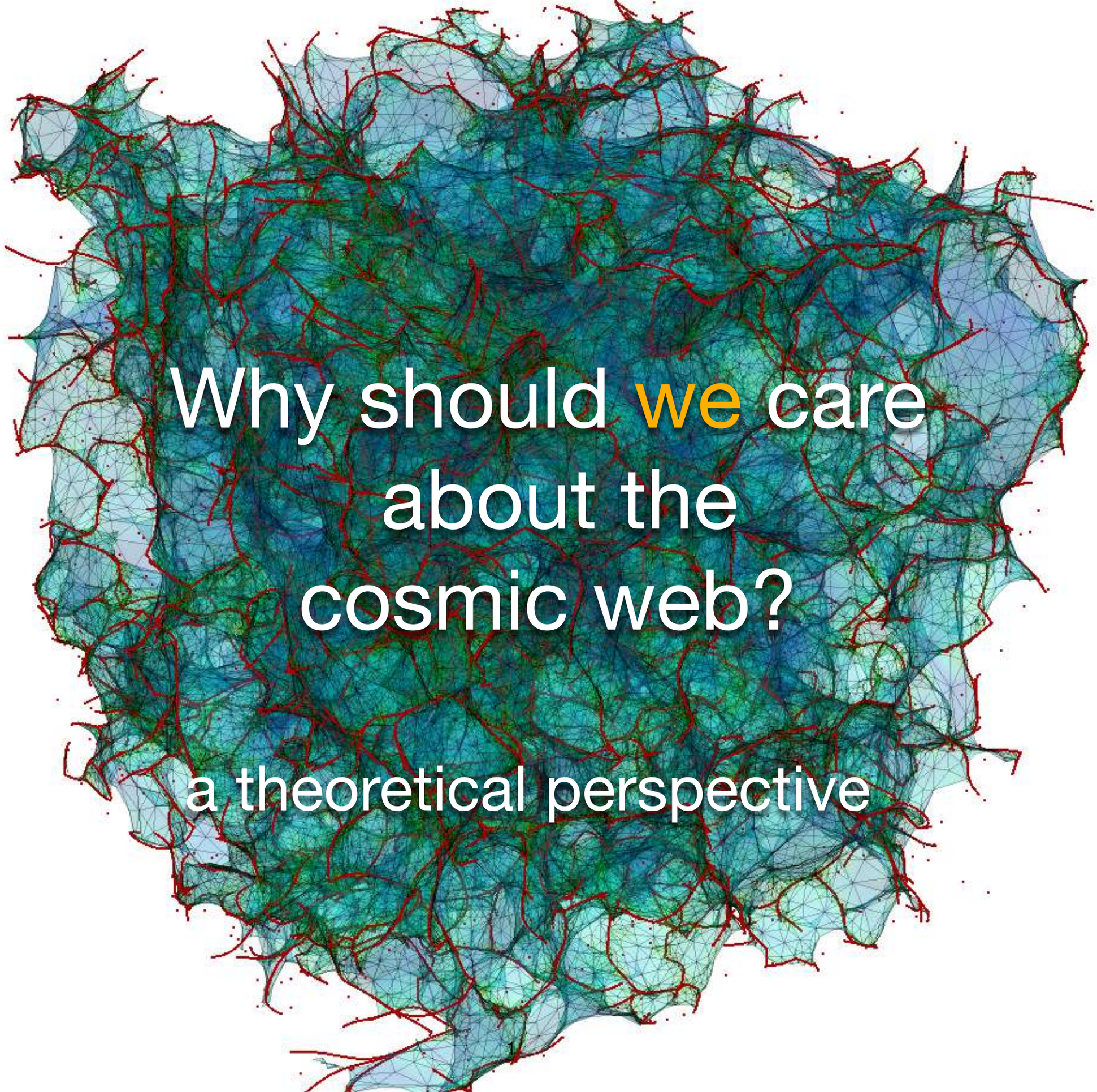
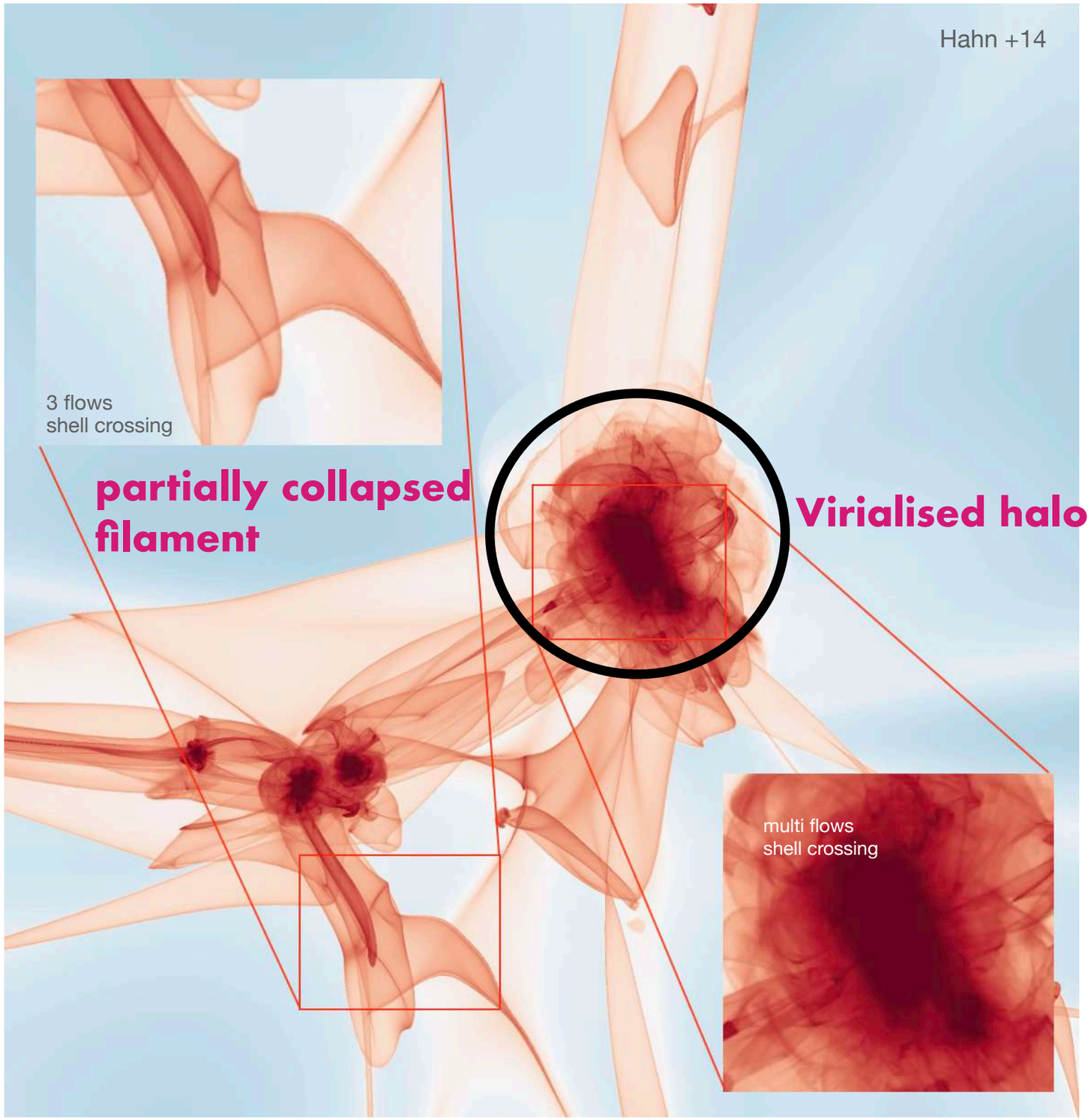


Why should we care about the cosmic web?

a theoretical perspective

1. What is the cosmic web?

The cosmic web is a dynamically relevant intermediate-density boundary between cosmology and galaxy formation.



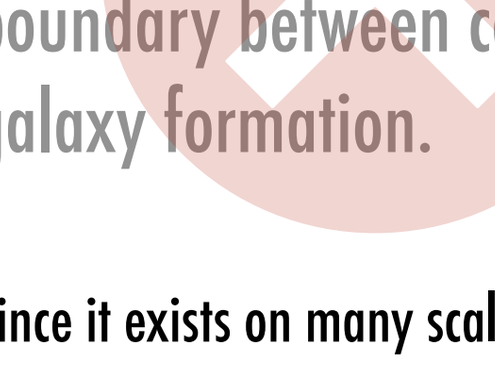
When halo collapse, neighbouring filaments+walls are in place.

1. What is the cosmic web?

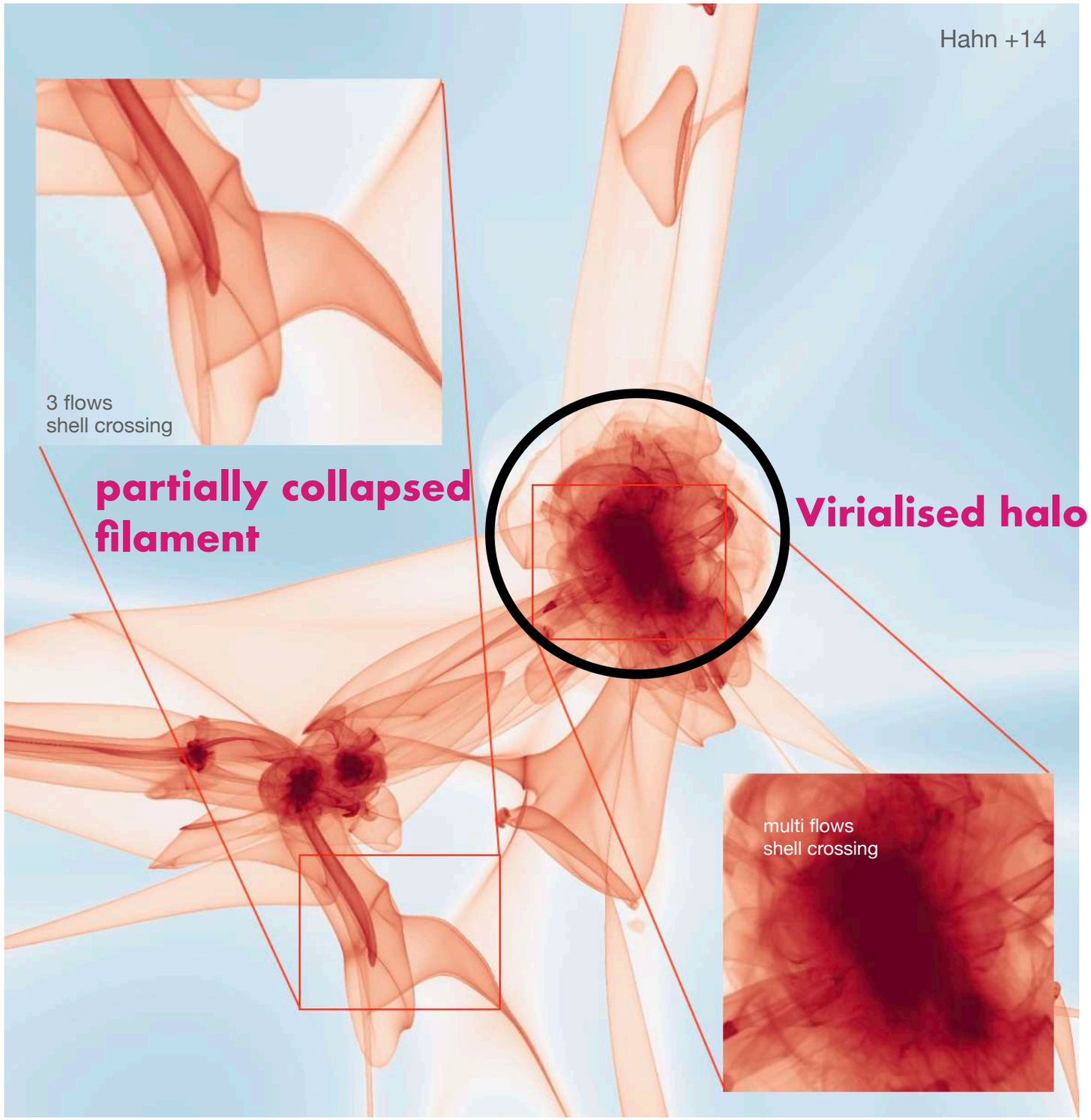
The cosmic web is a dynamically relevant intermediate-density boundary between cosmology and galaxy formation.

Since it exists on many scales

The cosmic web is a dynamically relevant anisotropic (=spin 2) boundary between a given scale and a larger scale.



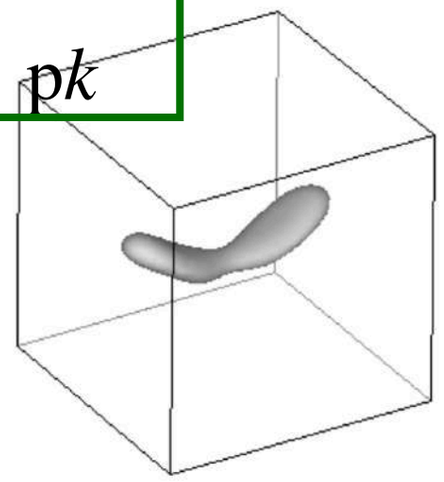




We must consider peaks rigged = dressed by their sets of (wall + filament) saddle critical pts.

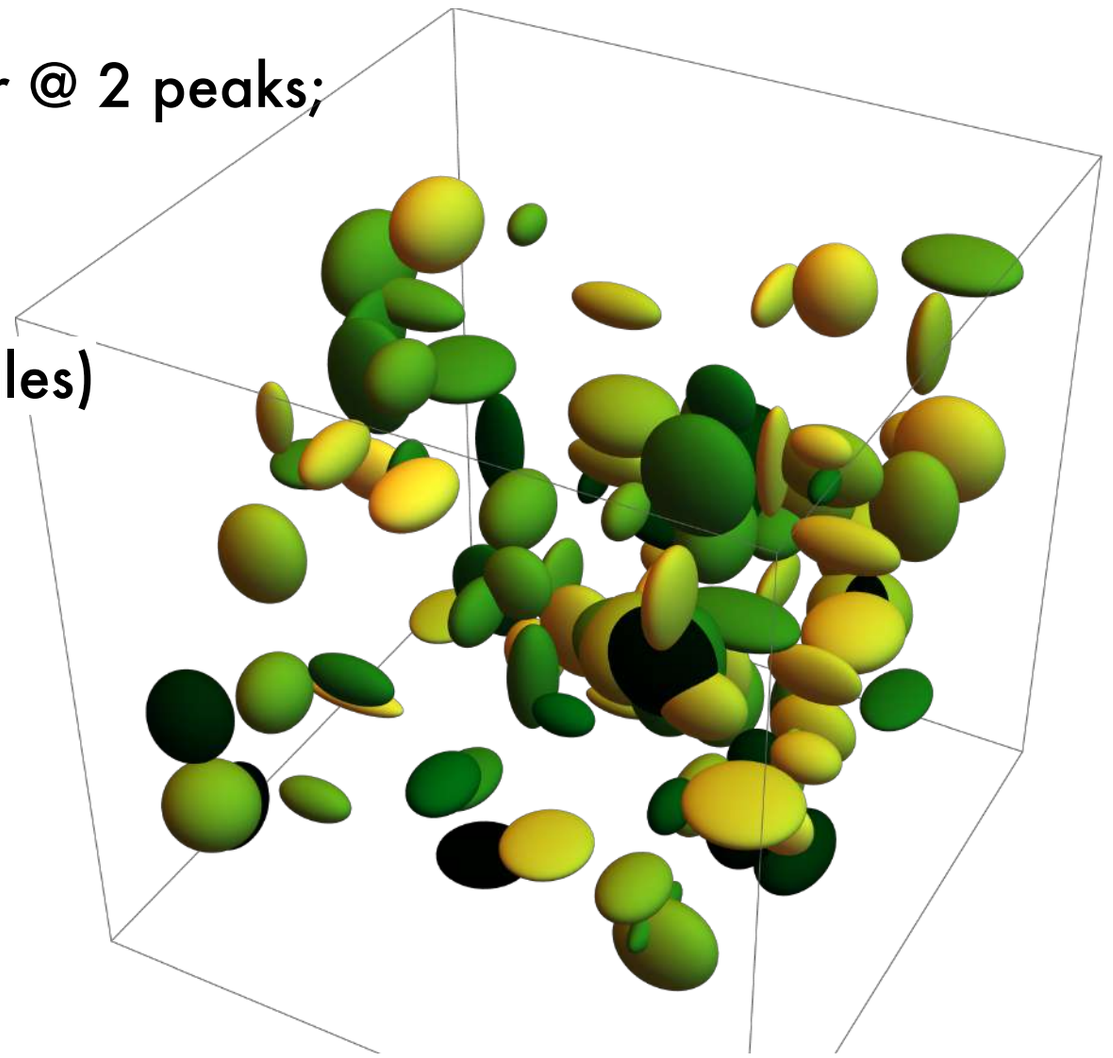
1. What is the cosmic web? a spin-2 two-point process

$$\textbf{cosmic web} = \text{alignment of eigframes} \left[\frac{\partial^2 \bar{\psi}}{\partial x_i \partial x_j} \right]_{pk}$$

