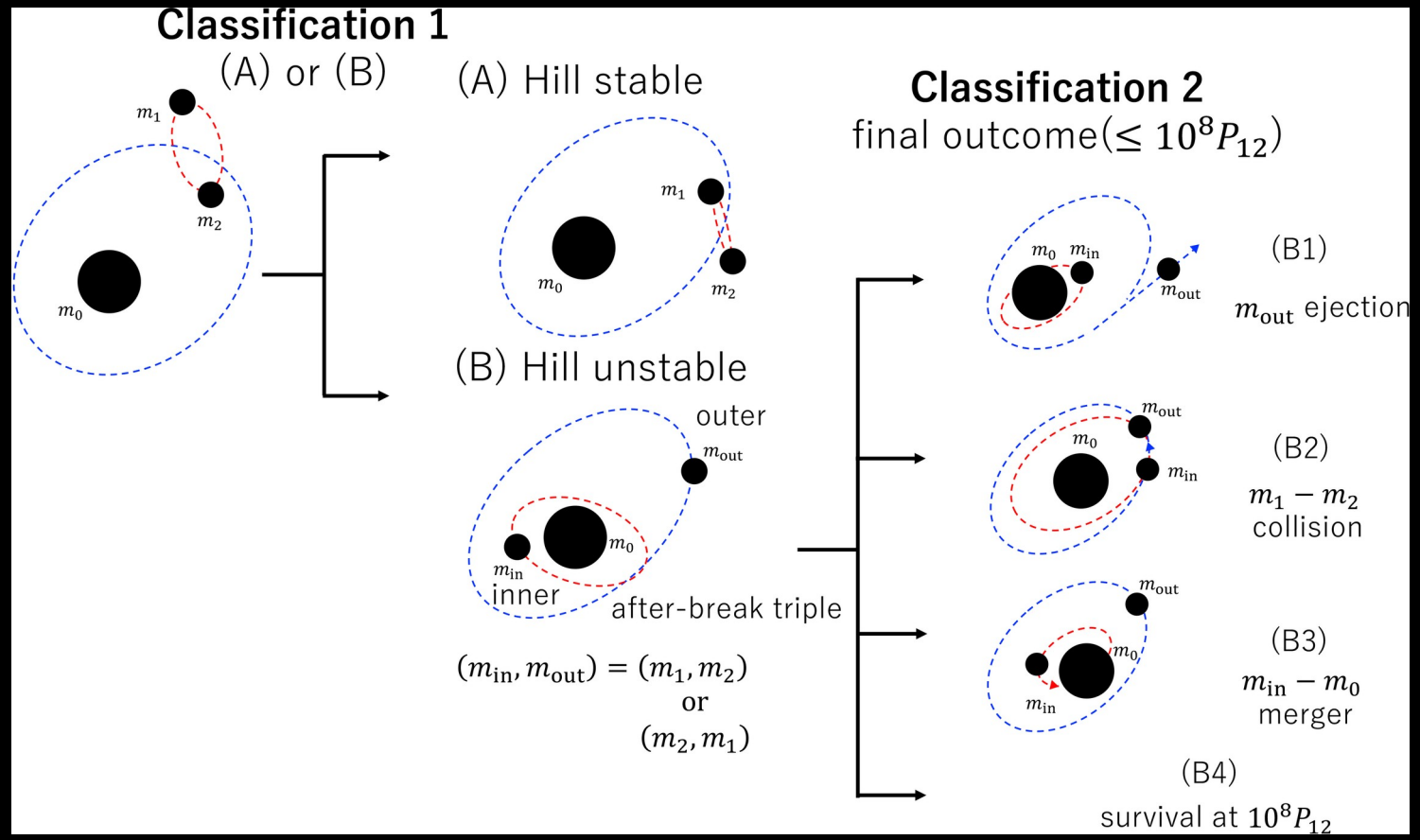
Dynamical instability criterion for hierarchical triple systems

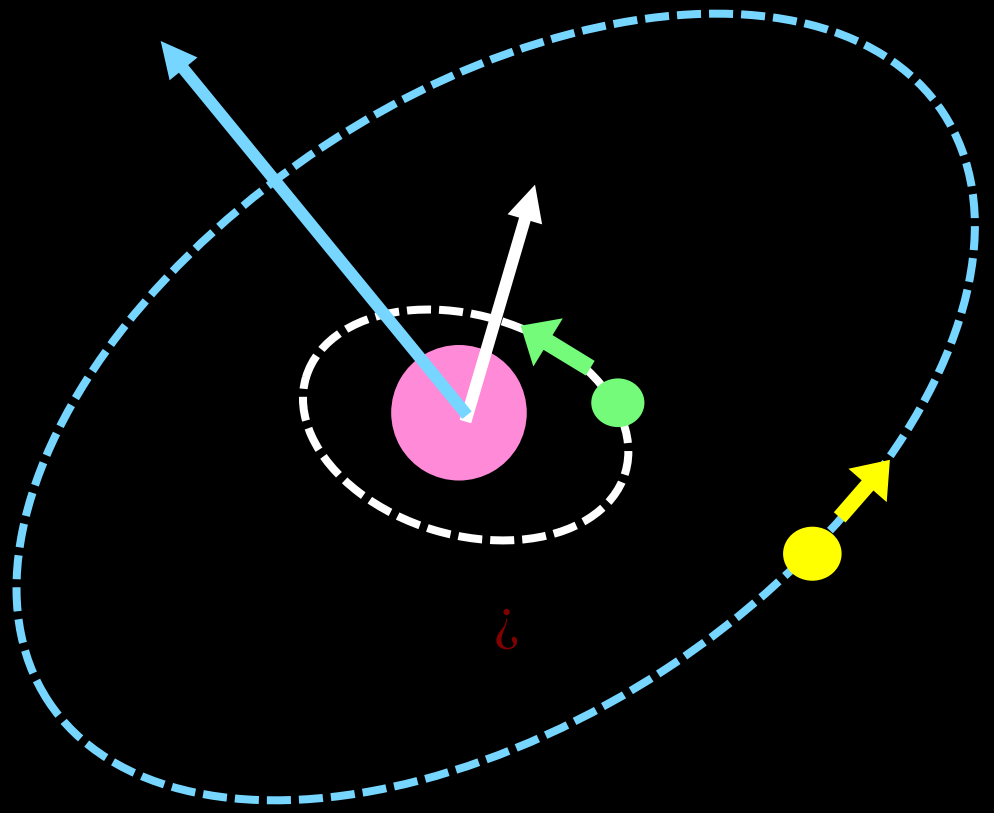


Yasushi Suto
Kochi University of Technology
高知工科大学
KOCHI UNIVERSITY OF TECHNOLOGY

The 11th KIAS workshop on cosmology and structure formation
@ Hilton Gyeongju, Korea, 11:20-11:40 on October 30, 2024

1 Introduction

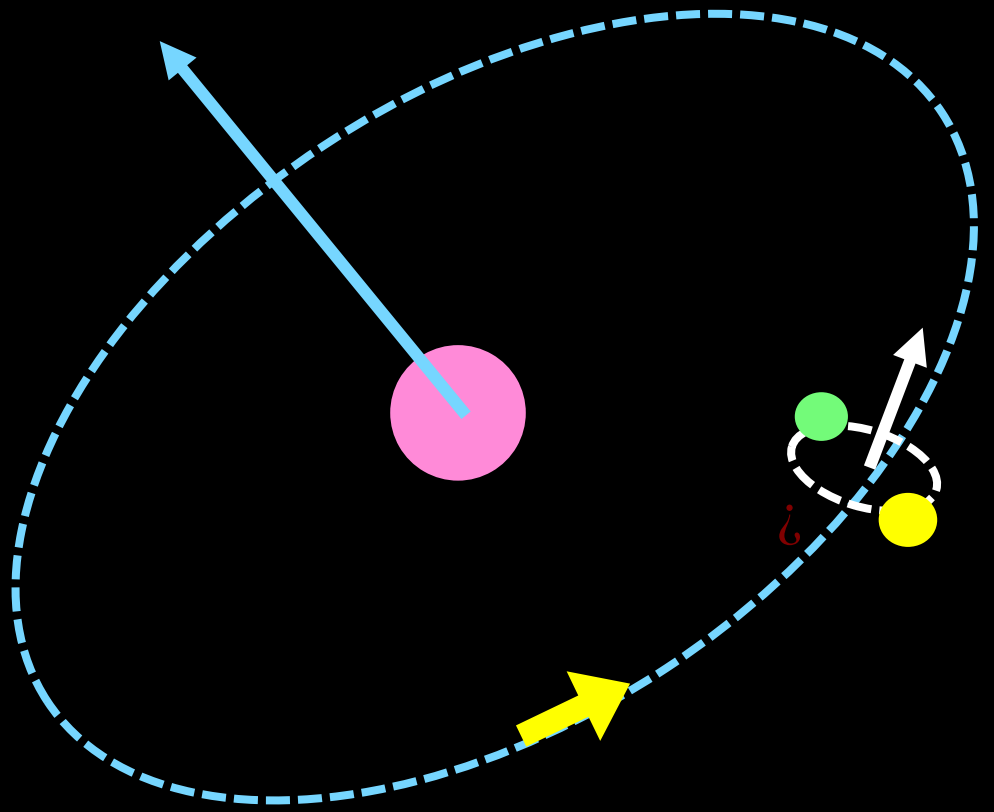
Hierarchical Triple Planet-type (HT-P)



m_1-m_2 binary orbited by tertiary m_3

- Diversity of planetary systems due to tertiary (e.g., spin-orbit misalignment via von-Zeipel – Kozai- Lidov effect)
  - Albrecht, Dawson, & Winn (2022)
- Search for binary blackholes in triples ⇒ talk by Toshinori Hayashi
  - Hayashi, YS & Trani (2023)
  - El-Badry et al. (2023)
  - Nagarajan et al. (2024)

