$\frac{\mathcal{L}_J}{\sqrt{-g_J}} = -\frac{1}{2}M_P^2\,\Omega R(g_J) + \mathcal{L}_{\mathrm{kin},J} - \Omega^2 V_E\,, \qquad \Omega = 1 - \frac{1}{3M_P^2}|H_1|^2 - \frac{1}{3M_P^2}|H_2|^2 - \frac{1}{3M_P^2}|\Phi|^2,$$

$$V_E = V_{\mathrm{PQ}} + V_{\mathrm{PQV}}; \qquad \begin{aligned} V_{\mathrm{PQ}} \;=\; & \lambda_\Phi|\Phi|^4 + \lambda_1|H_1|^4 + \lambda_2|H_2|^4 + \lambda_3|H_1|^2|H_2|^2 + \lambda_4\left(H_1^\dagger H_2\right)\left(H_2^\dagger H_1\right) \\ & + \lambda_{1\Phi}|H_1|^2|\Phi|^2 + \lambda_{2\Phi}|H_2|^2|\Phi|^2 + \left(2^{p/2-1}\kappa_p H_1^\dagger H_2 \Phi^p + \mathrm{h.c.}\right) \\ & + \mu_\Phi^2|\Phi|^2 + m_1^2|H_1|^2 + m_2^2|H_2|^2 + V_0, \end{aligned}$$

$$V_{\mathrm{PQV}} = \sum_{n,l}\sum_{k=0}^{[l/2]} \frac{c_{n,l,k}}{2M^{2n+l-4}}\,(H_1^\dagger H_2)^n |\Phi|^{2k}\Phi^{l-2k} + \mathrm{h.c.}$$

- Kinetic terms in Einstein frame.

$$H_1 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ h_1 e^{i\eta_1}\end{pmatrix}, \quad H_2 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ h_2 e^{i\eta_2}\end{pmatrix}, \quad \Phi = \frac{1}{\sqrt{2}}\,\rho\, e^{i\theta} \;:\; \text{inflaton background}$$

$$\frac{\mathcal{L}_{\mathrm{kin},E}}{\sqrt{-g_E}} = -\frac{1}{2}R + \frac{(\partial_\mu h_1)^2 + (\partial_\mu h_2)^2 + h_1^2(\partial_\mu\eta_1)^2 + h_2^2(\partial_\mu\eta_2)^2 + (\partial_\mu\rho)^2 + \rho^2(\partial_\mu\theta)^2}{2(1 - \frac{1}{6}h_1^2 - \frac{1}{6}h_2^2 - \frac{1}{6}\rho^2)} + \frac{3}{4}\frac{\left(\partial_\mu\left(\frac{1}{6}h_1^2 + \frac{1}{6}h_2^2 + \frac{1}{6}\rho^2\right)\right)^2}{(1 - \frac{1}{6}h_1^2 - \frac{1}{6}h_2^2 - \frac{1}{6}\rho^2)^2}.$$

Non-inflaton fields

- Effective masses for non-inflaton fields.

Pure PQ inflation with $h_1 = h_2 = 0$:

CP-odd Higgs mass - $$m^2_{\tilde{A},\text{eff}} = 6|\kappa_2| \frac{\sinh^2\left(\frac{\phi}{\sqrt{6}}\right)}{\cosh^4\left(\frac{\phi}{\sqrt{6}}\right)},$$

CP-even Higgs masses - $$m^2_{\pm,\text{eff}} \simeq \frac{3}{2}\left[\lambda_{1\Phi} + \lambda_{2\Phi} - 4\lambda_\Phi \pm \sqrt{(\lambda_{1\Phi} - \lambda_{2\Phi})^2 + 4\kappa_2^2}\right] \cdot \frac{\sinh^2\left(\frac{\phi}{\sqrt{6}}\right)}{\cosh^4\left(\frac{\phi}{\sqrt{6}}\right)}$$

- Effective action for inflation and axion.

$m^2_{\tilde{A},\text{eff}}$, $m^2_{\pm,\text{eff}} \gg H_I^2 \simeq 3\lambda_\Phi$: decoupled non-inflaton fields.

$$\frac{\mathcal{L}_{E,\text{eff}}}{\sqrt{-g_E}} = -\frac{1}{2}R + \frac{1}{2}(\partial_\mu\phi)^2 + 3\sinh^2\left(\frac{\phi}{\sqrt{6}}\right)(\partial_\mu\theta)^2 - V_E(\phi,\theta)\,, \qquad \rho = \sqrt{6}\tanh\left(\frac{\phi}{\sqrt{6}}\right)$$

$$V_E = V_{\text{PQ}} + V_{\text{PQV}}; \qquad V_{\text{PQ}}(\phi) = 9\lambda_\Phi \tanh^4\left(\frac{\phi}{\sqrt{6}}\right) = U(\phi),$$

$$V_{\text{PQV}}(\phi,\theta) = \sum_{k=0}^{[l/2]} \frac{3^{l/2}|c_{0,l,k}|}{M_P^{l-4}} \tanh^l\left(\frac{\phi}{\sqrt{6}}\right)\cos((l-2k)\theta + A_{0,l,k}).$$