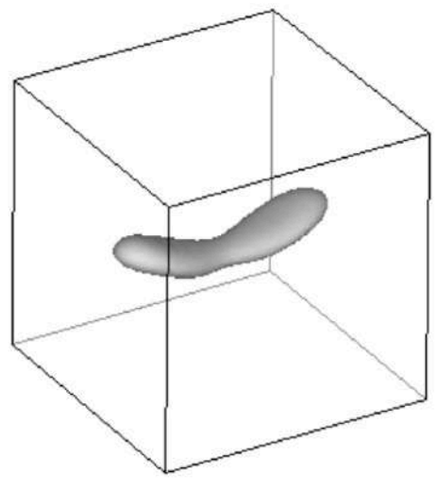


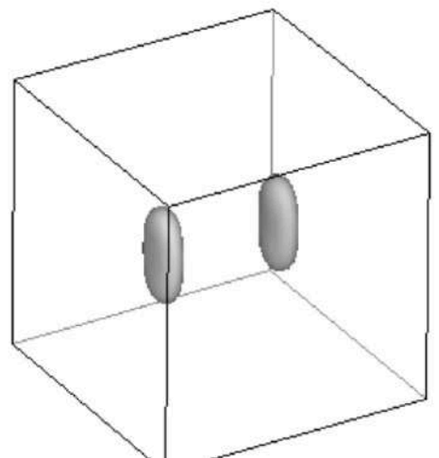
- Tides are longer range (than density) when aligned with something

BKP96: alignment of shear tensor @ 2 peaks;



- Predicts LSS in ICs (on large scales)
  unexpected result in 96
- Applicable on most scales
  important for this talk



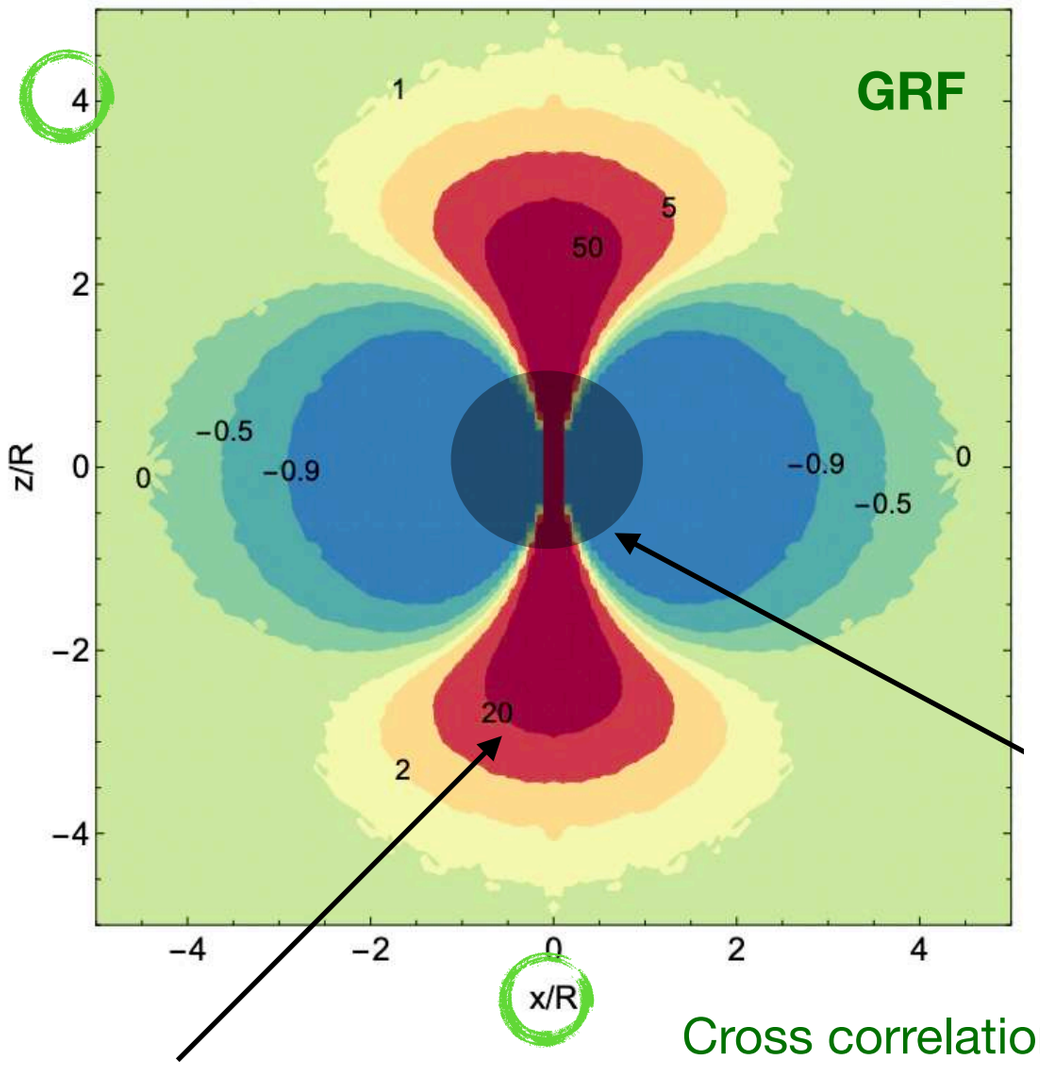


alignment → high degree of constructive interference → bridge

1. What is the cosmic web? a spin-2 one-point process

$$\textbf{cosmic web} \approx \text{metric set by eigframe} \left[\frac{\partial^2 \rho}{\partial x_i \partial x_j} \right]_{\text{sad}}$$

More recently, alignment w.r.t. (filament or wall) saddle eigen-frame = spin-2 one-point process.



upshot of talk: similar maps for

- tidal torque theory
- excursion set theory
- critical event theory
- morphology theory

Correlation zone of saddle

Cross correlation of peaks relative to a given saddle

long range: one should consider peaks dressed by neighbouring critical pts.

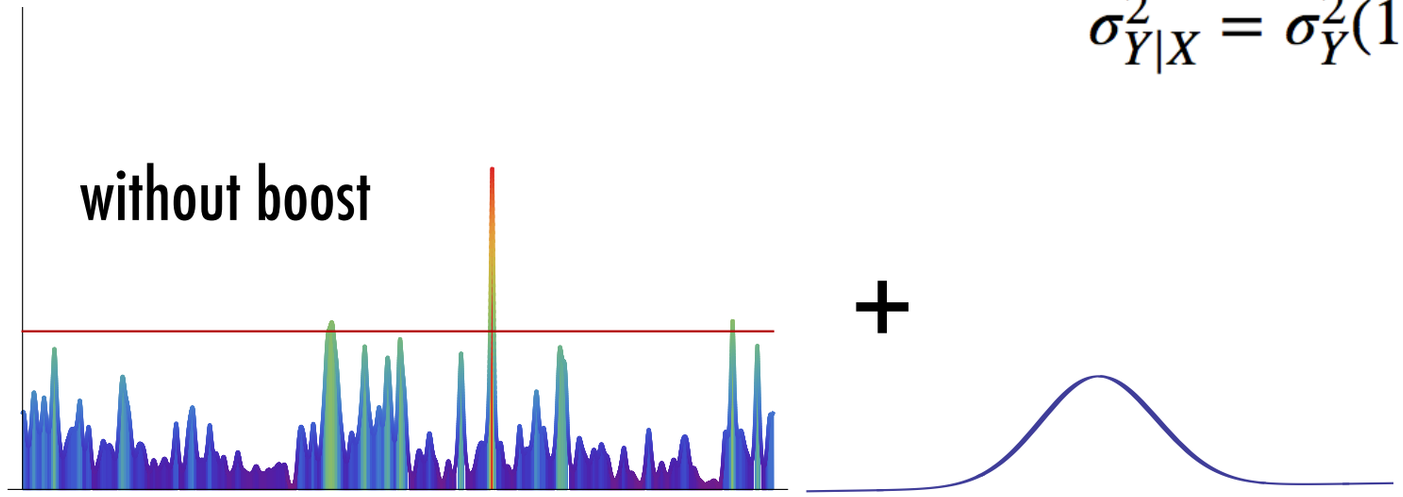
1. What is the cosmic web? a spin-2 one-point process

$$\textbf{cosmic web} \approx \text{metric set by eigframe} \left[\frac{\partial^2 \rho}{\partial x_i \partial x_j}\right]_{\text{sad}}$$

- partial alignment will change (=bias) anisotropically the mean and variance of things → specific signature of CW

$$E(Y|x) = \mu_Y + \rho\frac{\sigma_Y}{\sigma_X}(x - \mu_X)$$

$$\sigma^2_{Y|X} = \sigma^2_Y(1 - \rho^2)$$

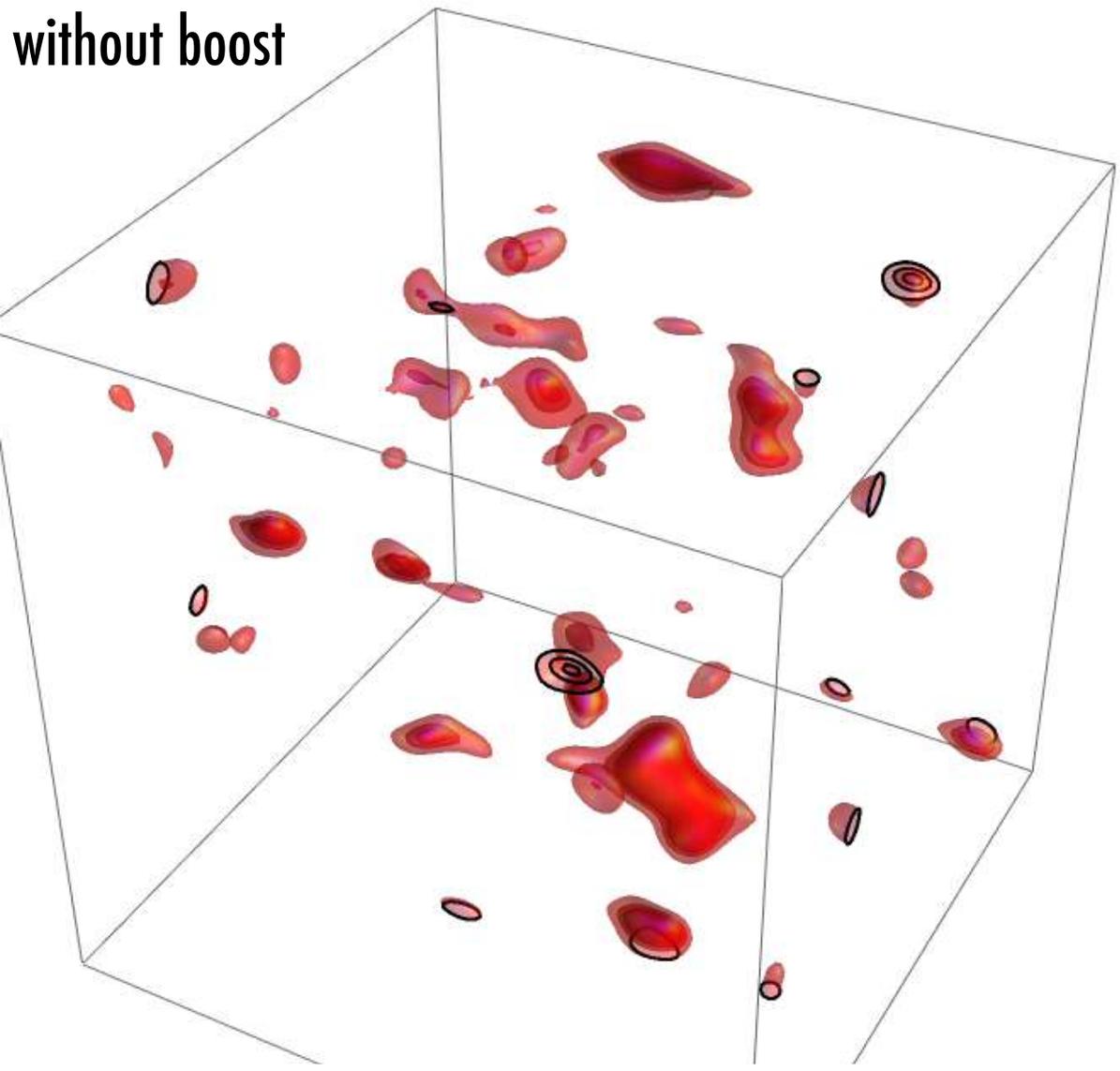


spin 0 pt constraint (=density) → isotropy (spherical collapse);

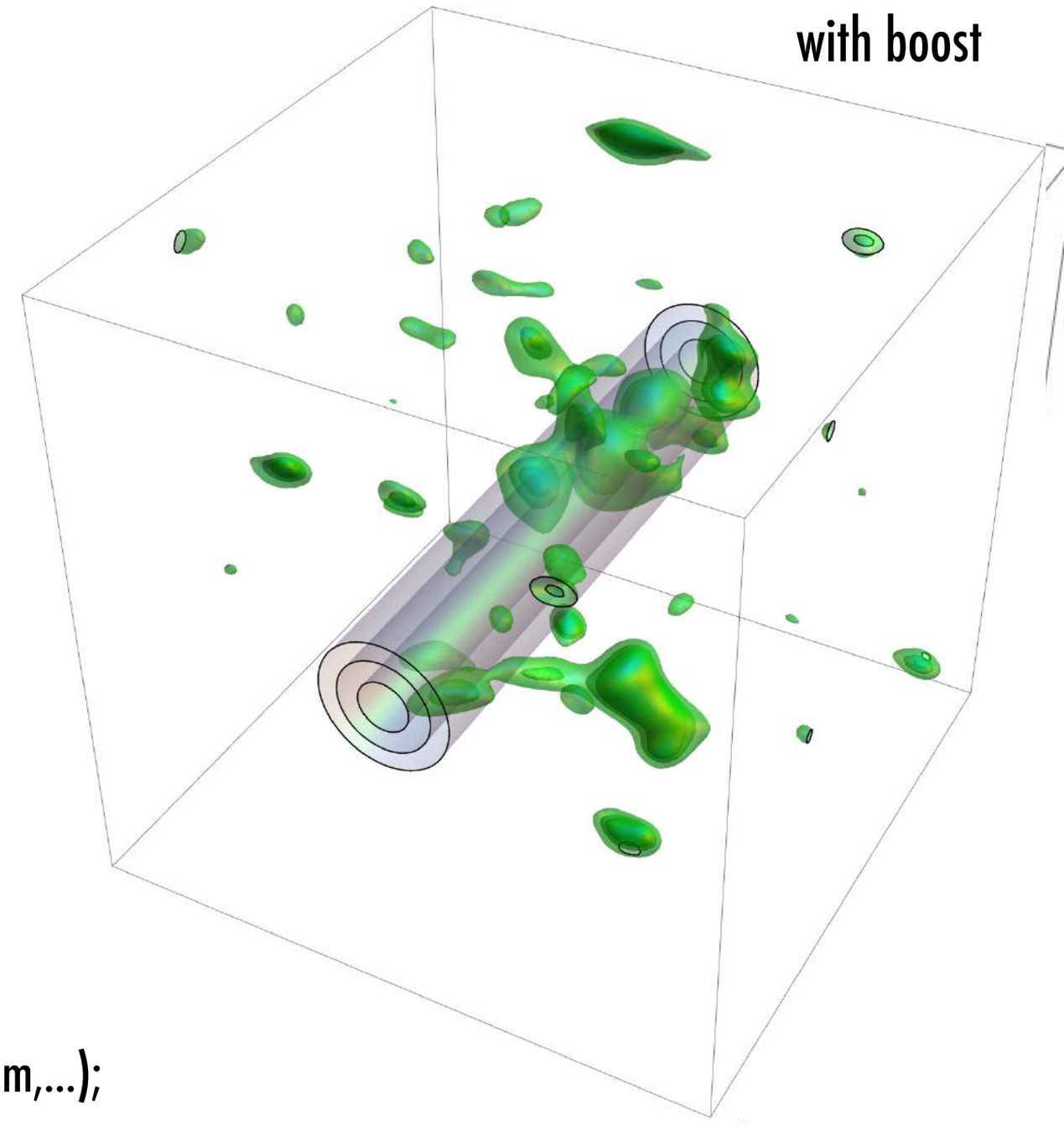
1. Kaiser bias on cosmic web

- partial alignment will change (=bias) anisotropically the mean and variance of things → specific signature of CW

without boost



with boost



spin 0 pt constraint (=density) → isotropy (spherical collapse);

spin 2 pt contraint (= CW) → anisotropy (e.g. Angular momentum,...);

1. What is the cosmic web? A spin 2 point process definition

$$\textbf{cosmic web} \approx \text{metric set by eigframe} \left[\frac{\partial^2 \rho}{\partial x_i \partial x_j} \right]_{\text{sad}}$$