Hierarchical Triple Satellite-type (HT-S)



central m_0 orbited by m_1-m_2 binary

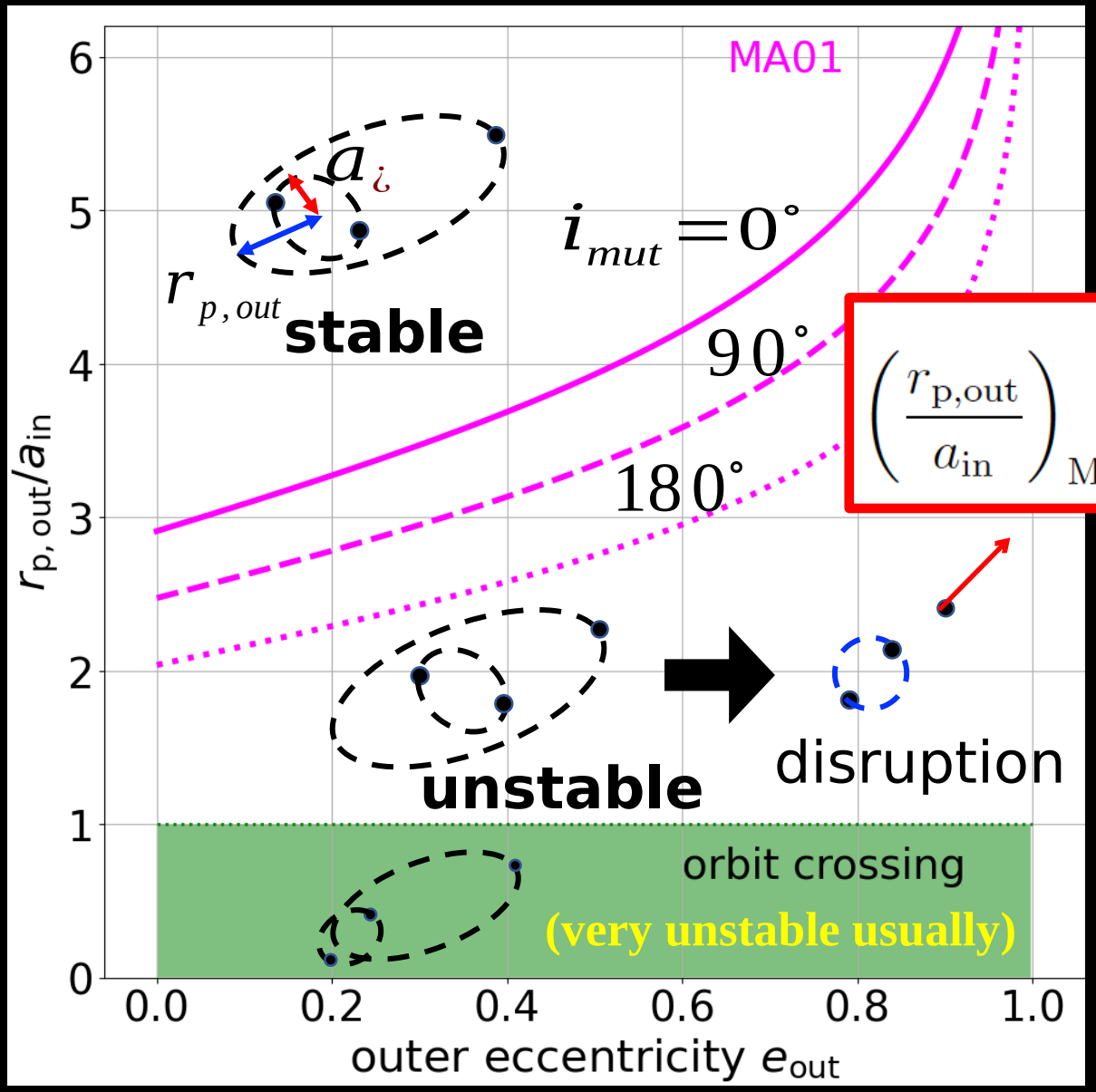
- Exomoon-planet/binary planet
  - Not yet unambiguously detected, but precited theoretically
  - Ochiai, Nagasawa & Ida (2014)
- Binary blackhole orbiting a central massive blackhole
  - potentially important

2 Lagrange instability timescales of HT-P systems

- Hayashi, Trani & YS (2023) “Lagrange versus Lyapunov Stability of Hierarchical Triple Systems: Dependence on the Mutual Inclination between Inner and Outer Orbits”, ApJ 943, 58

- Hayashi, Trani & YS (2022) “ Dynamical Disruption Timescales and Chaotic Behavior of Hierarchical Triple Systems”, ApJ, 939, 81

Stability of HT-P like systems

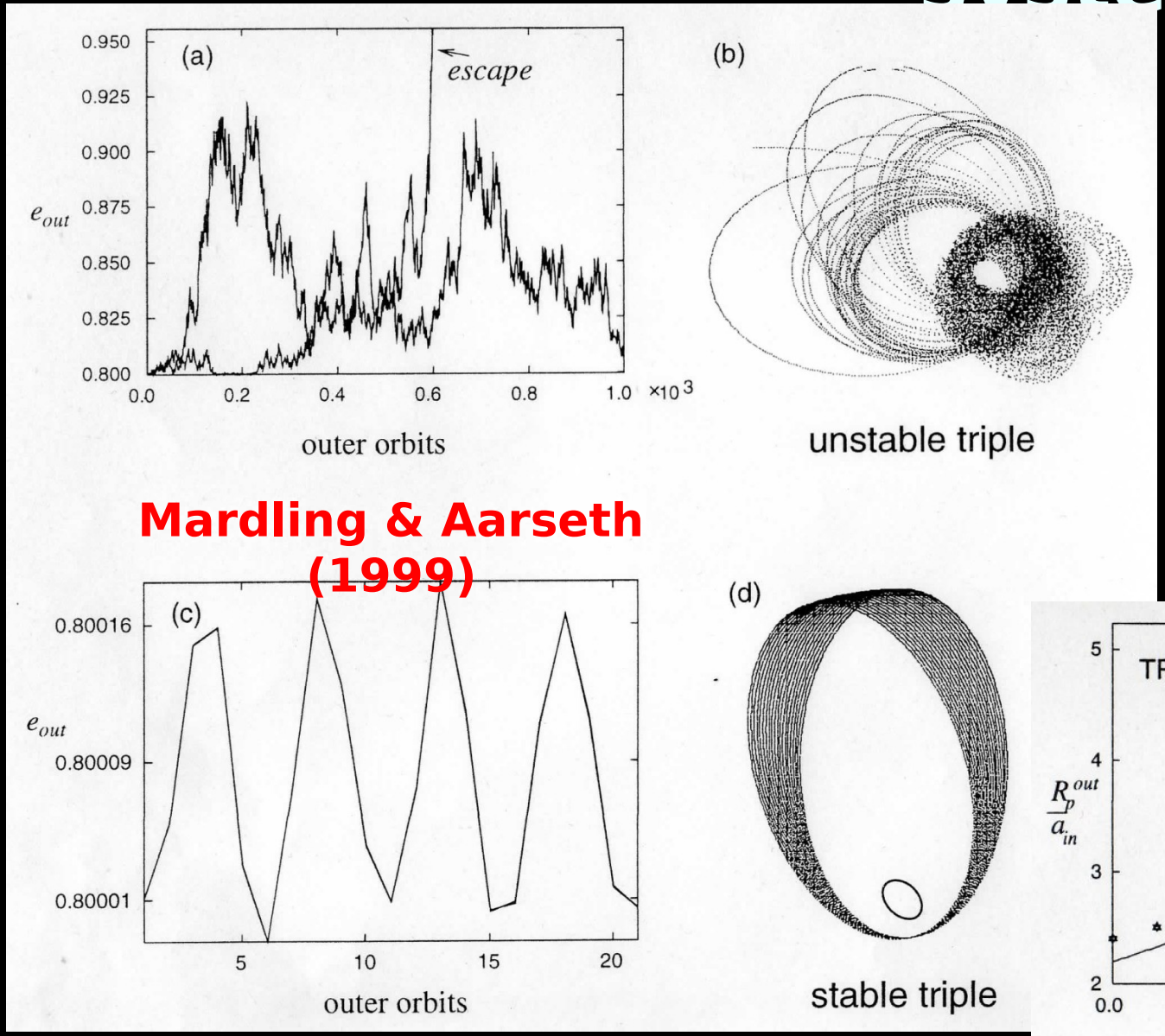


- Mardling & Aarseth (2001)
  - Well-known and widely used, classic stability criterion:

$$\left(\frac{r_{\rm p,out}}{a_{\rm in}}\right)_{\rm MA} \equiv 2.8\left(1-0.3\frac{i_{\rm mut}}{\pi}\right)\left[\left(1+\frac{m_3}{m_{12}}\right)\frac{(1+e_{\rm out})}{\sqrt{1-e_{\rm out}}}\right]^{2/5}$$

- Lyapunov (chaoticity of local trajectory) vs. Lagrange (escape of a body from the system) stability
- ***Disruption timescale,*** instead of the stable-unstable boundary (Hayashi, Tsuchida & YS

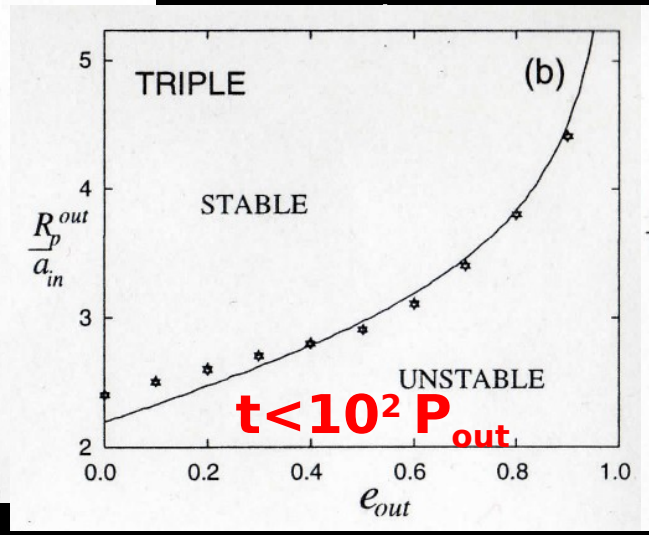
Normal and chaotic evolution of triple systems

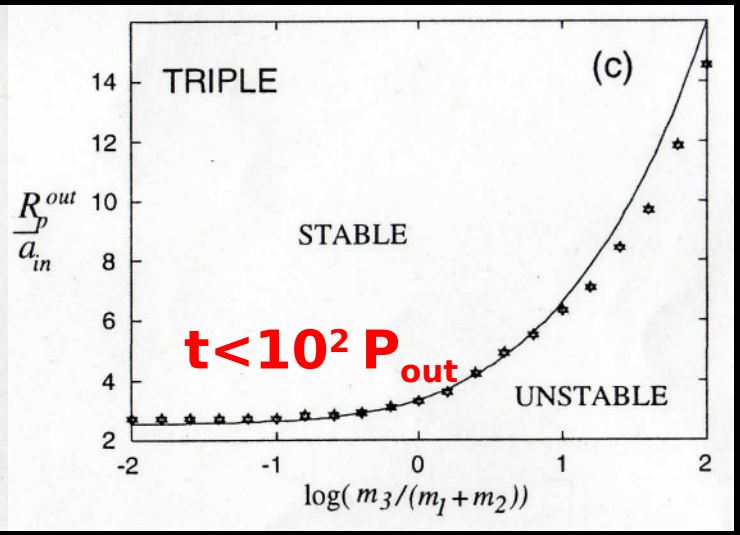


Mardling & Aarseth (1999)

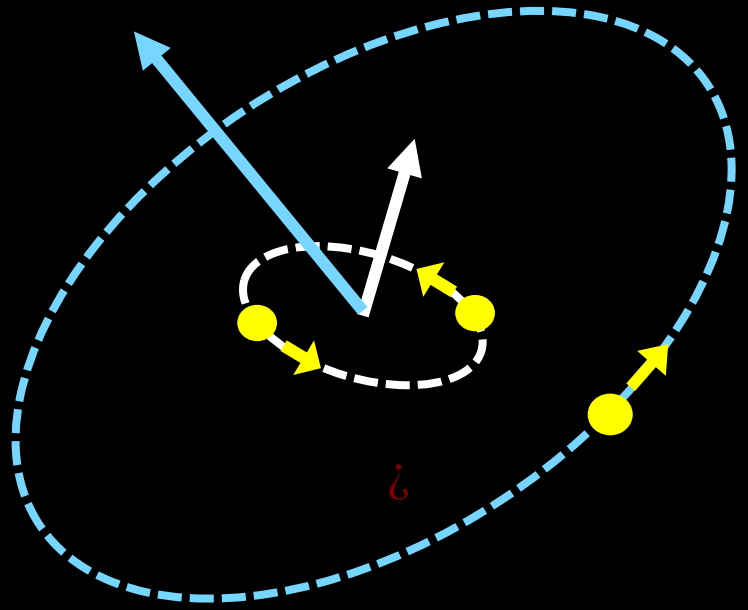
"A system was deemed stable if two orbits, initially differing by 1 part in 10^5 in the eccentricity, remained close after 100 orbits." (Mardling & Aarseth 1999)

"We deem a triple system stable if it remains bound for 100 outer orbits and if the semimajor axes of both inner and outer orbits do not change by more than 10 percent of the initial value." (Vynatheya et al. 2022 MNRAS 516,





Method: Longer-term N-body simulations in Newtonian dynamics *neglecting GR effects*



(circular)

N-body code
TSUNAMI
(Trani and Spera)

Integration time
up to
(roughly $10^{6\text{-}7}$ P_{out})

Disruption time

:

Coplanar: prograde, retrograde
Orthogonal

Mass ratio (inner binary)

Mass ratio (tertiary)

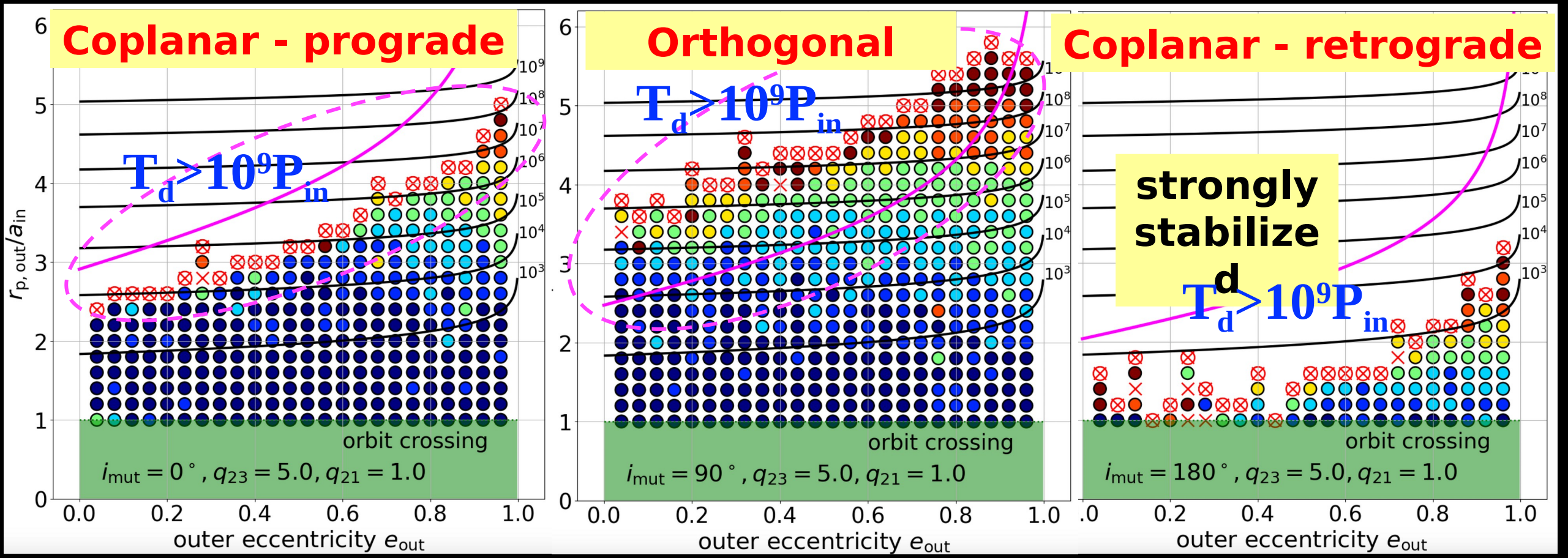
Hayashi, Trani & YS,
arXiv:2207.12672 ApJ 939(2022)81
arXiv:2209.08487 ApJ 943(2023)58

Disruption timescales on $a_{out}(1-e_{out})/a_{in}$ – e_{out} plane

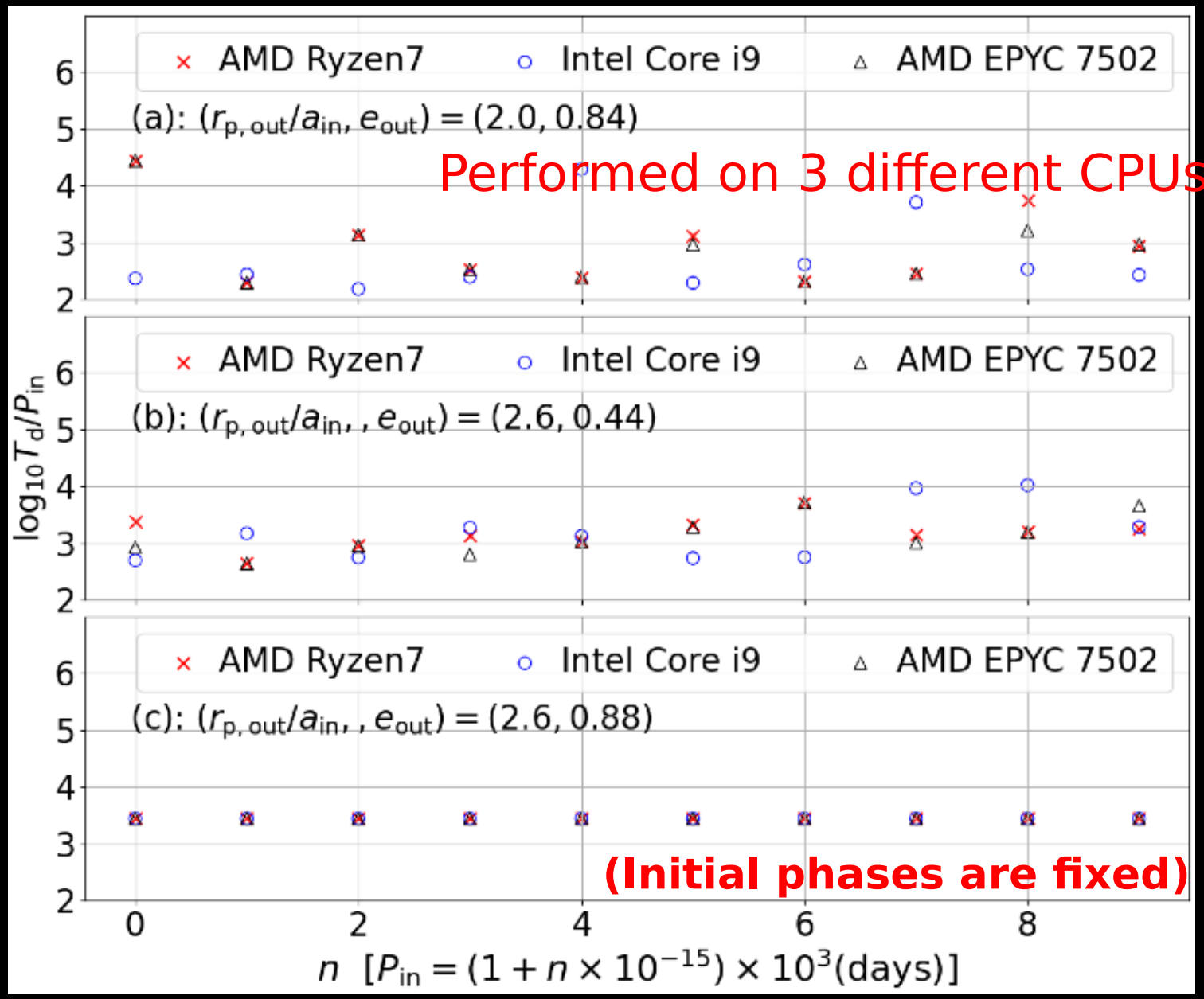
$m_1=m_2=5m_3$

no sharp boundary

——Dynamical stability boundary by Mardling & Aarseth (2001



Chaotic nature of disruption timescale distribution



- Tiny difference in the input value of P_{in} leads to one or two order-of-magnitude difference of disruption timescales
- Initial phase difference of the three bodies also leads to one or two order-of-magnitude difference of disruption timescales
- We do not understand why just one or two order-of-magnitudes …

Hayashi, Trani & YS
ApJ 939(2022)81

3 Hill instability (binary break-up) timescales of HT-S systems

- Hayashi, Trani & YS (2024) , submitted to ApJ

Stability of HT-S like systems

- Instantaneous Hill stability condition for circular orbit

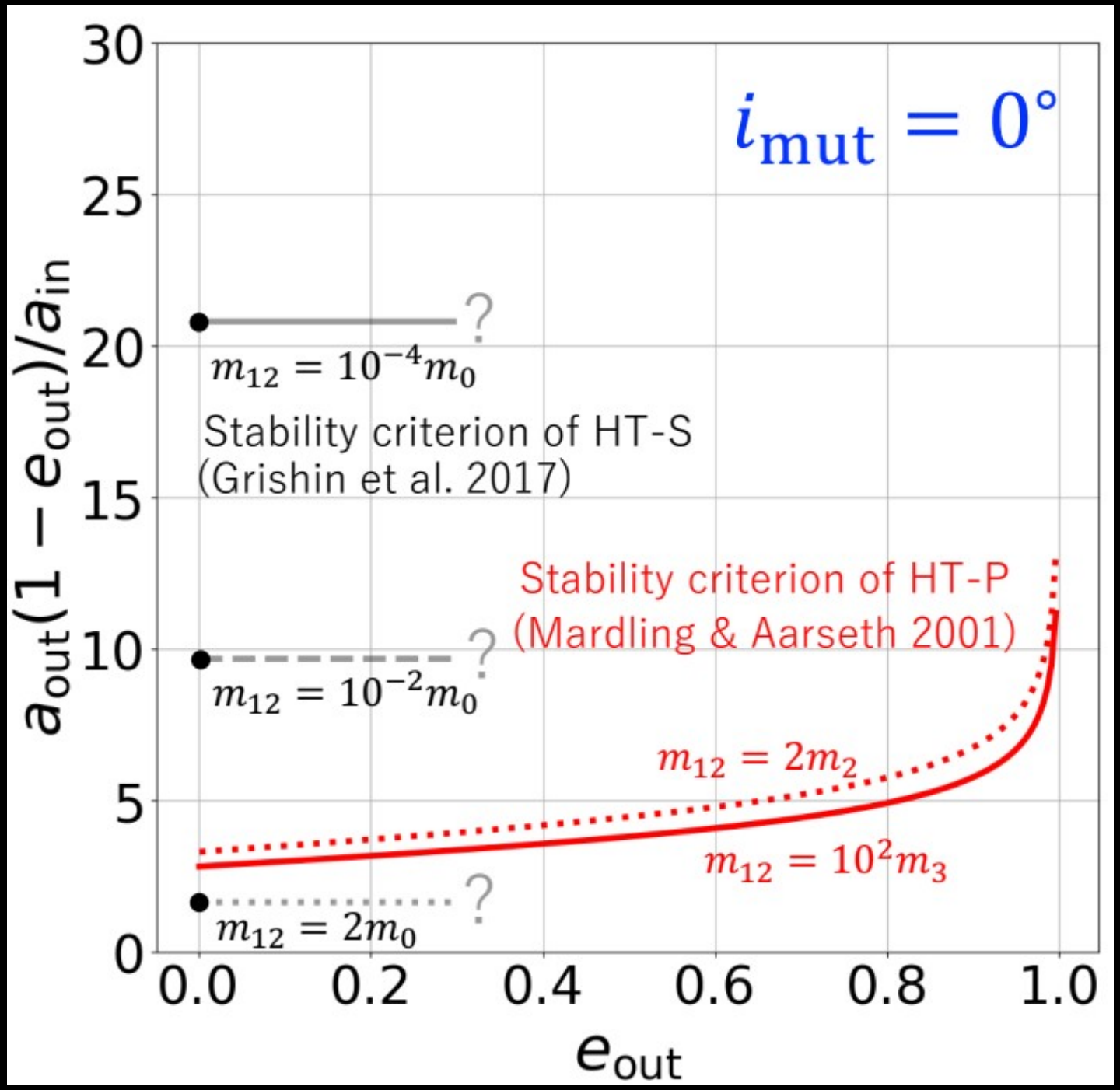
$$a_{12} < a_{\rm Hill} \equiv a_{012} \left(\frac{m_1}{3m_0} \right)^{1/3}$$