- partial alignment will change (=bias) anisotropically the mean and variance of things = specific signature of CW

$$E(Y|x) = \mu_Y + \rho \frac{\sigma_Y}{\sigma_X}(x - \mu_X)$$

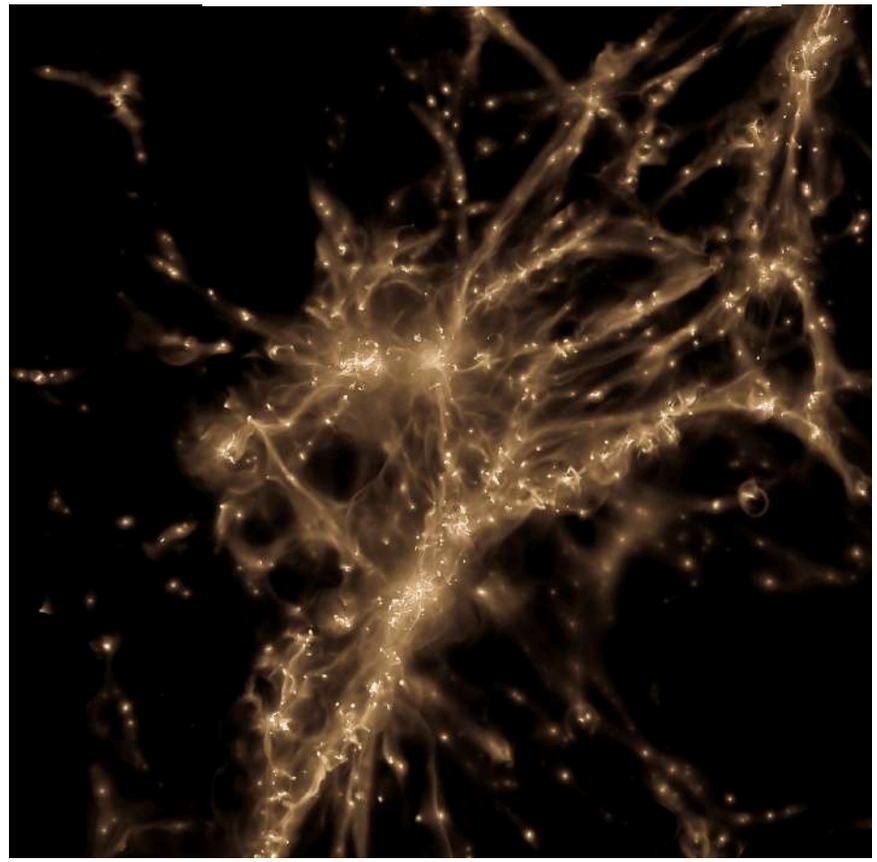
$$\sigma^2_{Y|X} = \sigma^2_Y(1 - \rho^2)$$

revisit

- tidal torque theory
- excursion set theory
- critical event theory
- morphology theory

} Sometimes **small** for DM

subject to saddle

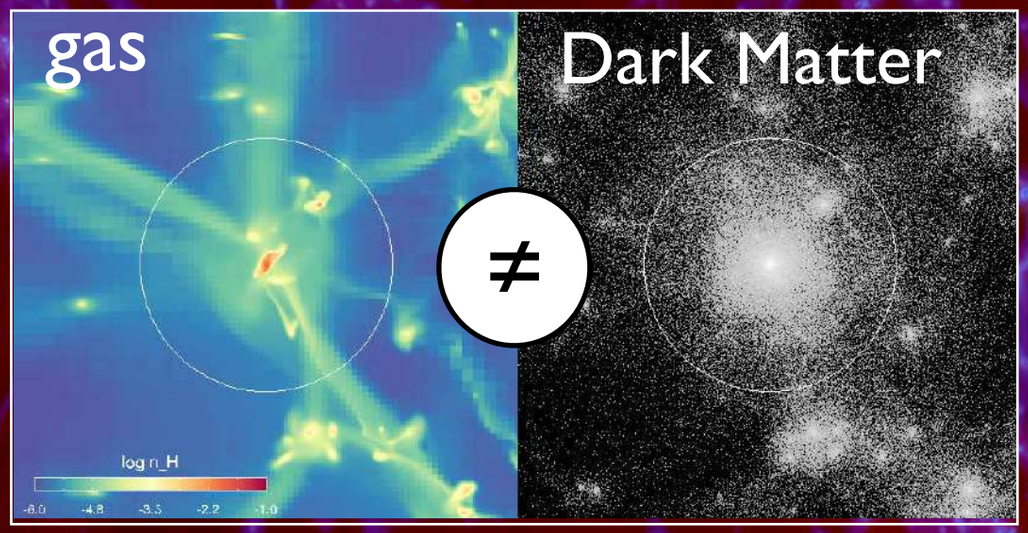


BUT It really matters for baryons

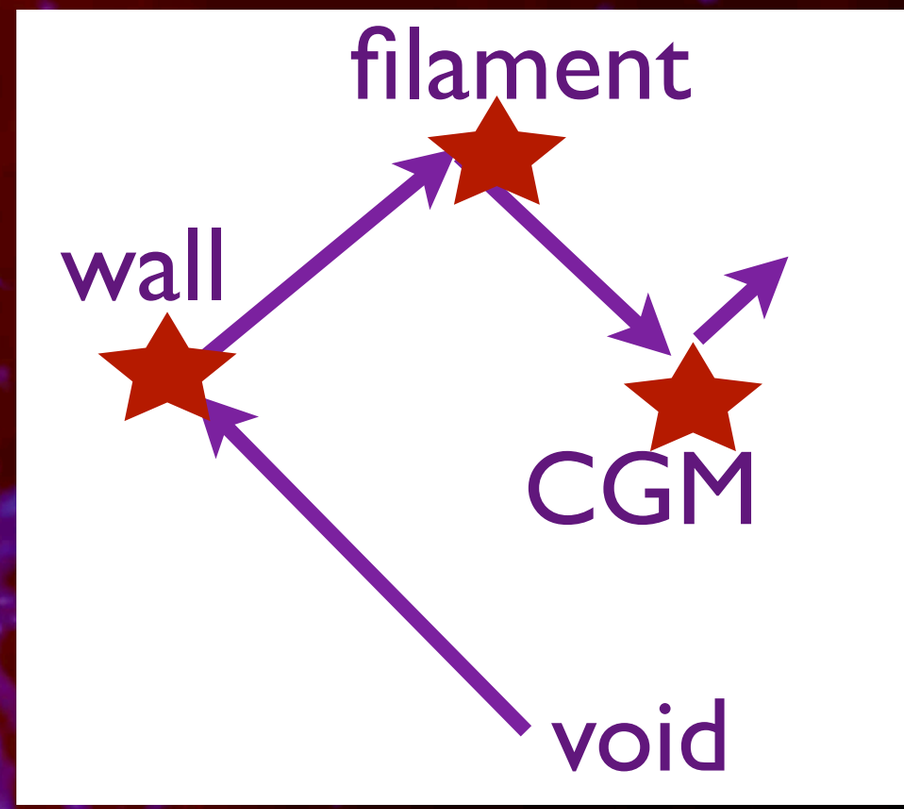
alignments funnel gas along CW : small scales inherit coherence and stability

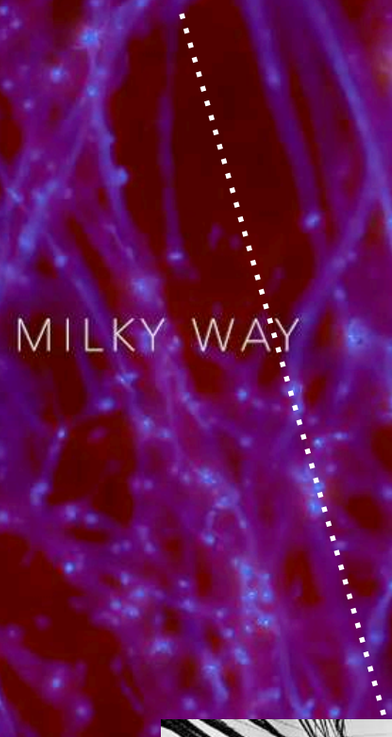
1. The impact of shocks in gaseous cosmic web

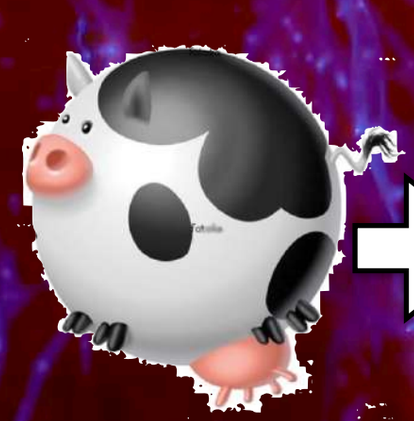
CW drives secondary infall :



$$t_{\rm dyn} \sim 1/\sqrt{\rho}$$







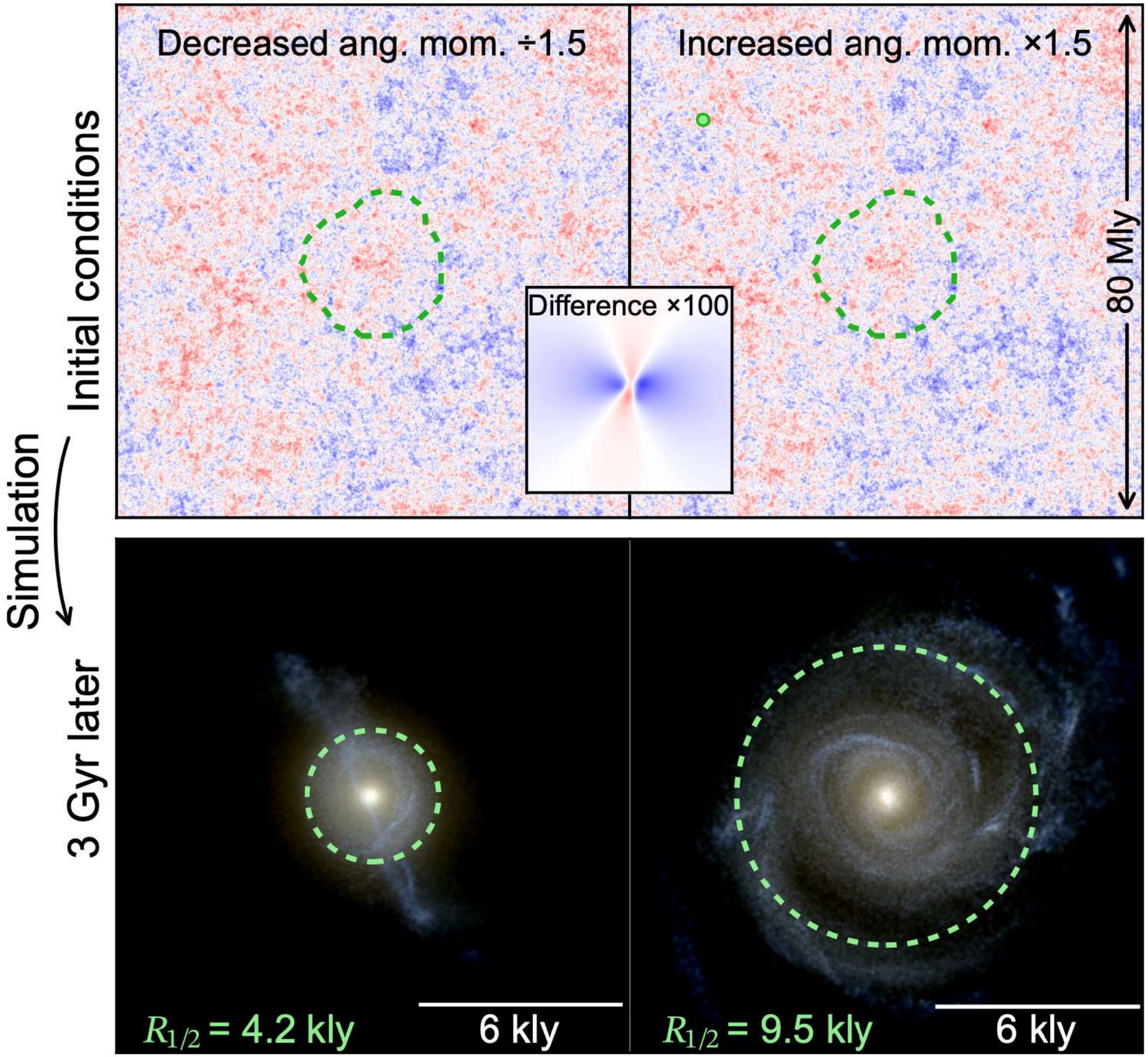




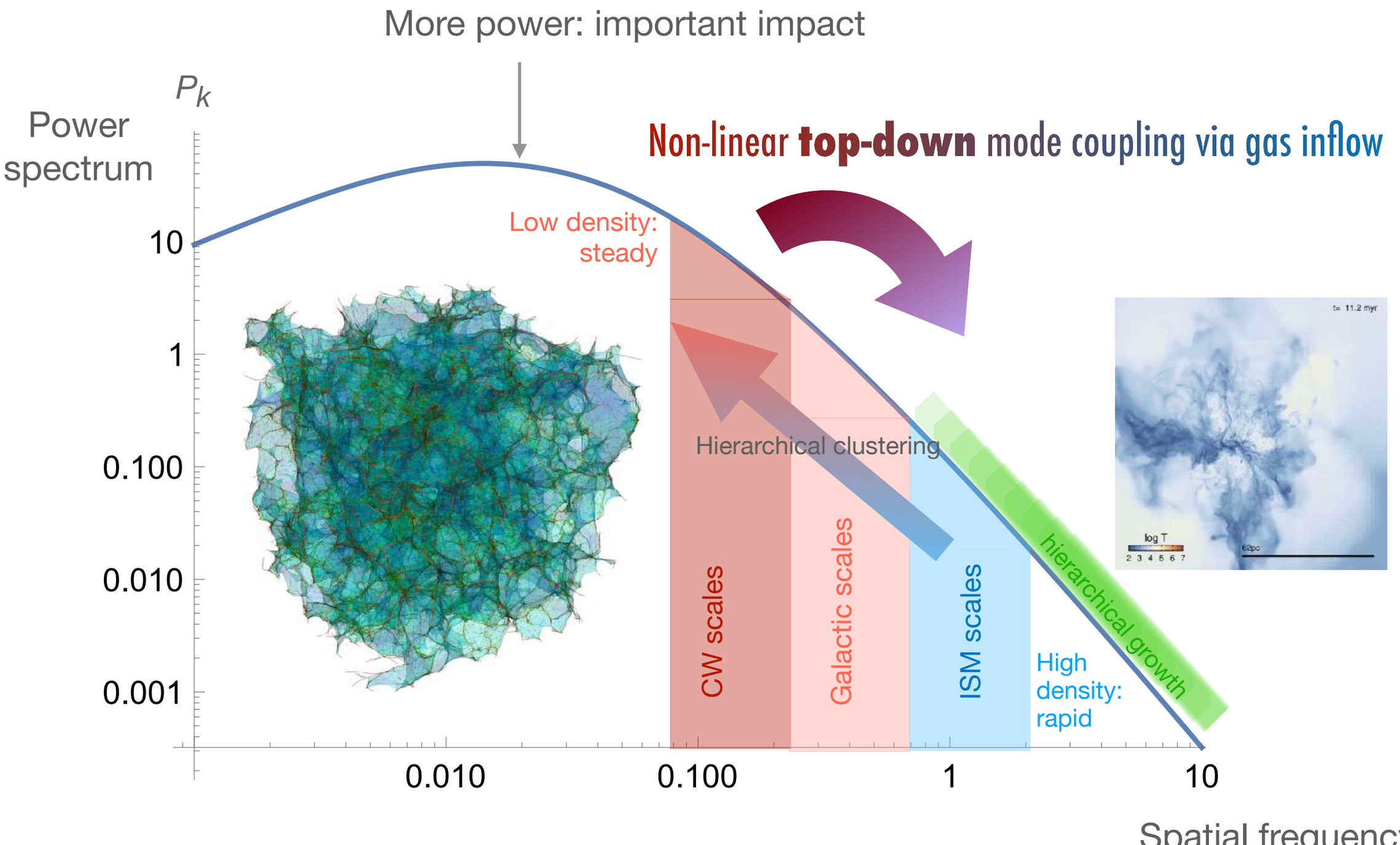
Disks (re)form because LSS are large (dynamically young)
and (partially) an-isotropic :
they induce persistent angular momentum advection of gas along filaments which stratifies accordingly.