- Grishin, Perets, Zenati & Michaely (2017)
  - generalized Hill-stability criteria to arbitrarily inclined tri[illegible]ct

$$\frac{a_{012}}{a_{12}} > \frac{1}{3^{1/3}} \left(\frac{3m_0}{m_1+m_2} \right)^{1/3} (\cos i_{\rm mut} + \sqrt{3+\cos^2 i_{\rm mut}})^{2/3} \times \begin{cases} 1 & (\cos^2 i_{\rm mut} > 3/5) \\ \dfrac{9-5\cos^2 i_{\rm mut}}{6} & (\cos^2 i_{\rm mut} \leq 3/5) \end{cases}$$

Eccentricity dependence

- Stability boundaries of HT-P and HT-S systems against outer eccentricity
- the corresponding instability timescales
- Fate of HT-S



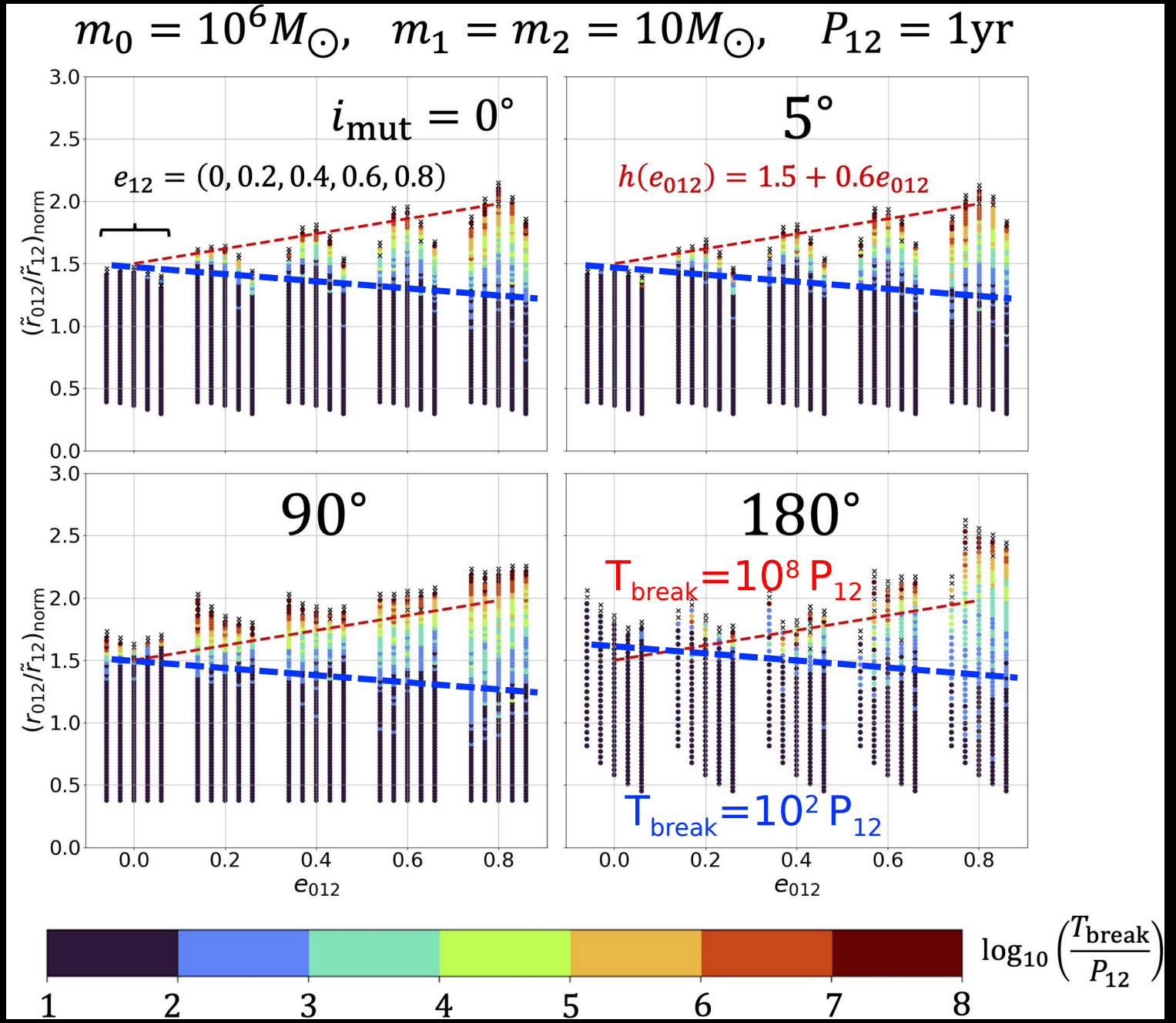
Method

- Longer-term direct N-body simulations with Tsunami
  - without orbit-averaged secular approximation
  - under Newtonian gravity (neglecting general relativity)
  - i_{mut}=0°, 5°, 90°, 180°
  - varying inner and outer eccentricities
  - break-up timescales of initial binary up to t=10^8 P_{12}
  - generalized Hill-like stability boundaries (binary

Dependence of T_{break} on the mutual inclination i_{mut} and the outer eccentricity e_{out}

$$(\cos i_{\rm mut} + \sqrt{3 + \cos^2 i_{\rm mut}})^{2/3} h(e_{012})$$

Note that $h(e_{012})$ is sensitive to the choice of the value of



Binary stability boundary *at 10^8 P_{12}*

$$\Upsilon \equiv \frac{\tilde{r}_{012}}{\tilde{r}_{12}} > \Upsilon_{\rm crit} \equiv \left(\frac{m_0}{m_1+m_2}\right)^{1/3} (\cos i_{\rm mut} + \sqrt{3+\cos^2 i_{\rm mut}})^{2/3} h(e_{012})$$

$$\tilde{r}_{012} \equiv a_{012}(1-e_{012})$$

$$h(e_{012}) = 1.5 + 0.6 e_{012}$$

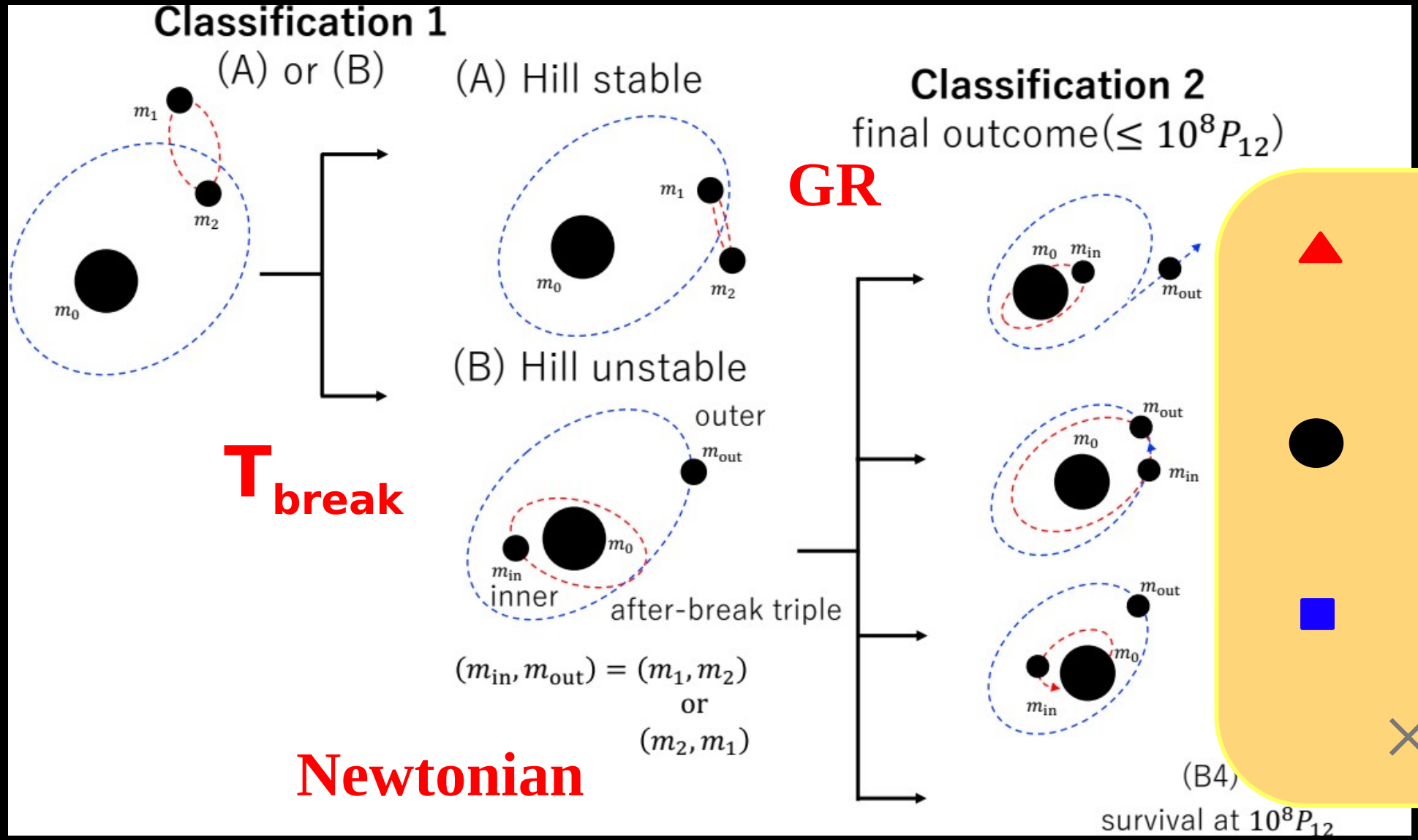
$$\tilde{r}_{12} = \begin{cases} a_{12}(1+0.5e_{12}^2) & (i_{\rm mut} = 0^\circ, 5^\circ) \\ a_{12}[1+0.5e_{\rm max}^2(i_{\rm mut})] & (i_{\rm mut} = 90^\circ) \\ a_{12}(1+e_{12}) & (i_{\rm mut} = 180^\circ) \end{cases}$$

$$e_{\rm max} = \sqrt{1 - \frac{5}{3}\cos^2 i_{\rm mut}} \qquad (\cos^2 i_{\rm mut} \leq 3/5)$$