Galaxies are multi-scale non-linearly asymmetrically coupled systems

Cadiou et al 2023



On galactic scales, the Shape of initial P_k is such that galaxies inherit stability from LSS via cold flows

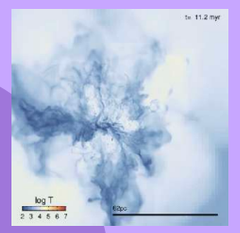


Galaxies are multi-scale non-linearly asymmetrically coupled systems

Non-linear mode coupling brings coherence from LS to MS

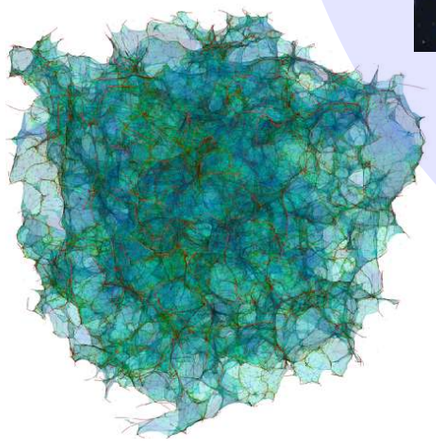
$$t_{\rm dyn} = \frac{1}{\sqrt{\rho}_{\rm LSS}} \gg 1$$

$$t_{\rm dyn} = \frac{1}{\sqrt{\rho}_{\rm gal}} \sim 1$$



$$t_{\rm dyn} = \frac{1}{\sqrt{\rho}_{\rm ISM}} \ll 1$$





Non-linear mode coupling brings stochasticity from SS to MS

Why (naive) subgrid physics is a bad idea…

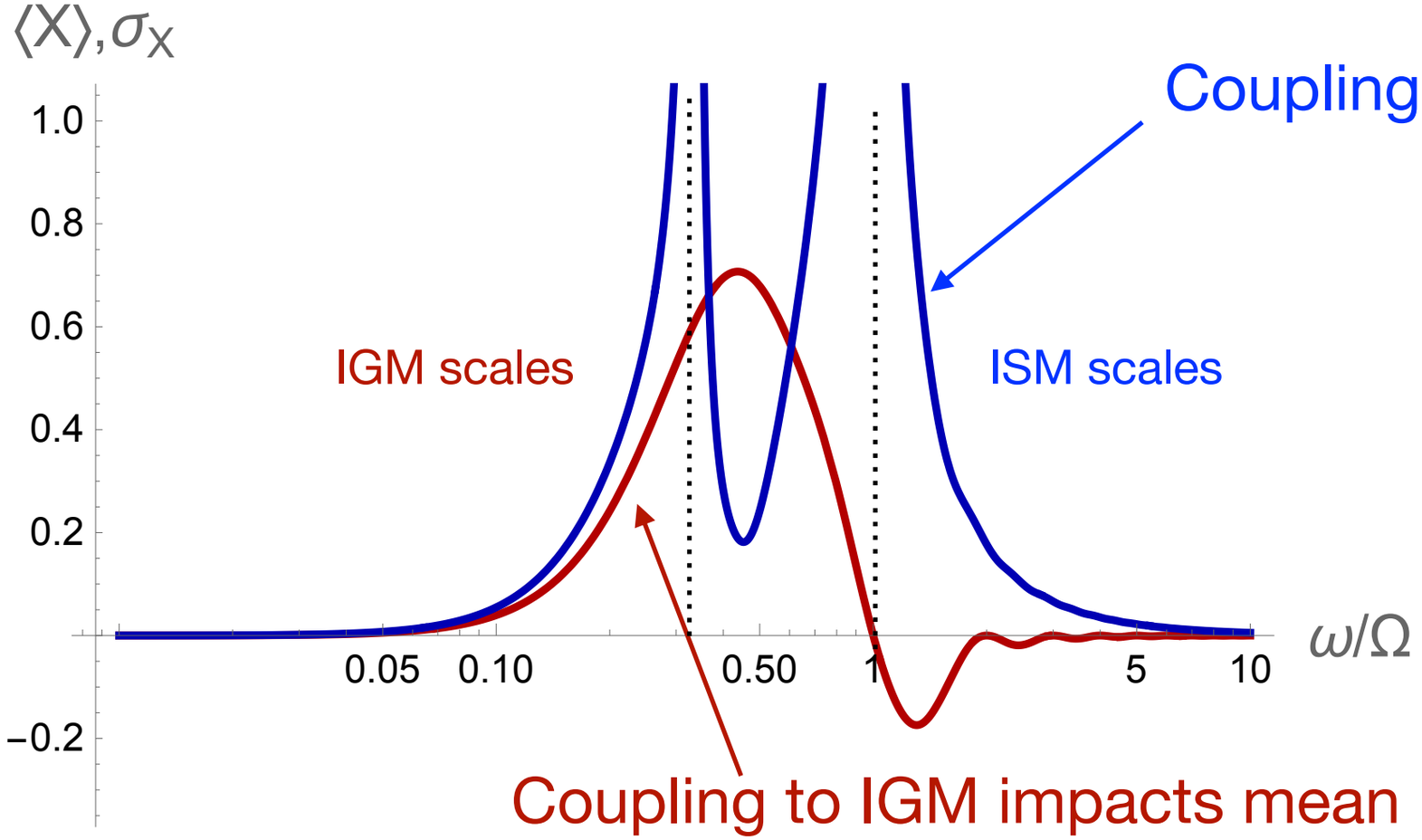
Galaxies are multi-scale non-linearly asymmetrically coupled systems

$$\ddot{X} + \Omega^2 X = \epsilon X_{\rm env}^3 \quad \text{given} \quad \ddot{X}_{\rm env} + \omega^2 X_{\rm env} = 0$$

Natural frequency of galaxy

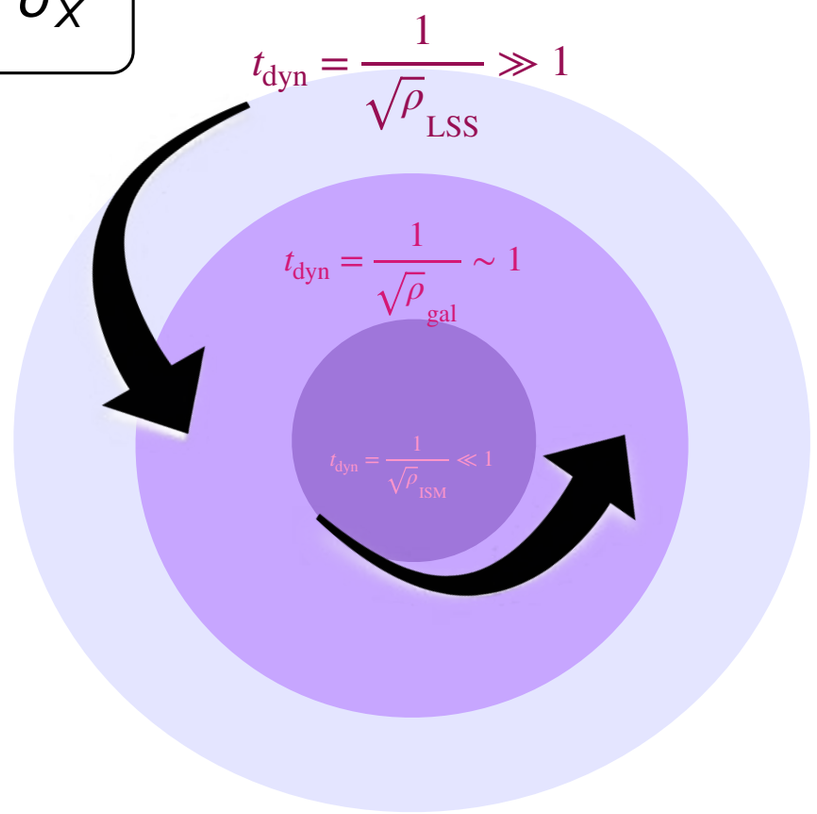
perturbative coupling to environment

Natural frequency of environment

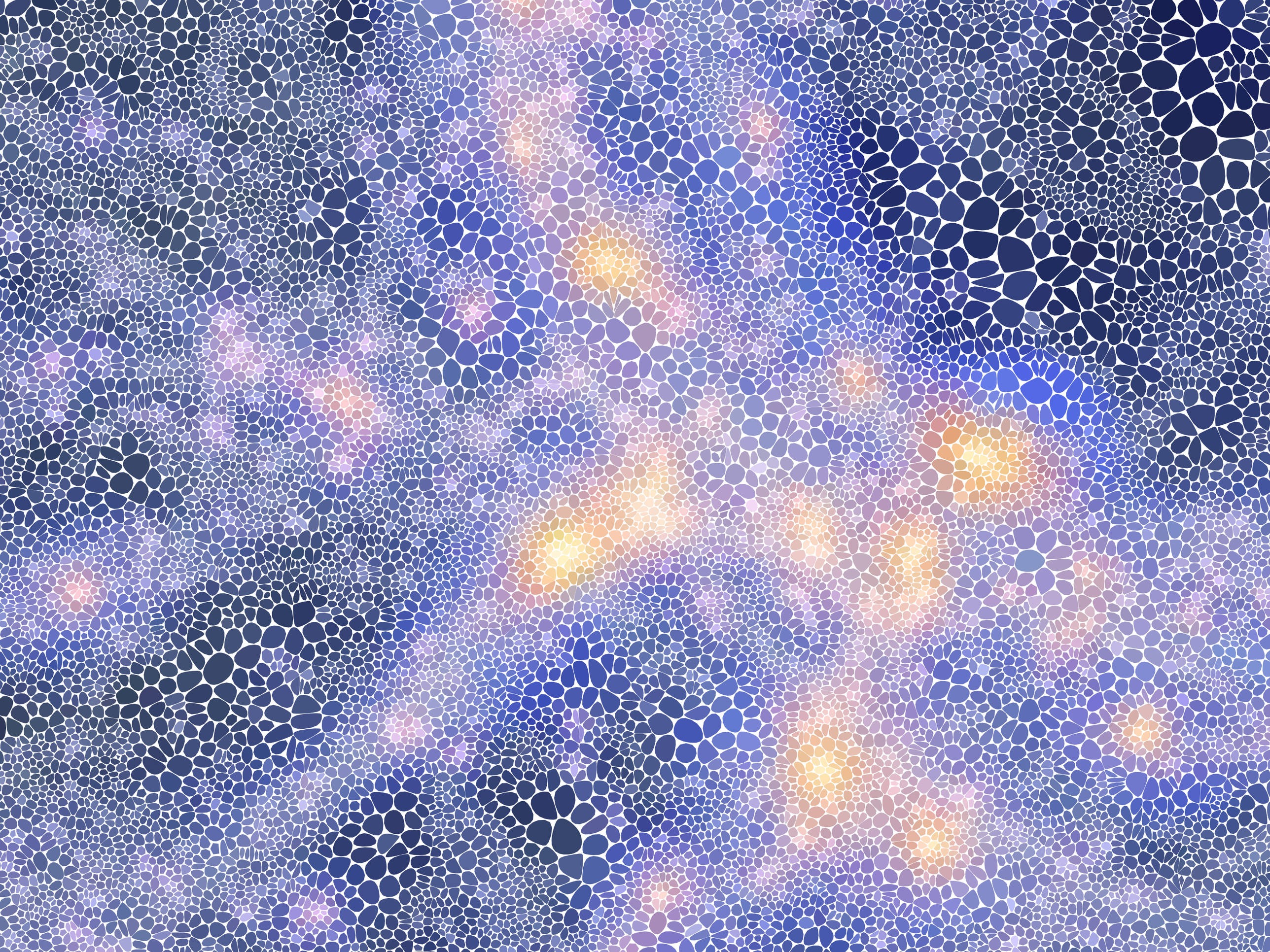


$$\langle X \rangle = \frac{\sin^2(3\pi\omega)}{12\pi\omega(3\omega-1)(3\omega+1)} - \frac{3\sin^2(\pi\omega)}{4\pi(\omega-1)\omega(\omega+1)}$$

$$\sigma_X = \frac{\sin^4(\pi\omega)\left(-75\omega^2+8\left(\omega^2-1\right)\cos(2\pi\omega)+4\left(\omega^2-1\right)\cos(4\pi\omega)+3\right)}{16\pi^2\omega^2\left(\omega^2-1\right)^2\left(9\omega^2-1\right)} - \frac{8\sin^4(3\pi\omega)+3\pi\omega\sin(12\pi\omega)}{1152\pi^2\omega^2\left(1-9\omega^2\right)^2} + \frac{8\pi\omega\left(365\omega^4-82\omega^2+5\right)-3\left(261\omega^4-74\omega^2+5\right)\sin(4\pi\omega)+3\left(9\omega^4-10\omega^2+1\right)\sin(8\pi\omega)}{128\pi\omega\left(9\omega^4-10\omega^2+1\right)^2}$$



Why (naive) subgrid physics is a bad idea…



2.1 Revisiting tidal torque theory subject to CW

revisit

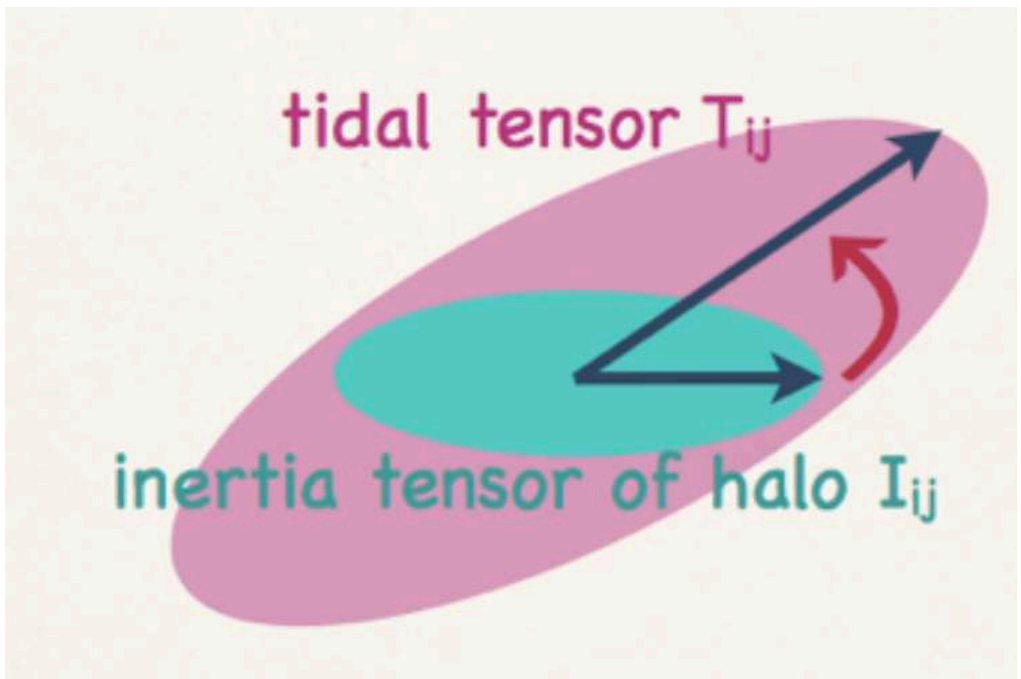
- tidal torque theory
- excursion set theory
- critical event theory
- morphology theory

- CW metric changes (=biases) anisotropically the mean and variance of tides = specific signature of CW
- implication for halo spin orientation?

Tidal torque theory reflects the mis alignment of two tensors on different scales

Angular momentum = anti symmetric contraction of two tensors

$$L_k = \epsilon_{ijk} I_{kl} \psi_{,lj}$$



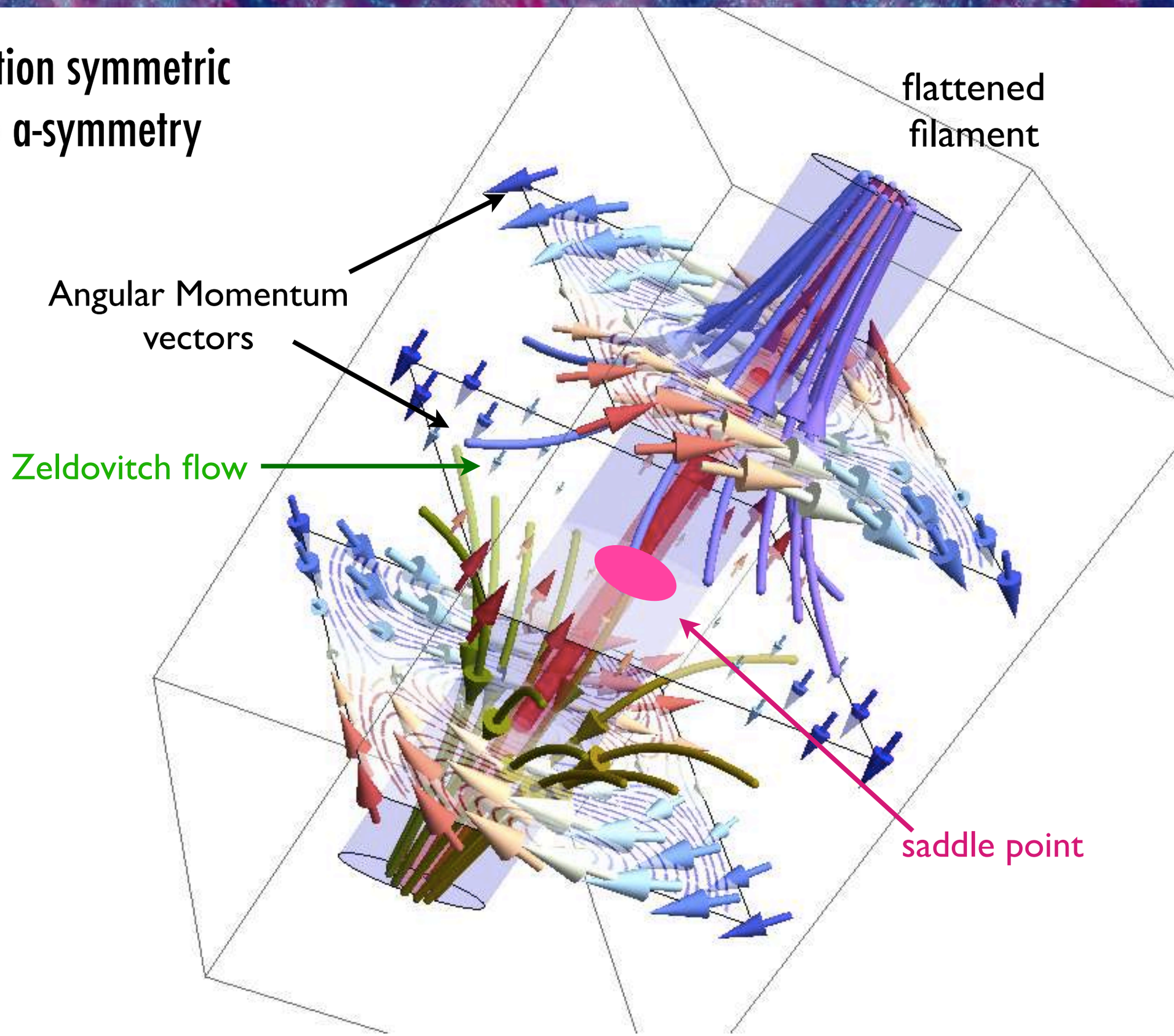
2.1 Revisiting tidal torque theory subject to CW