4 Fate of HT-S systems

- Hayashi, Trani & YS (2024) , submitted to ApJ

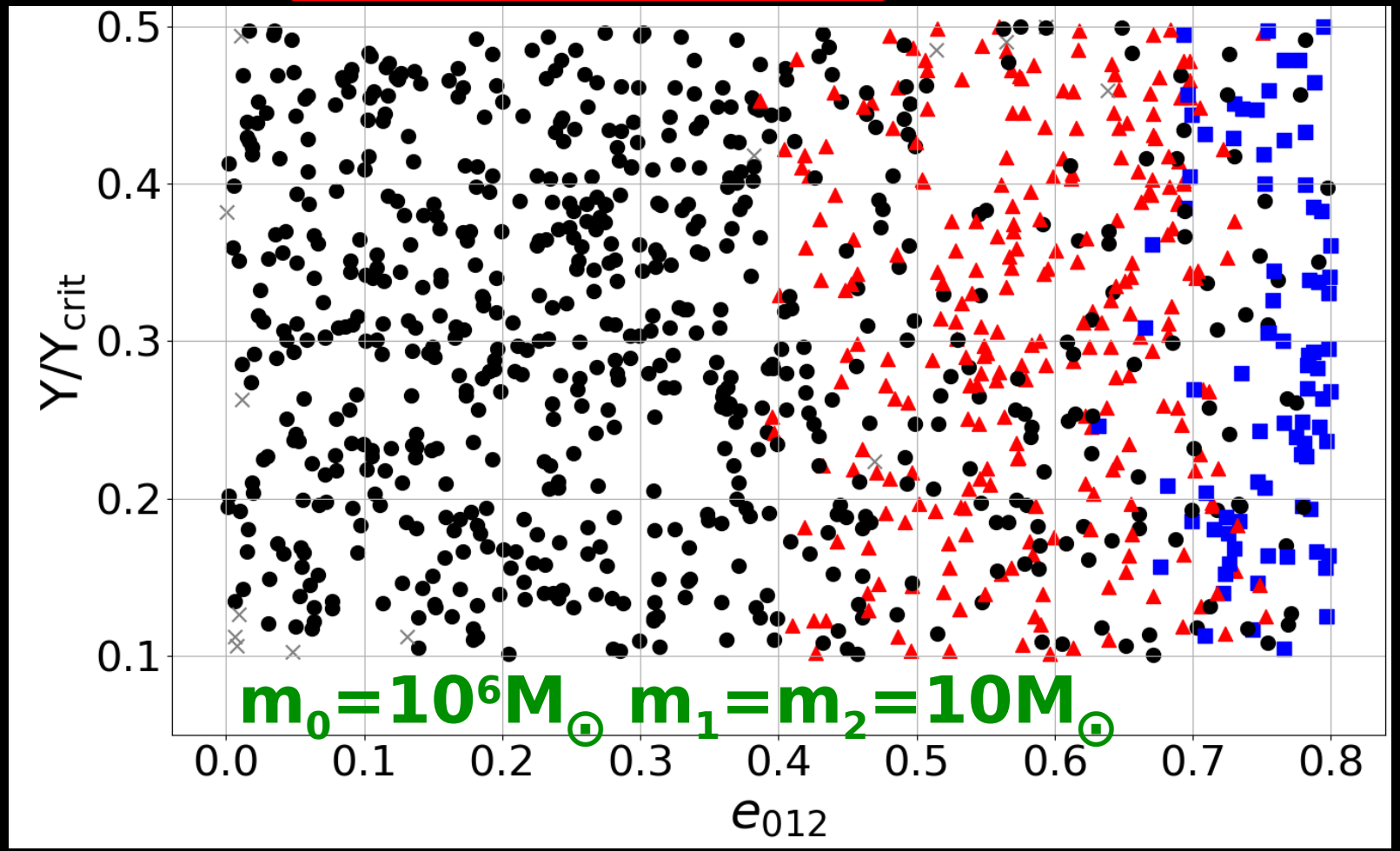
Fate of the HT-S systems

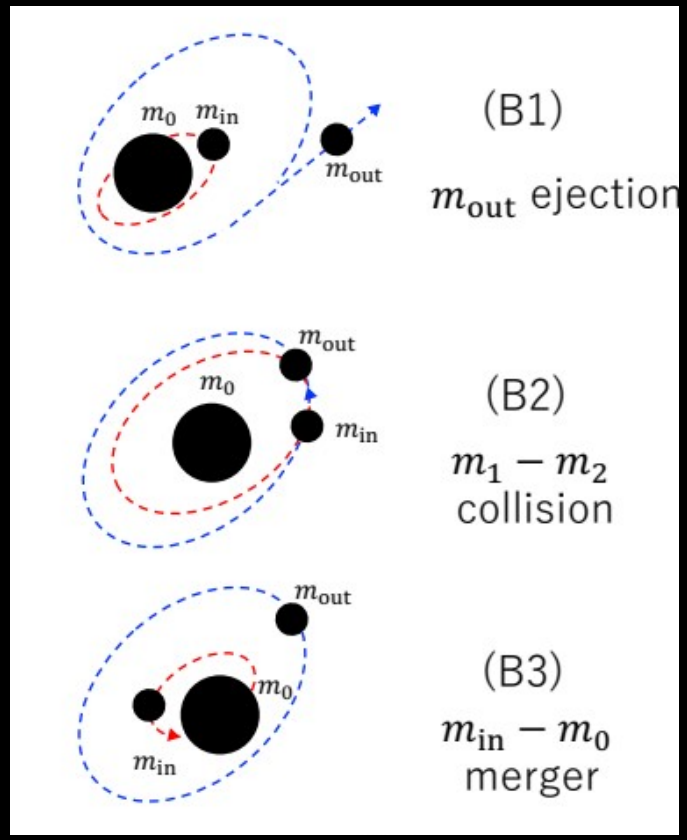


classification of the fates of HT-S systems

$i_{mut} = 0^\circ$

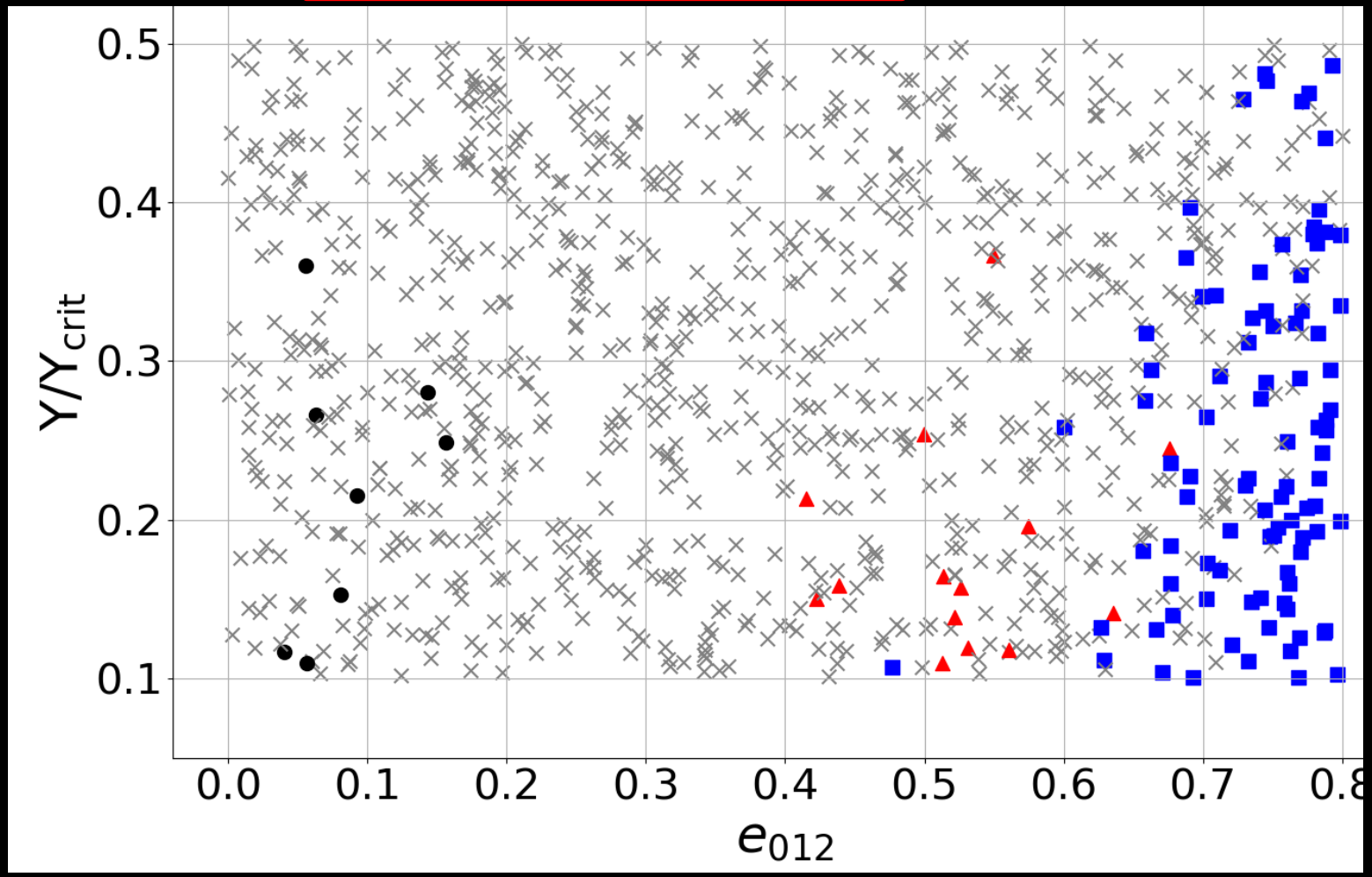
- collision
- ejection
- merger
× stable <