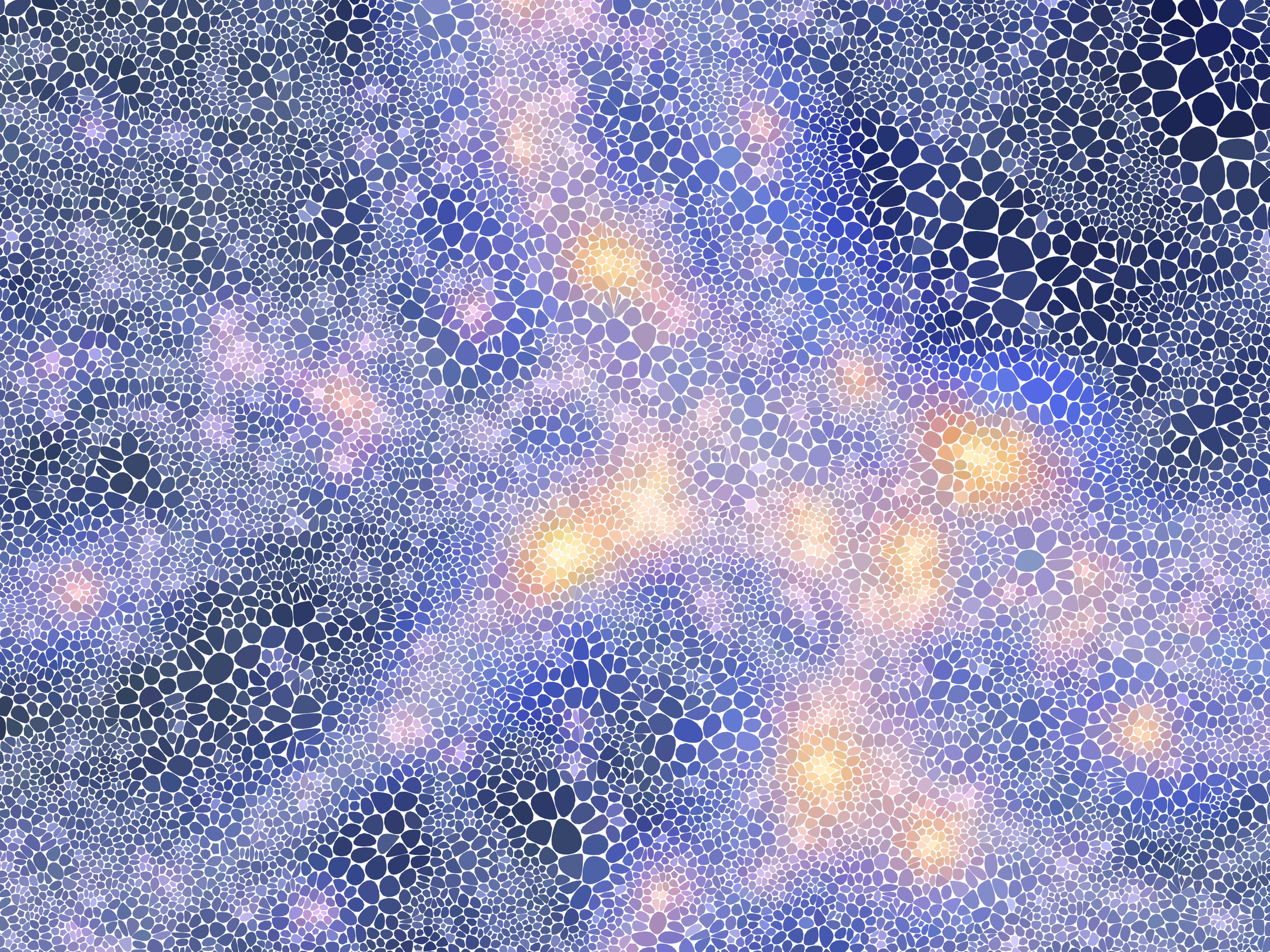
- point reflection symmetric
- vanish if no α-symmetry

perp. = along e_Φ

spin //
to filament

perp. = along e_Φ





2.2 Revisiting (up-crossing) excursion set theory subject to CW

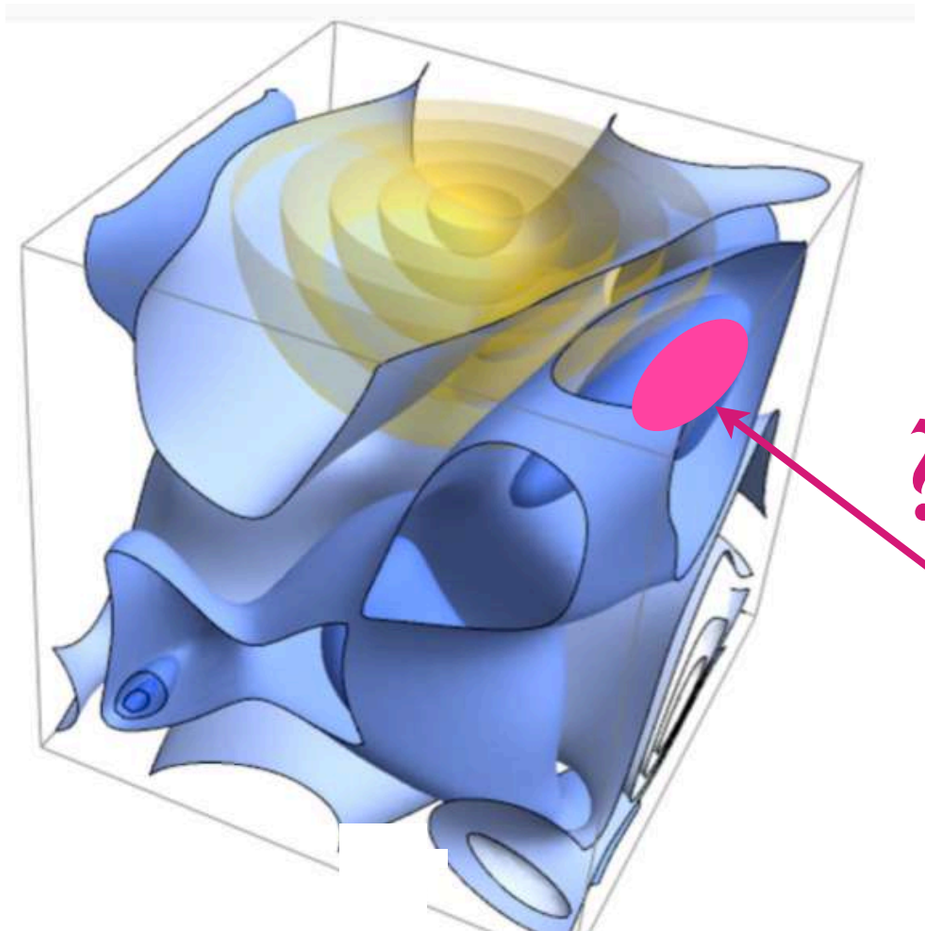
revisit

- tidal torque theory
- excursion set theory
- critical event theory
- mophology theory

- CW metric changes (=biases) anisotropically the excursion = specific signature of CW
- implication for diversity of accretion rate at fixed mass?

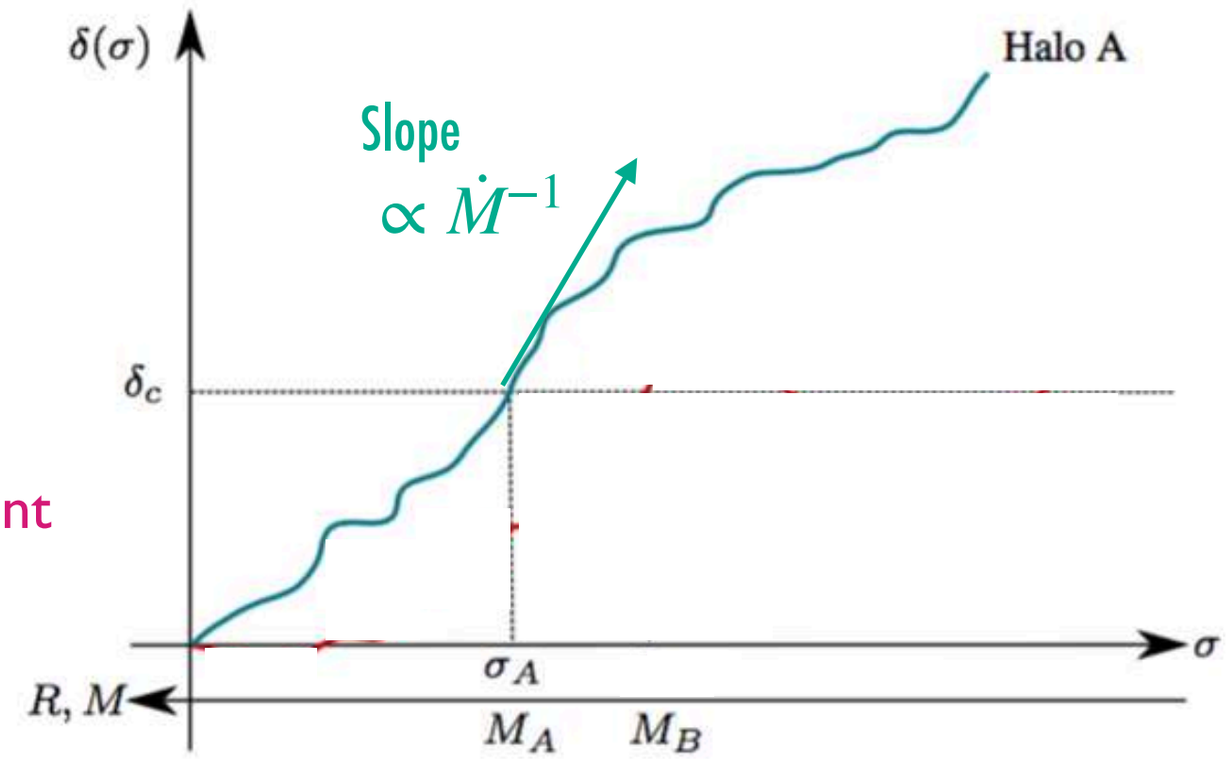
Excursion set theory quantifies barrier crossing

$$\mathcal{P}(\delta, \partial_R \delta | \mathcal{S}addle)$$



?