classification of the fates of HT-S systems

$$i_{\mathrm{mut}} = 5^\circ$$

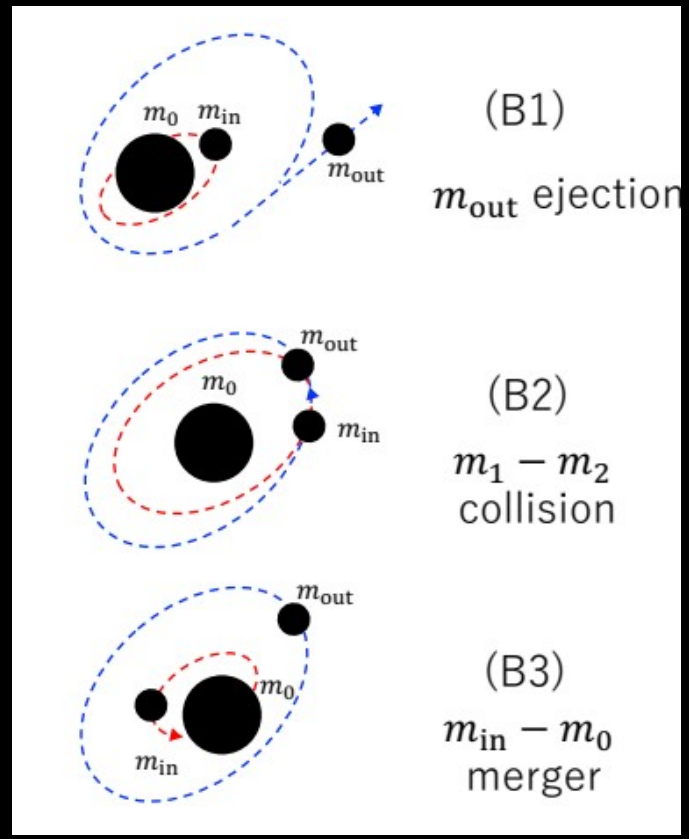


● - collision

▲ - ejection

■ - merger

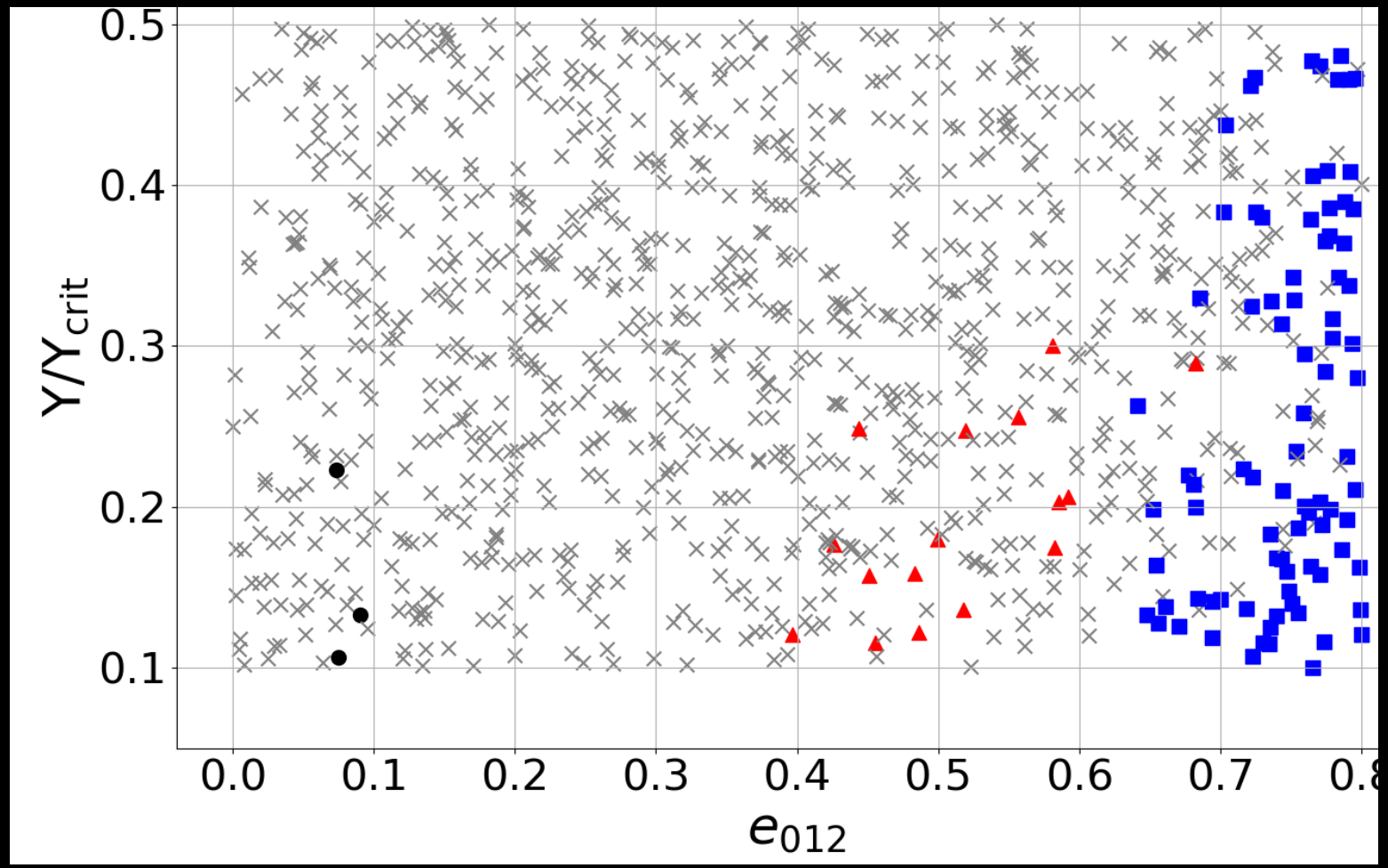
× stable <

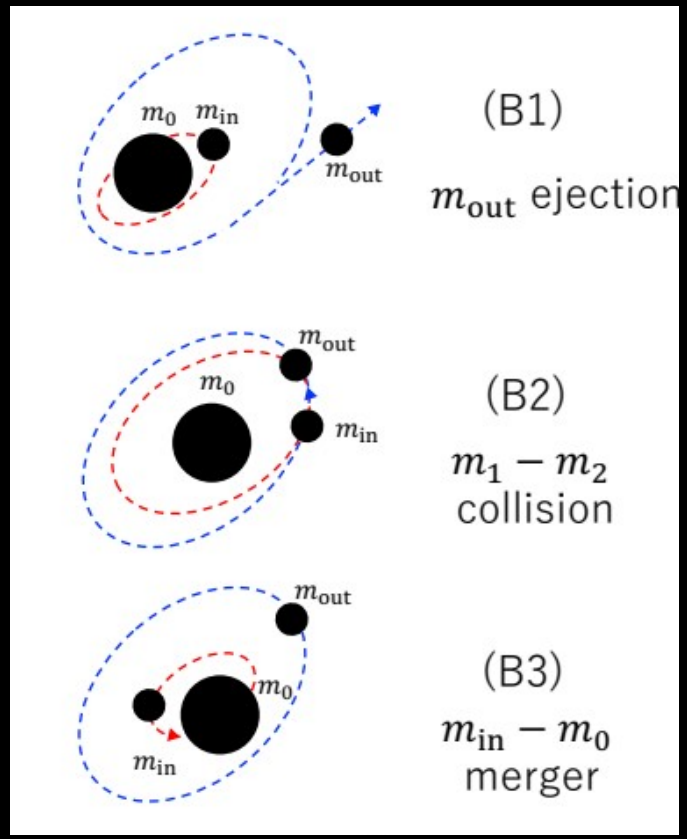


classification of the fates of HT-S systems

$i_{\mathrm{mut}} = 90^\circ$

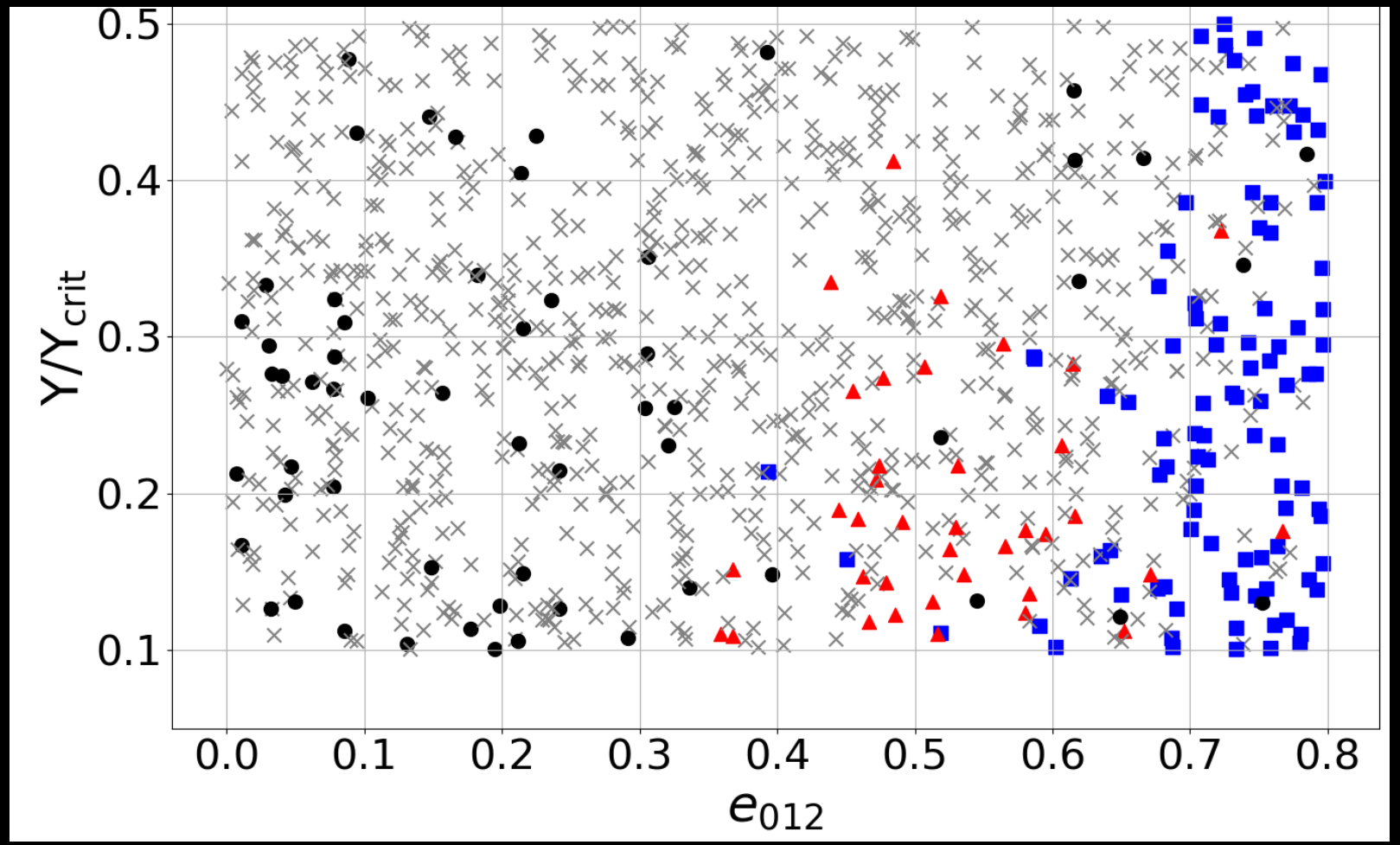
- collision
- ejection
- merger
× stable <