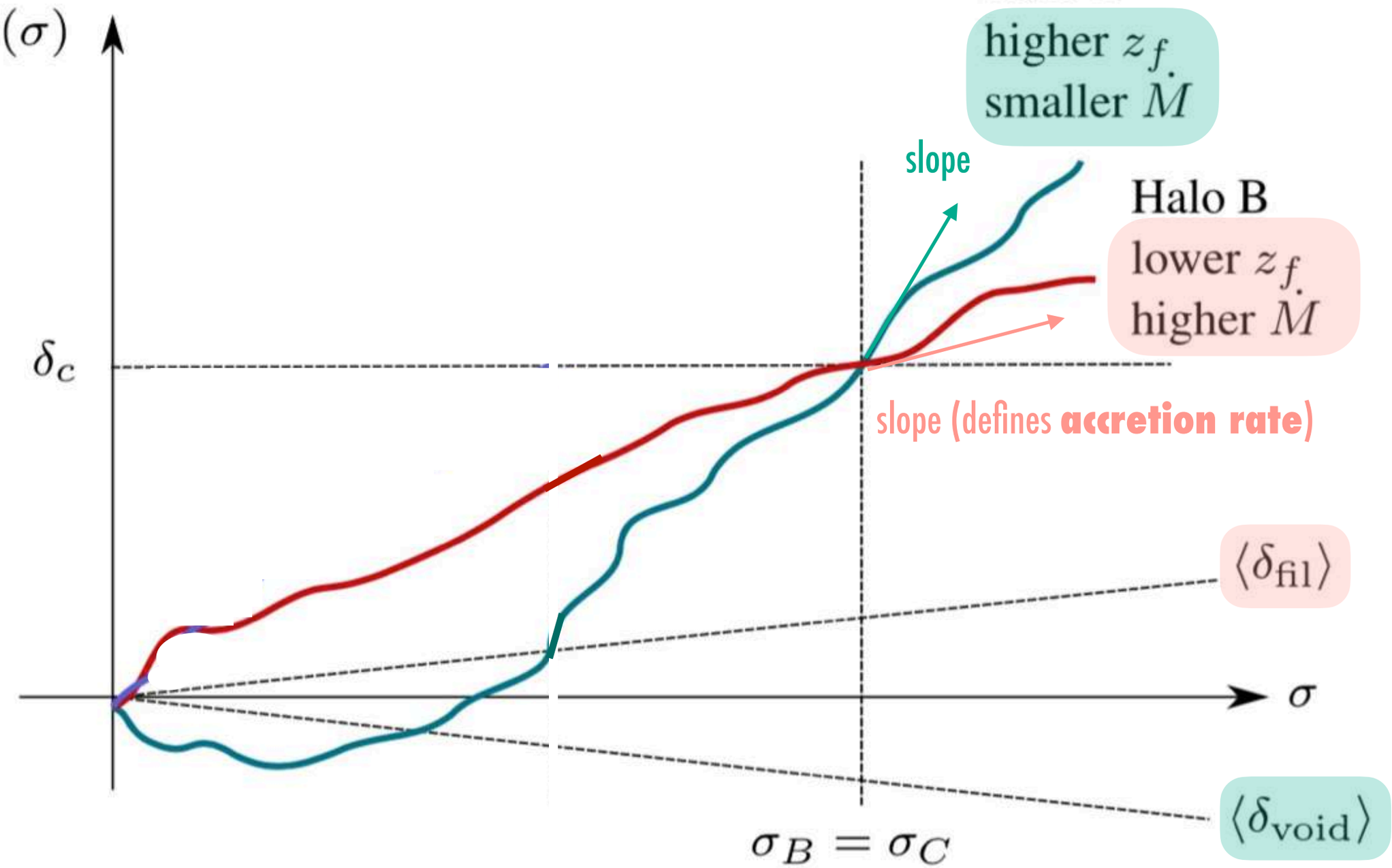
saddle point



2.2 Revisiting (up-crossing) excursion set theory subject to CW

$$\mathcal{P}(\delta, \partial_R \delta | \mathcal{S}addle)$$

set of paths (=excursion) compatible with saddle tidal field

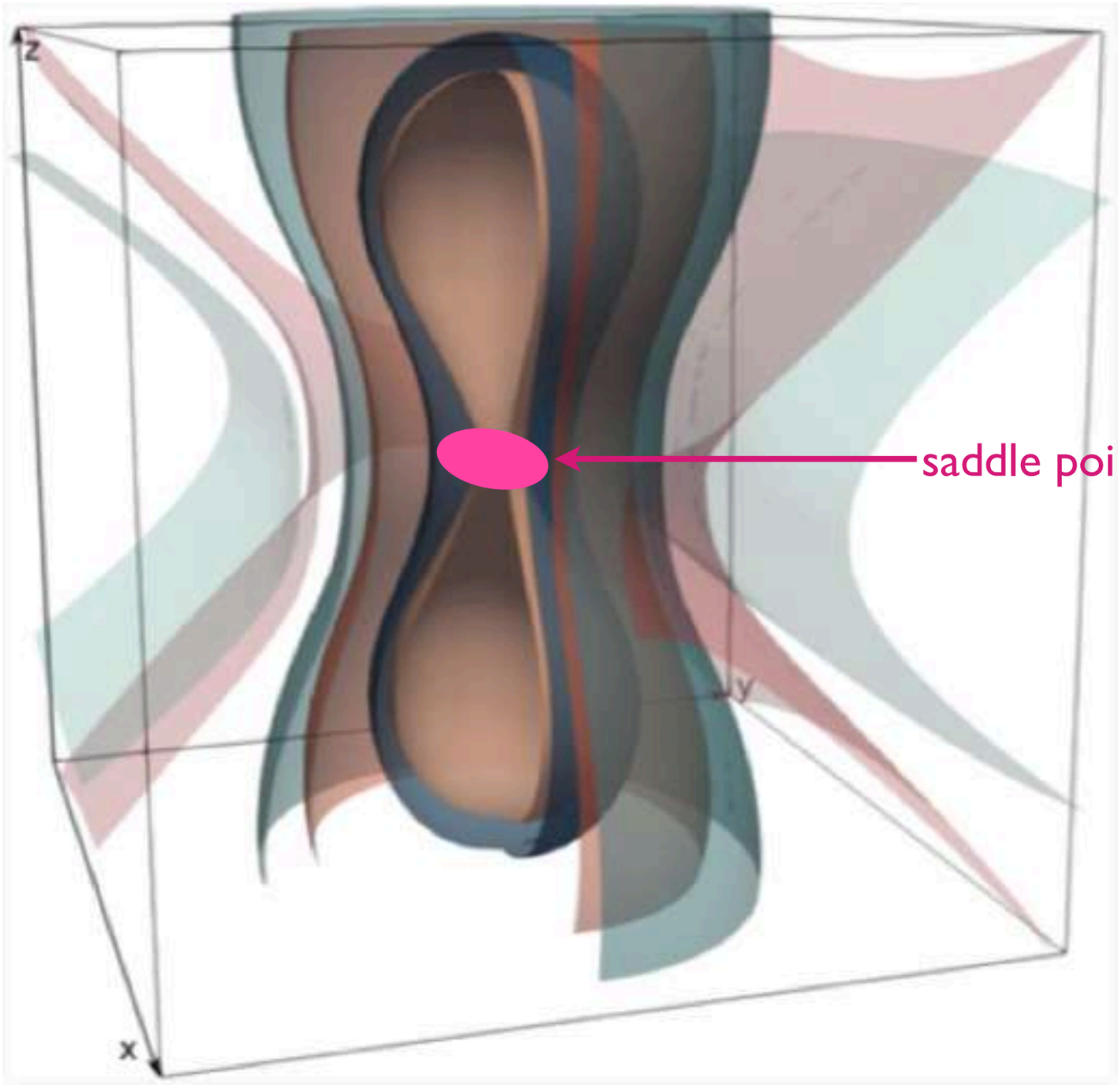


Halos with same mass can have different slope because of tides

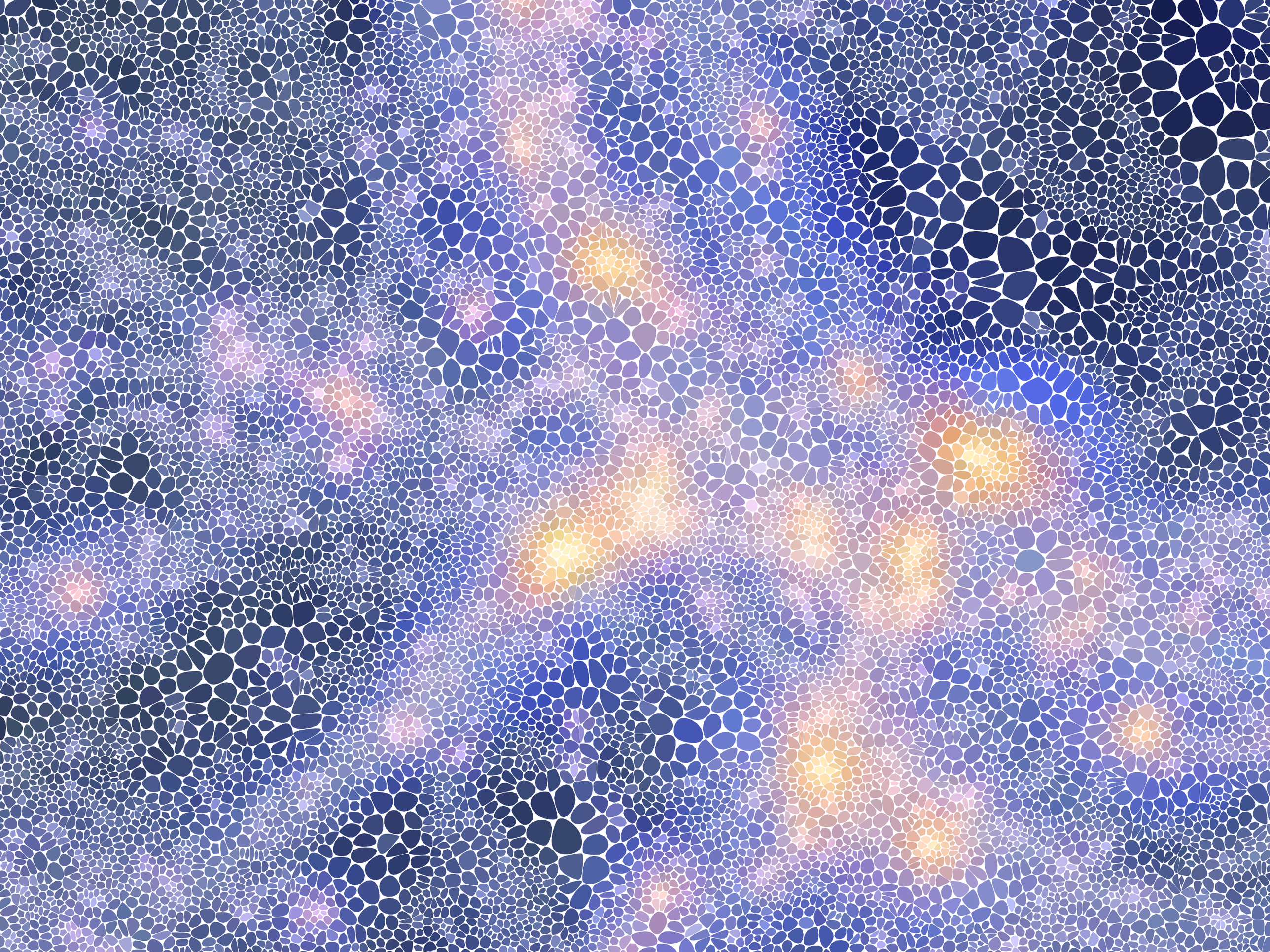
$$\mathcal{P}(\delta, \partial_R \delta | \mathcal{S}addle)$$

→ Accretion rate is not a function of mass and density alone.

isocontours of accretion rate, mass, and density



applies also to formation time, concentration (?), kinetic anisotropy...

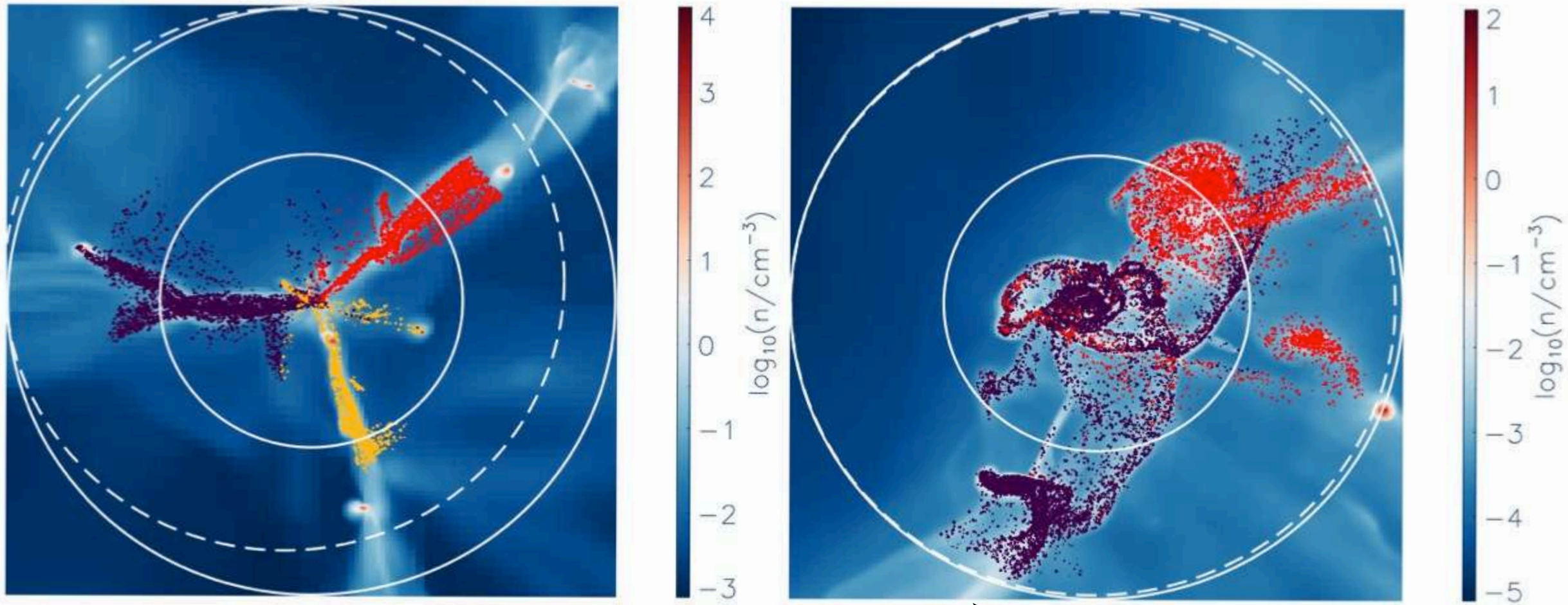


2.3 Critical events:Galactic motivation

revisit
- tidal torque theory
- excursion set theory
- critical event theory
- morphology theory

- CW metric changes (=biases) anisotropically the mean and variance of mergers = specific signature of CW
- implication for halo spin amplitude PDF + connectivity?

filament disconnect = cold gas inflow truncation



cosmic time

2.3 Synopsis of merger events

What happens to neighbouring critical pts?

Peak merger

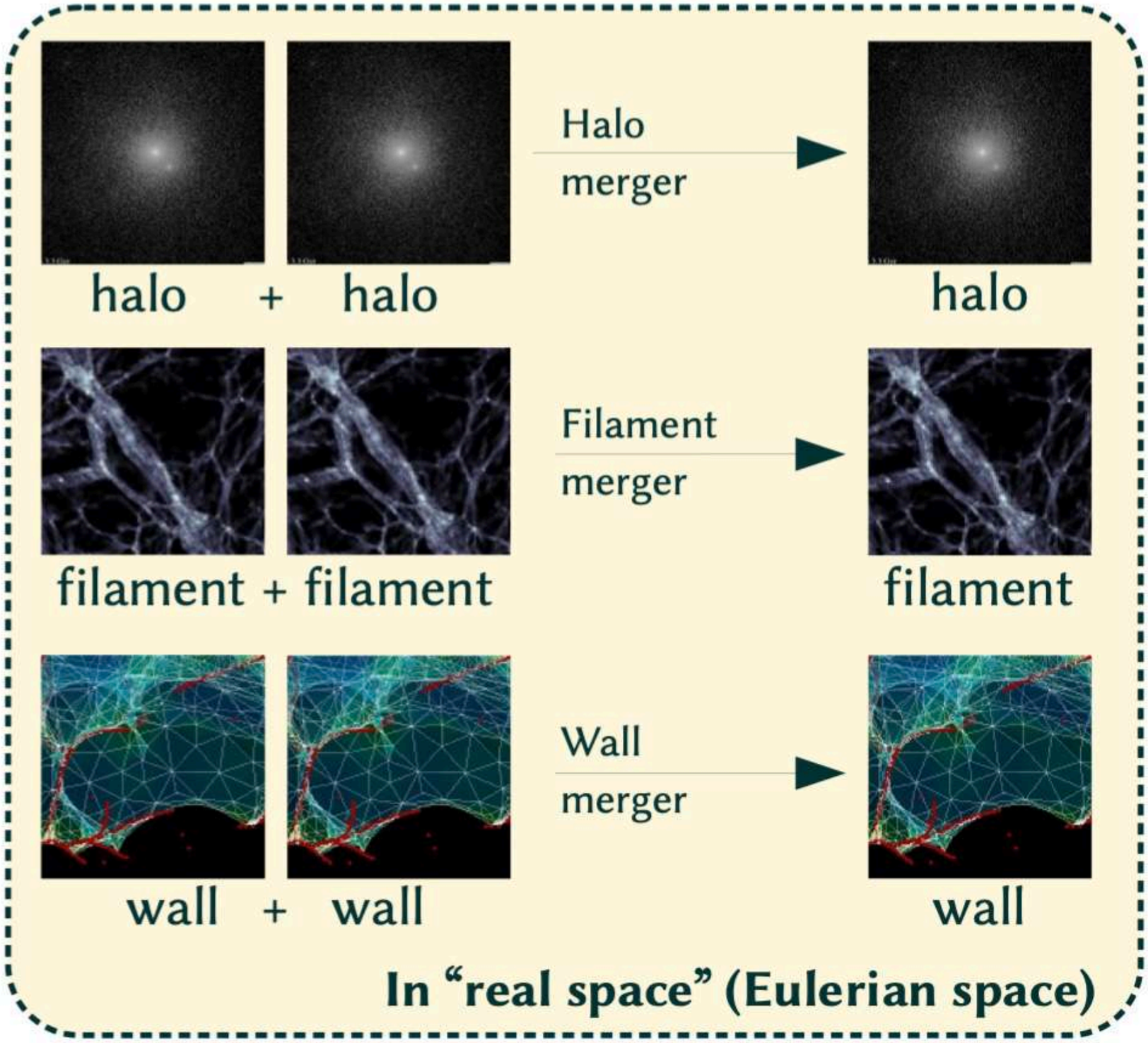
Filament vanishing

Filament merger

Wall vanishing

Wall merger

Void vanishing

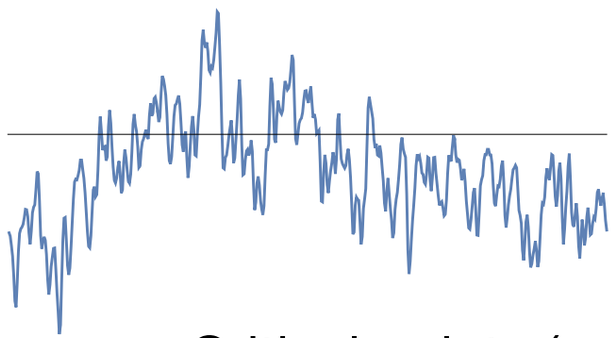


2.3 smoothing-cosmic evolution duality

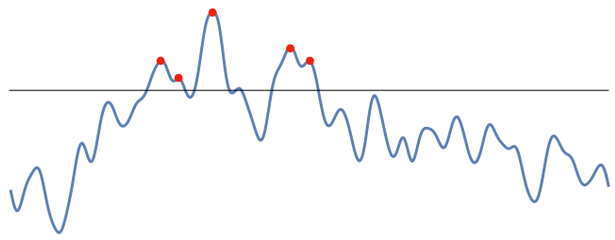
We consider the average density in spheres of increasing radius

This amounts to **smoothing** the field with a **top-hat filter**.

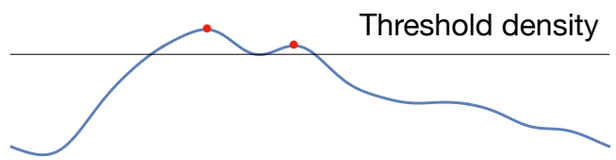
Initial field



Lightly smoothed field

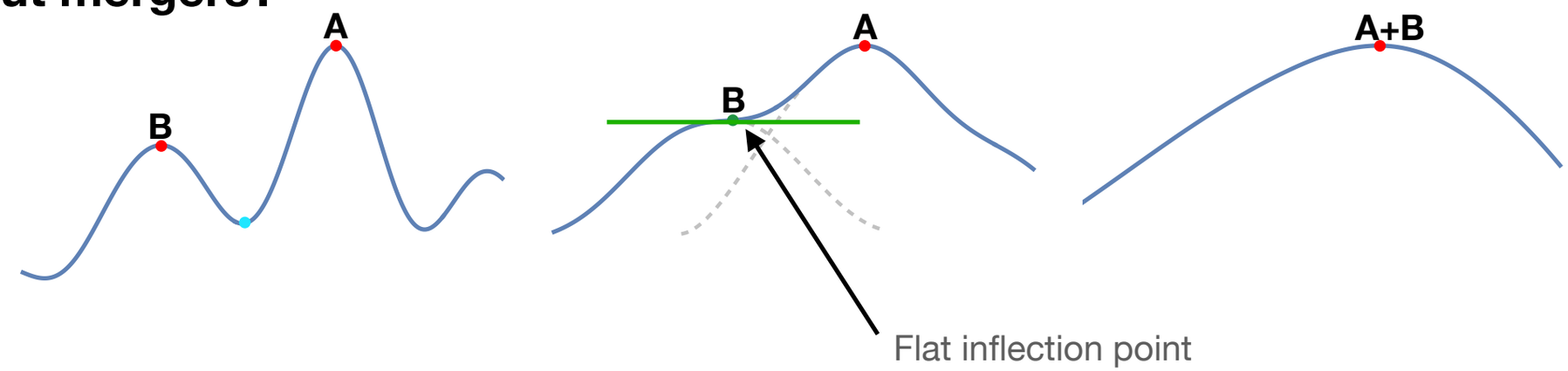


Strongly smoothed field

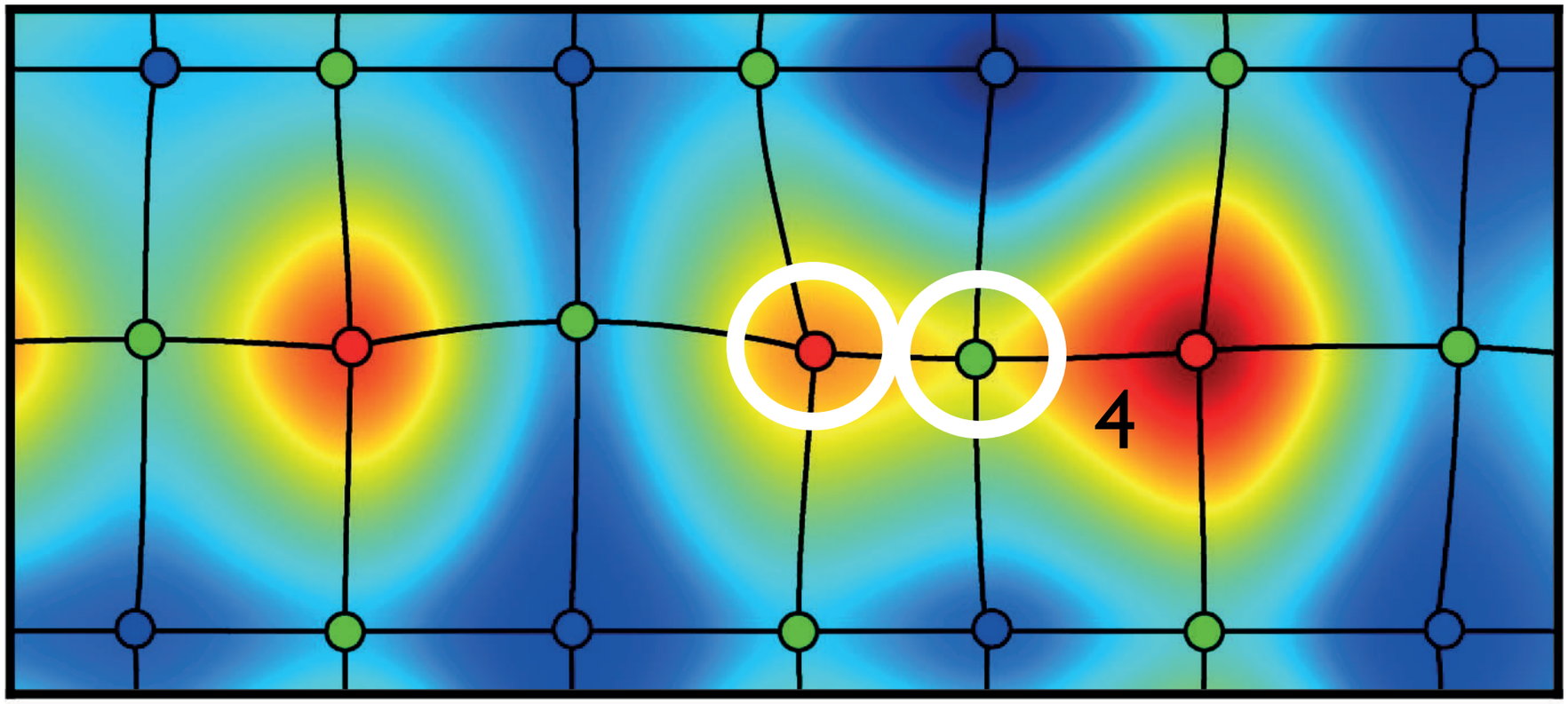


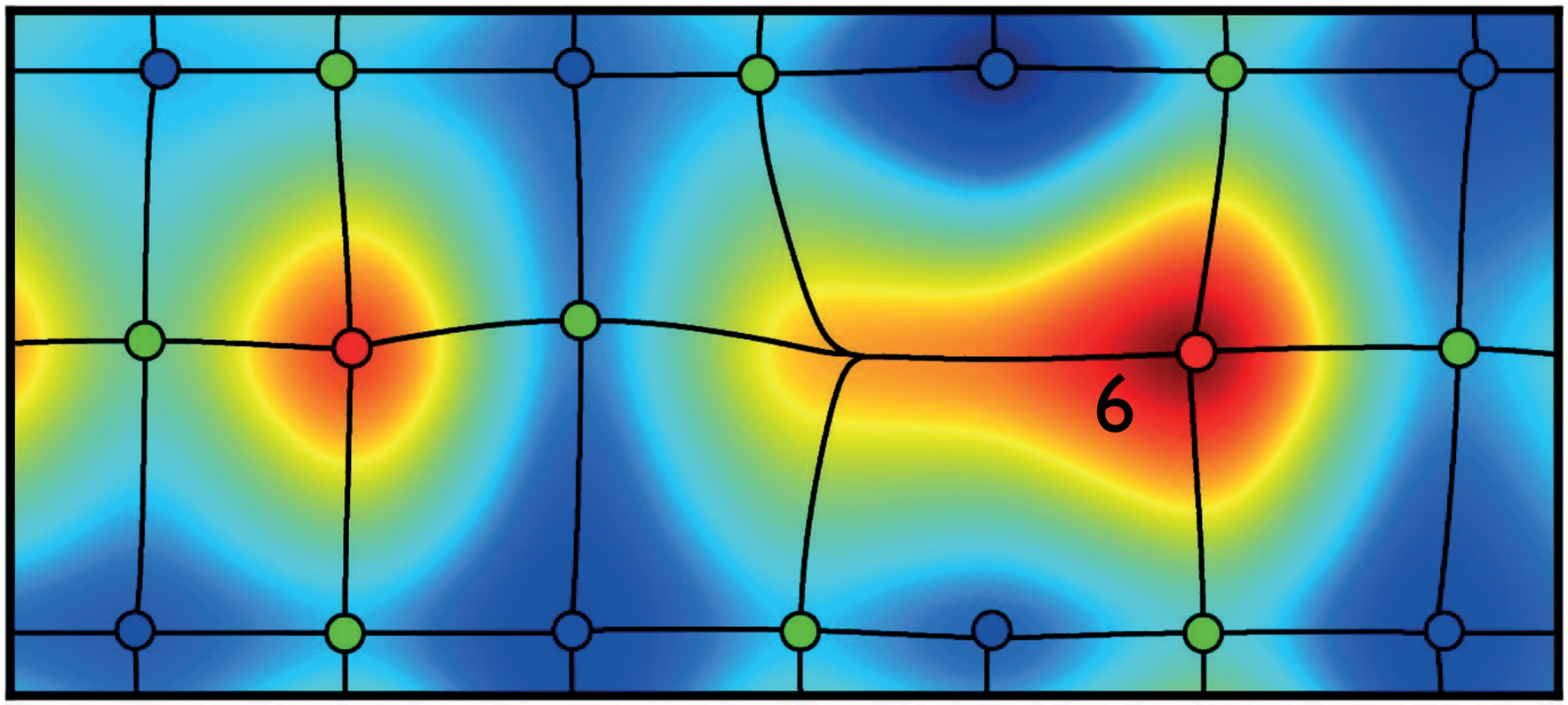
Critical points (maxima, saddle points, minima) of the smoothed field can be linked to particular structures of the **cosmic web** (peaks, filaments, walls, voids).

What about mergers?