classification of the fates of HT-S systems

$i_{\mathrm{mut}} = 180^\circ$

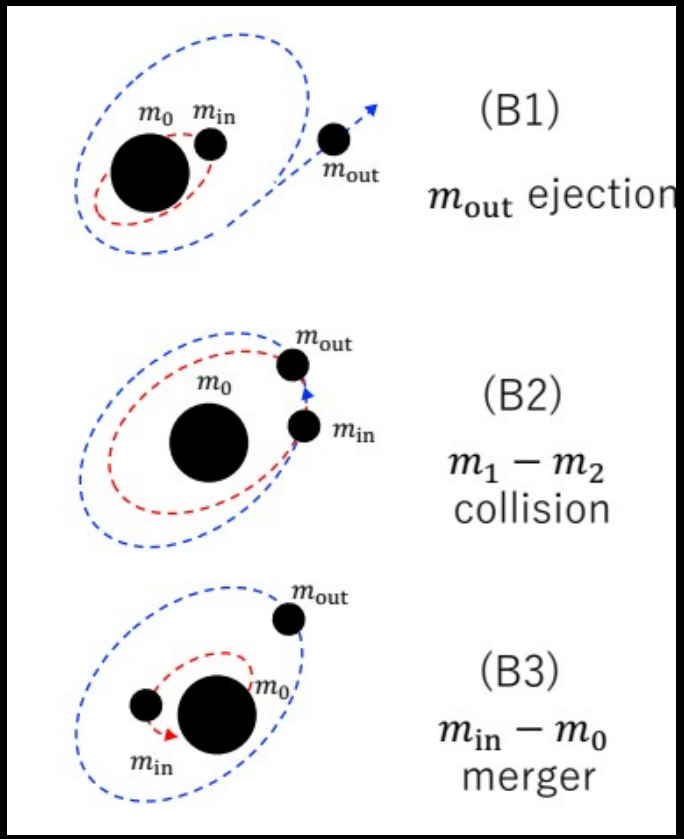


● - collision

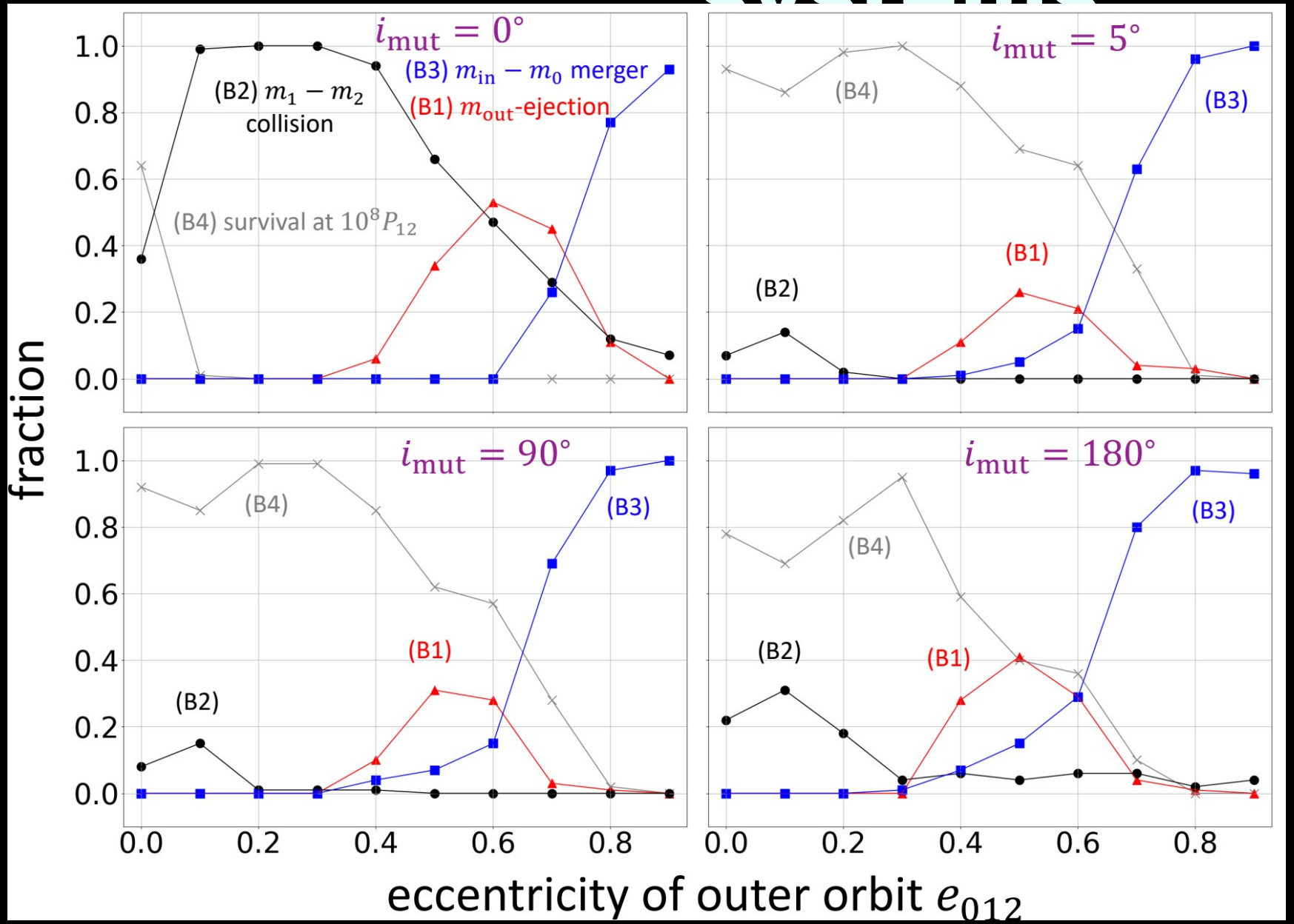
▲ - ejection

■ - merger

× stable <



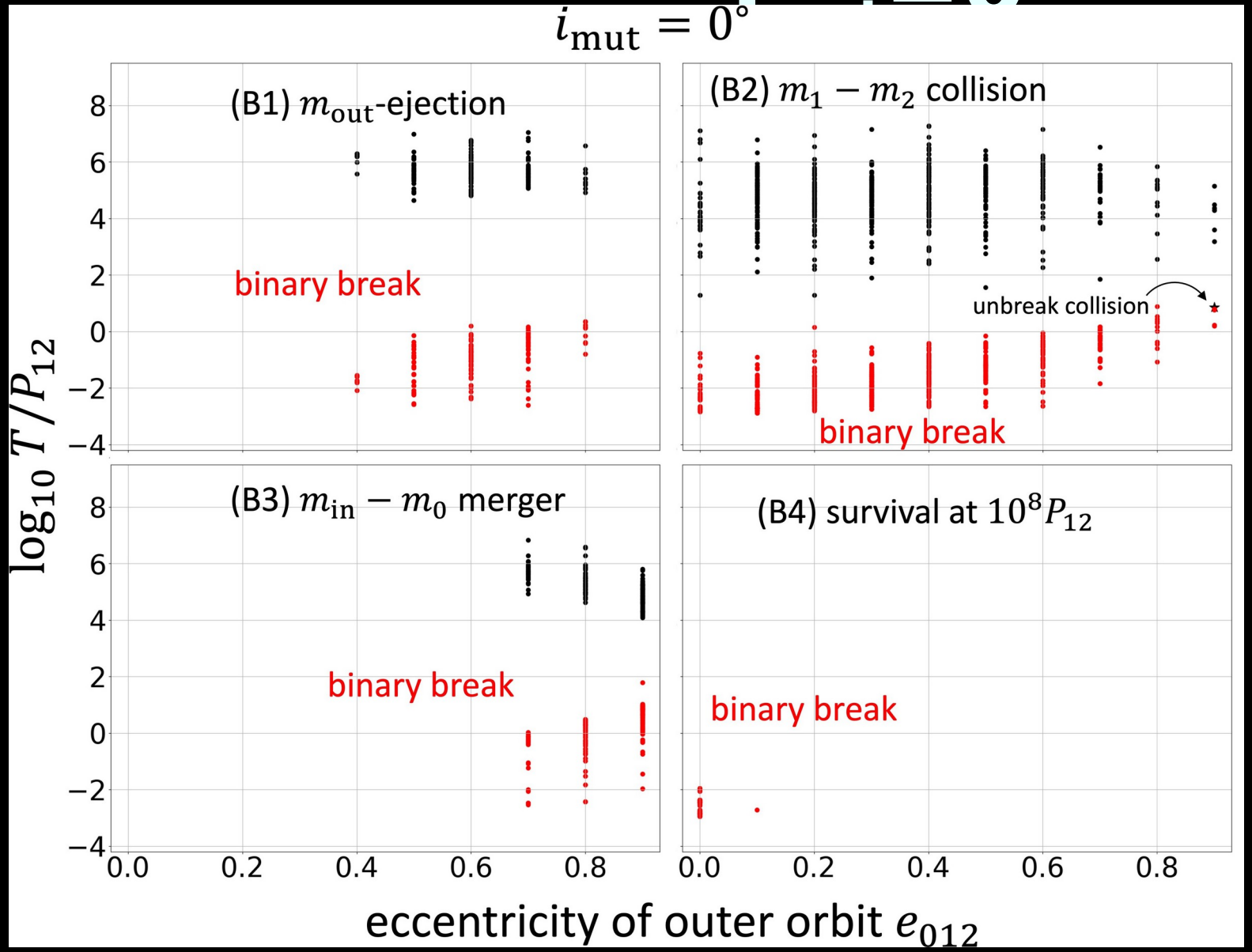
Fraction of different fates of HT-S systems



100 random phase realizations evolved up to $t=10^8\,P_{12}$ for models with and $e_{12}=0.5$ initially

($m_0=10^6 M_\odot$

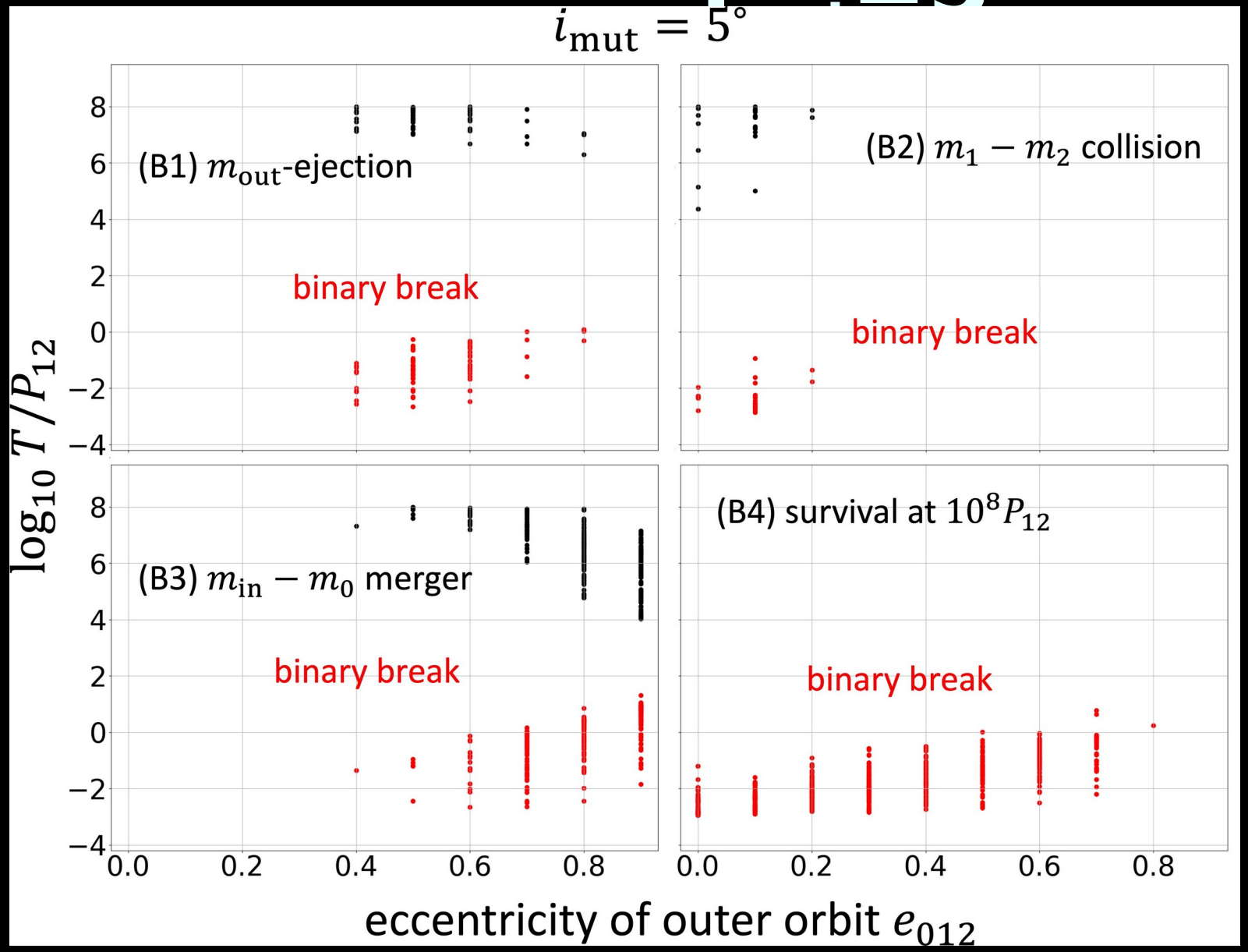
Timescales for different fates : $i_{mut}=0°$



100 random phase realizations evolved up to $t=10^8\,P_{12}$ for models with and $e_{12}=0.5$ initially

($m_0=10^6 M_\odot$

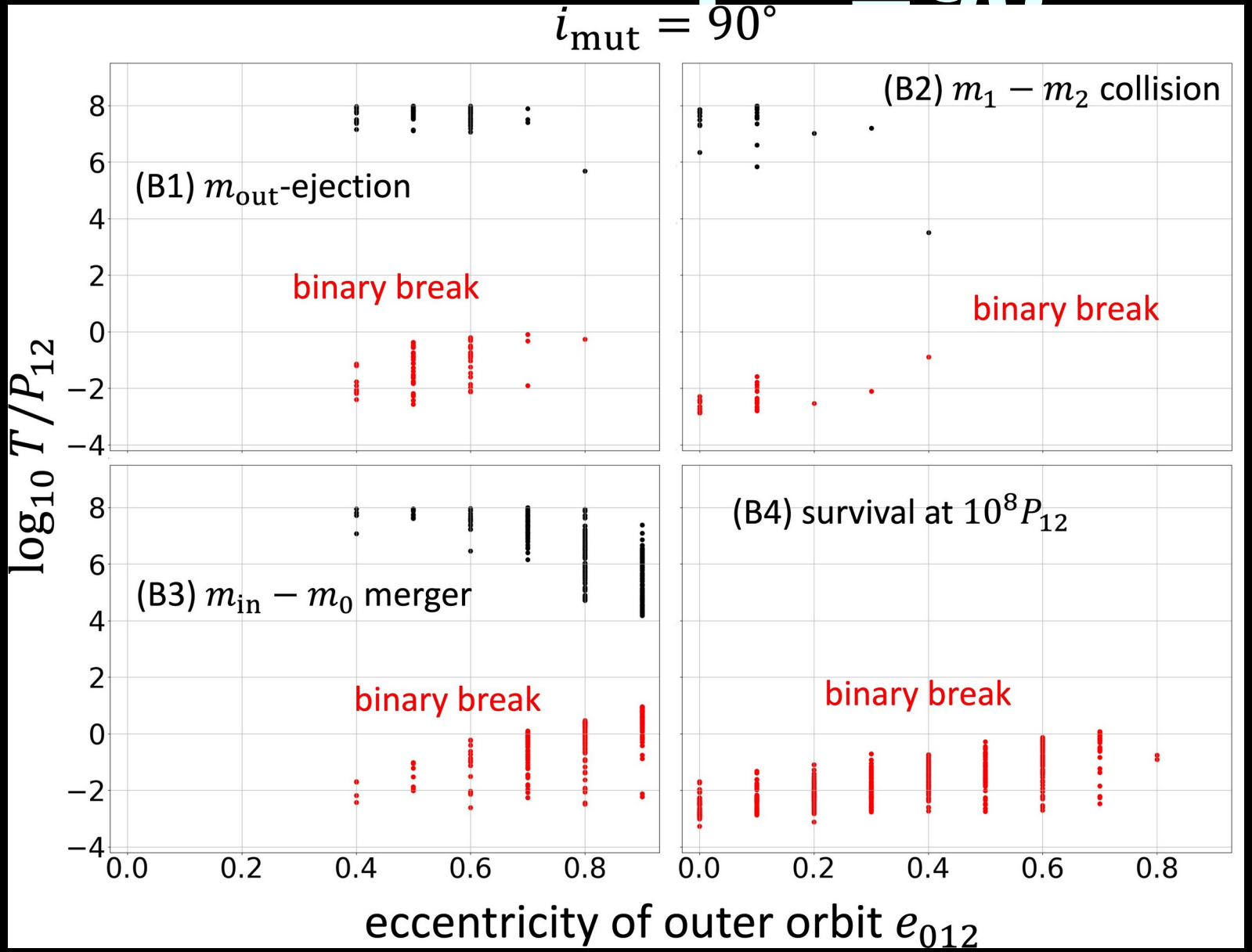
Timescales for different fates : i_{mut}=5°



100 random phase realizations evolved up to $t=10^8\,P_{12}$ for models with and e_{12}=0.5 initially

($m_0=10^6 M_\odot$

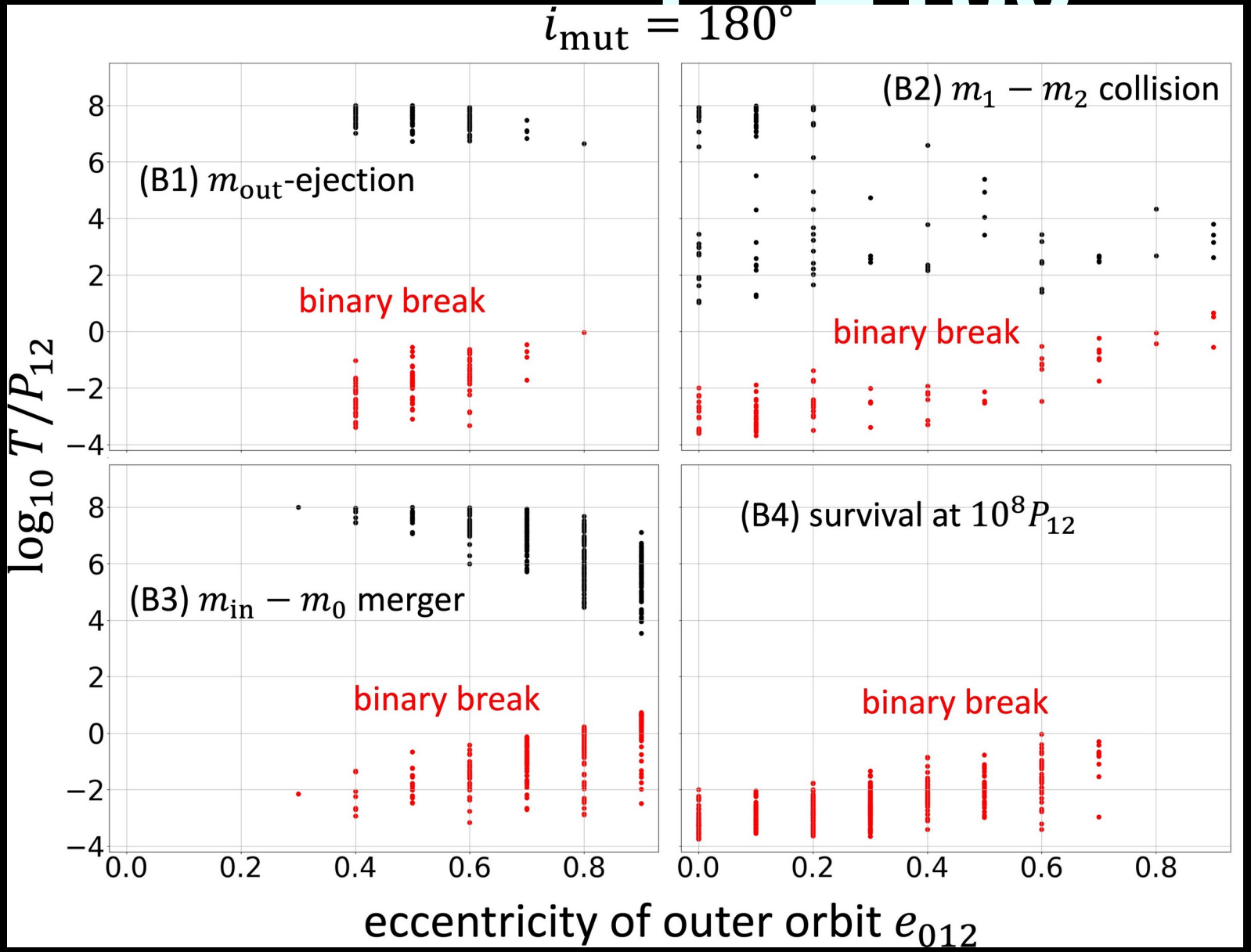
Timescales for different fates : $i_{mut}=90°$



100 random phase realizations evolved up to $t=10^8\,P_{12}$ for models with and $e_{12}=0.5$ initially

($m_0=10^6 M_\odot$

Timescales for different fates : $i_{mut}=180°$



100 random phase realizations evolved up to $t=10^8\,P_{12}$ for models with and $e_{12}=0.5$ initially

($m_0=10^6 M_\odot$

Conclusions

- Improved stability criteria for HT-P (Mardling & Aarseth 2001) and for HT-S (Grishin et al. 2017)
  - incorporated the dependence on eccentricities and mutual inclination of inner and outer orbits
  - sensitive to the timescales in mind (no sharp boundary), especially for inclined and eccentric triple systems
- Fate of the HT triples
  - HT-P: mostly ejection of the initially outer body ⇒ stable binary of the central and initially inner bodies
  - HT-S: quick break-out of the initial binary ⇒ quasi-HT-P type configuration ⇒ after all, one of the initial binary body merges to the central body ⇒ stable binary of the central and the outer bodies

Fate of the HT-S systems