2.3 smoothing cancels low persistence pairs

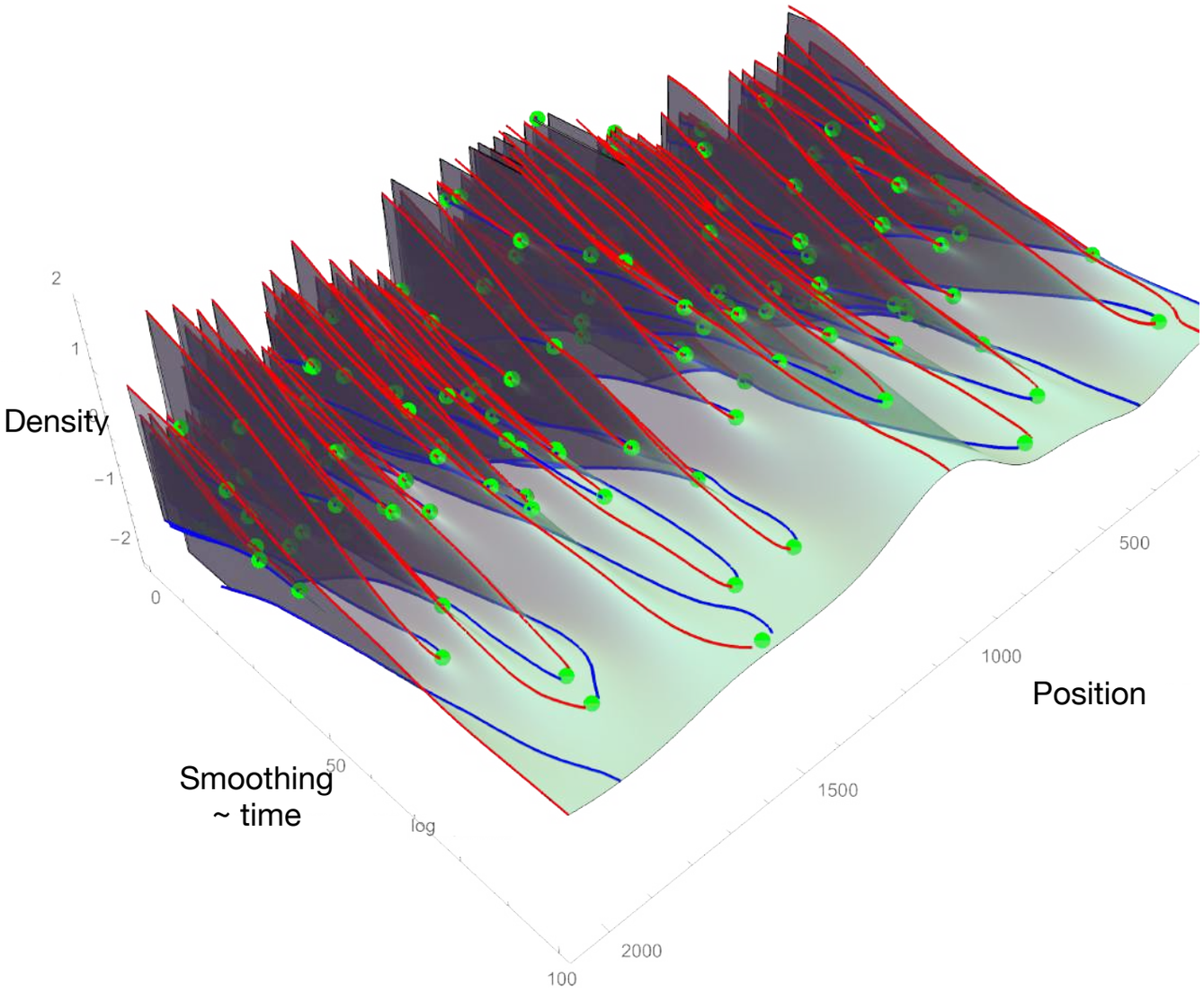




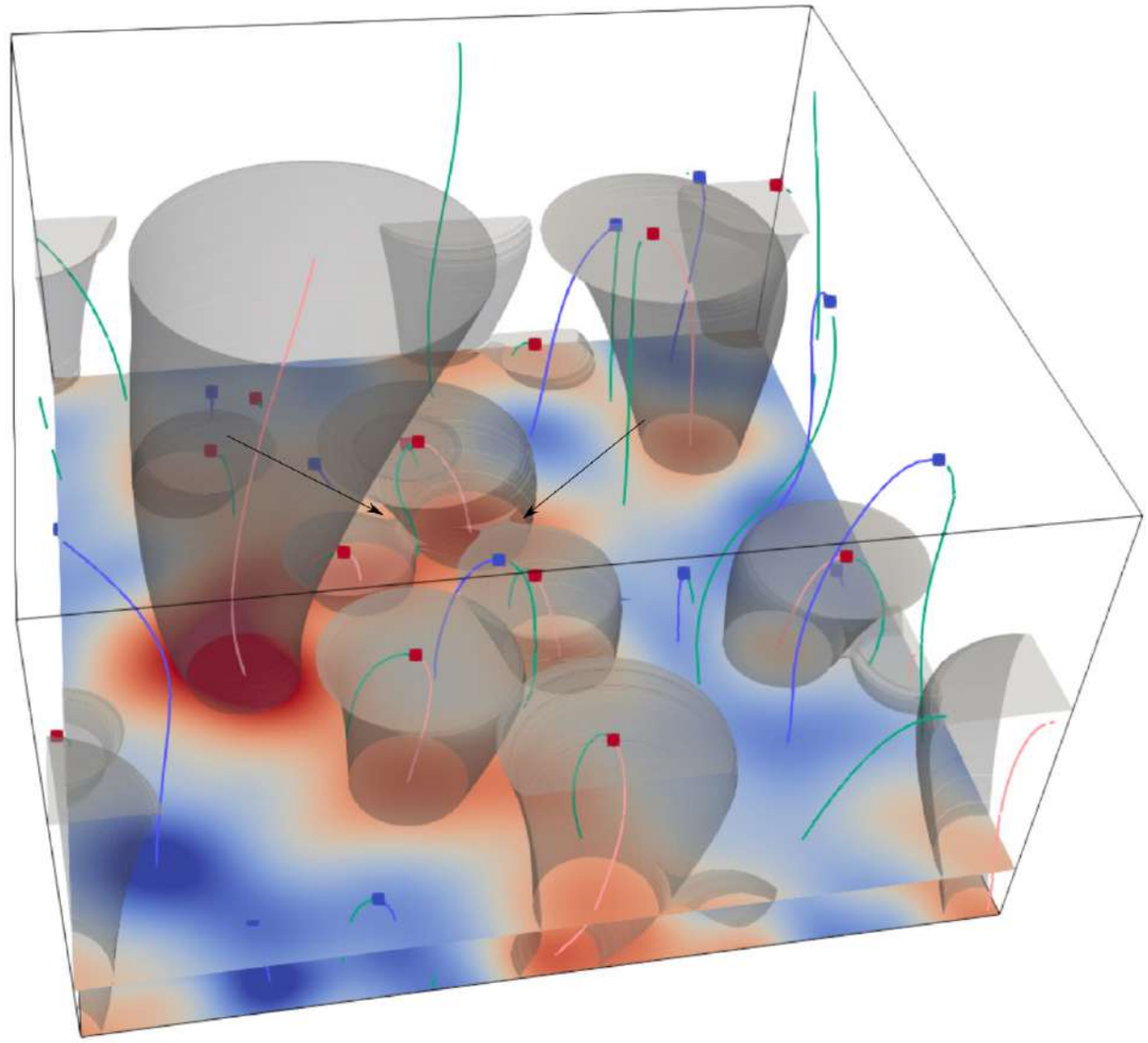


2.3 Critical event examples 1D & 2D

Critical events



In one …

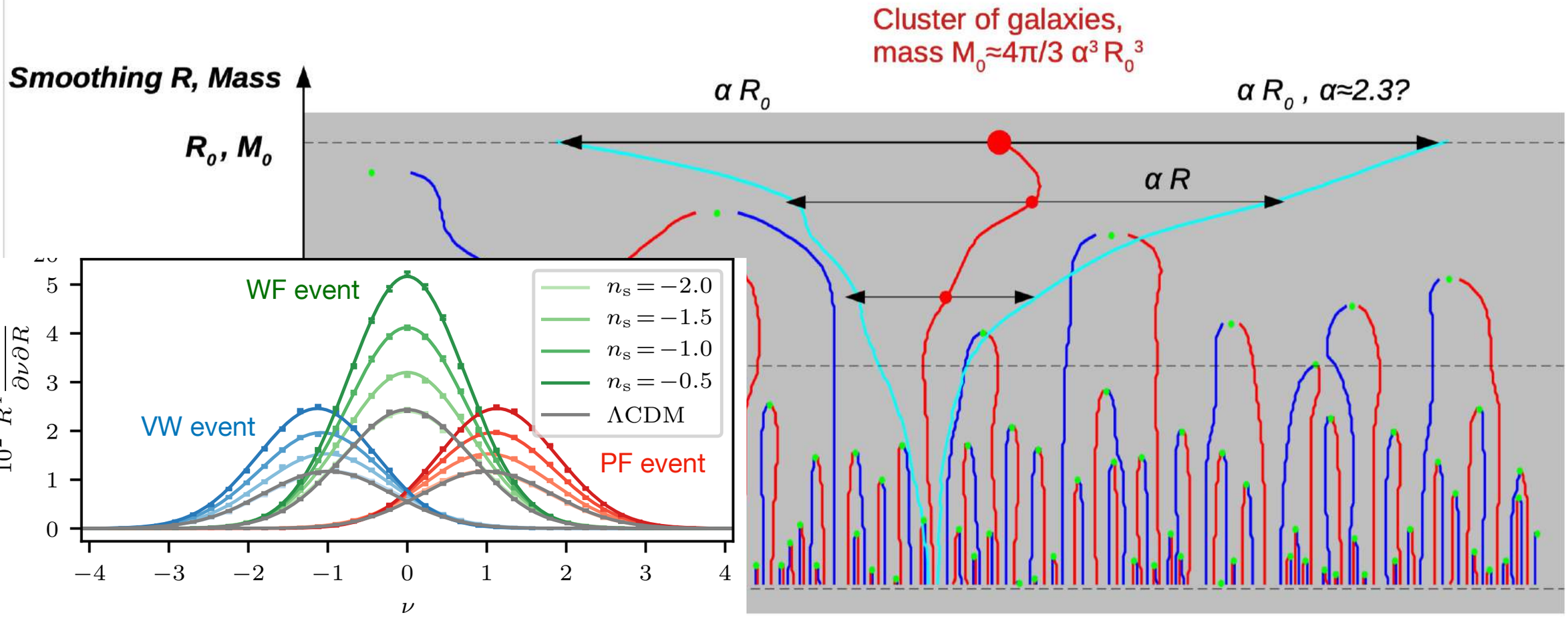


… and two dimensions

2.3 number of mergers / halo 1D theory

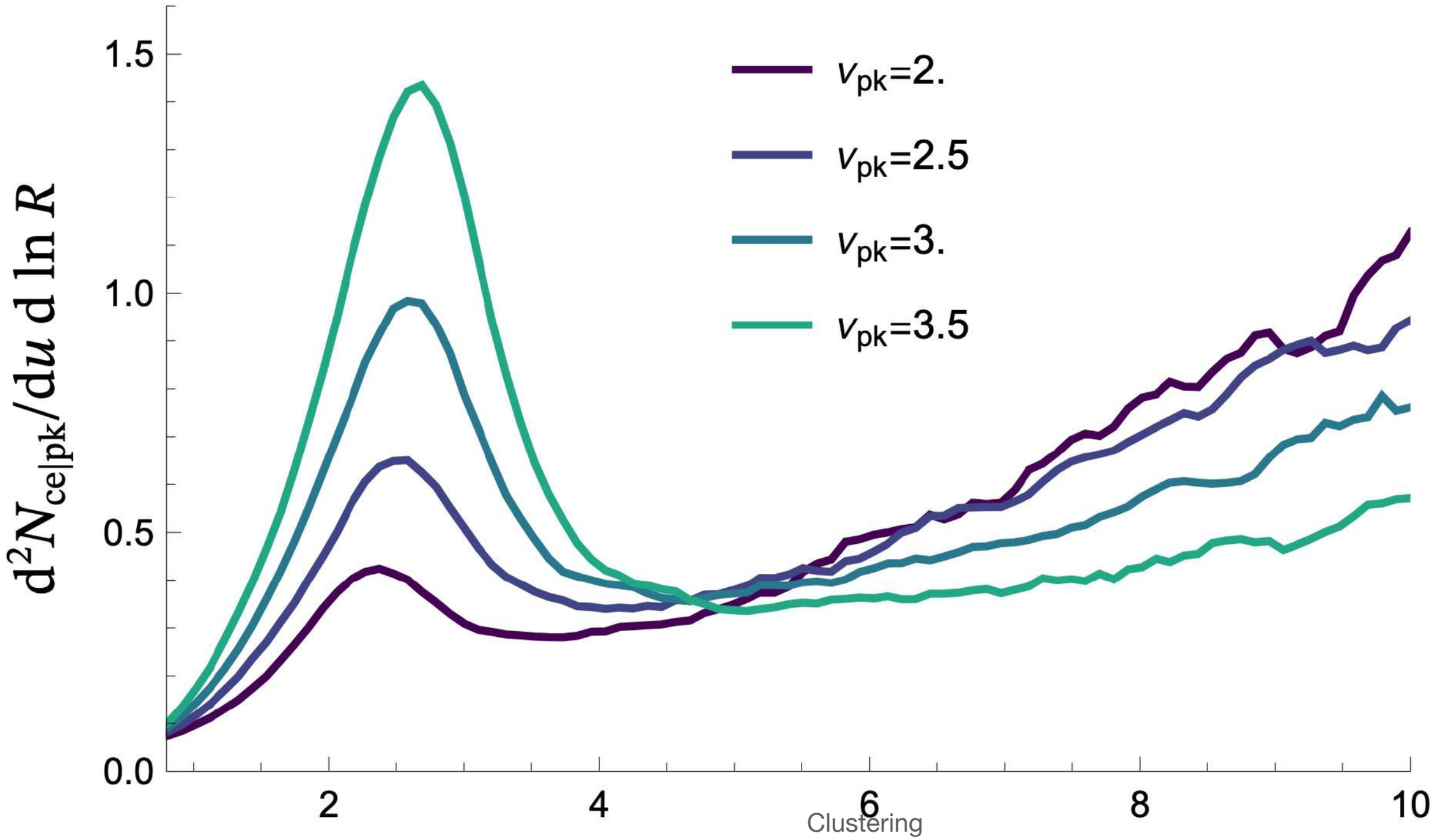
$$\mathrm{cond(event)} \propto \delta_{\mathrm{D}}(x')\delta_{\mathrm{D}}(x'')\,|\,x'''\,|^2$$

Past merger capture cone one



- Build straight “capture cone” of cross-section αR, centered on the final Lagrangian position.
- Count the number of critical events inside this cone – easy when critical event density is known. And it is ! (Cadiou et al 2020)

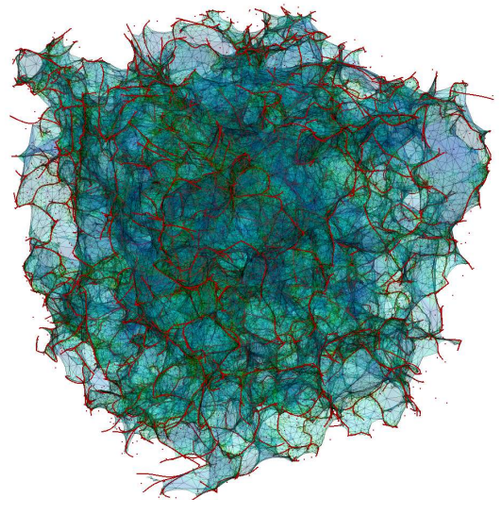
2.3 number of mergers / halo



$$\mathrm{cond(peak)} = \frac{1}{R_\star}\delta_{\mathrm{D}}(x')\,|x''|$$

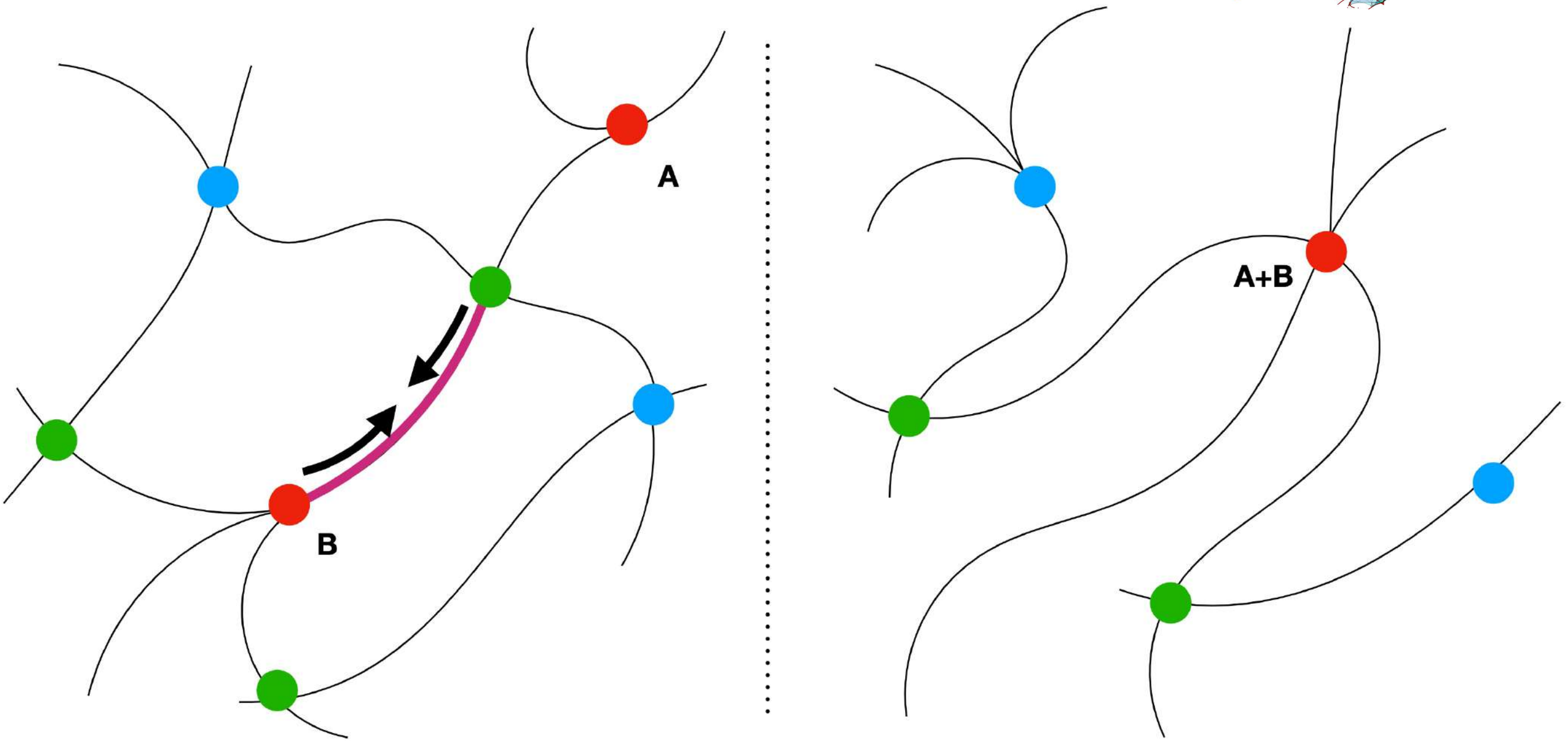
$$\mathrm{cond(event)} = \frac{R}{R_\star \tilde{R}^2}\delta_{\mathrm{D}}(x')\delta_{\mathrm{D}}(x'')\,|x'''|^2$$

2.3 cloud in cloud problem: merging along filaments



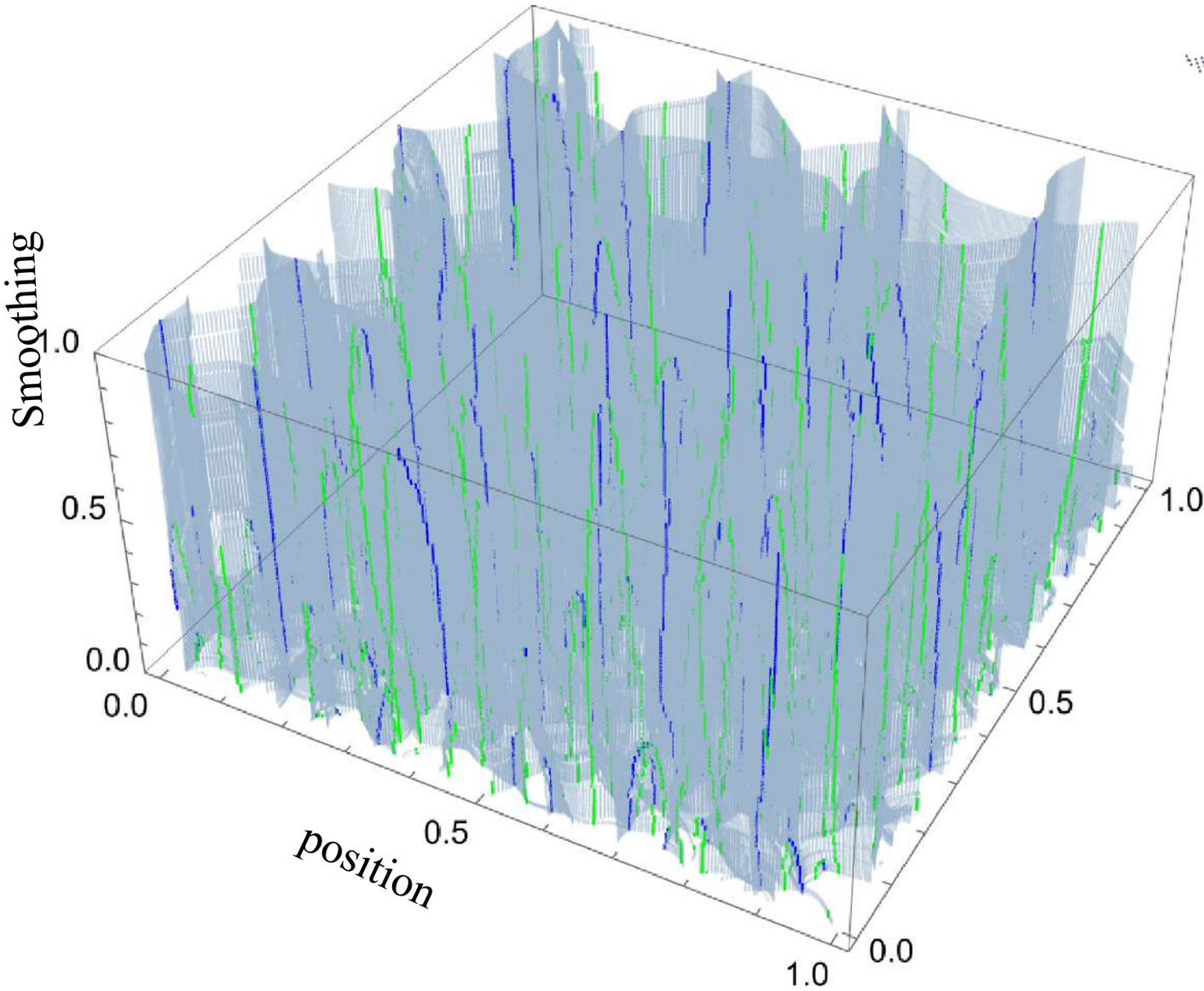
- Use disperse to trace the ridges along which the merger occurs
- Rely on persistence to **pairwise connect** merger to relevant peak

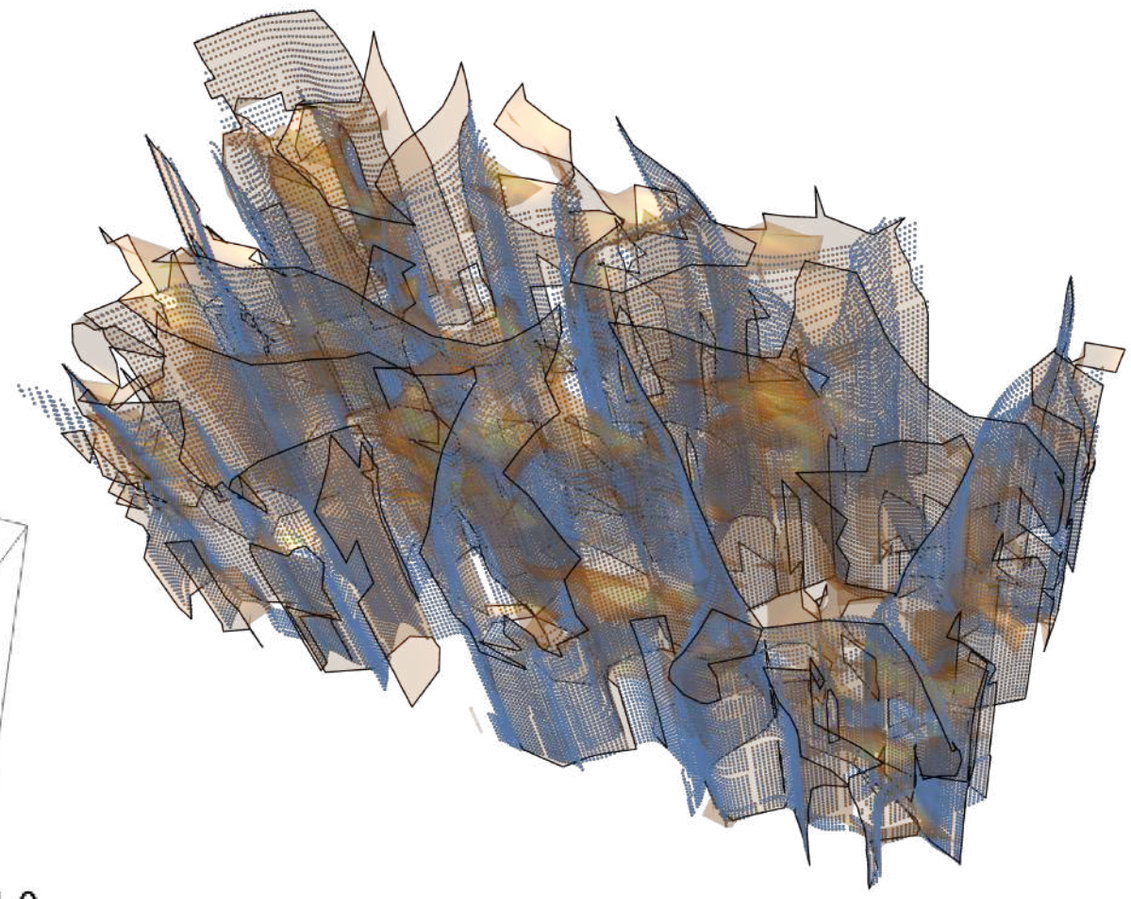
Topological mergers in 2D (and 3D), using the skeleton



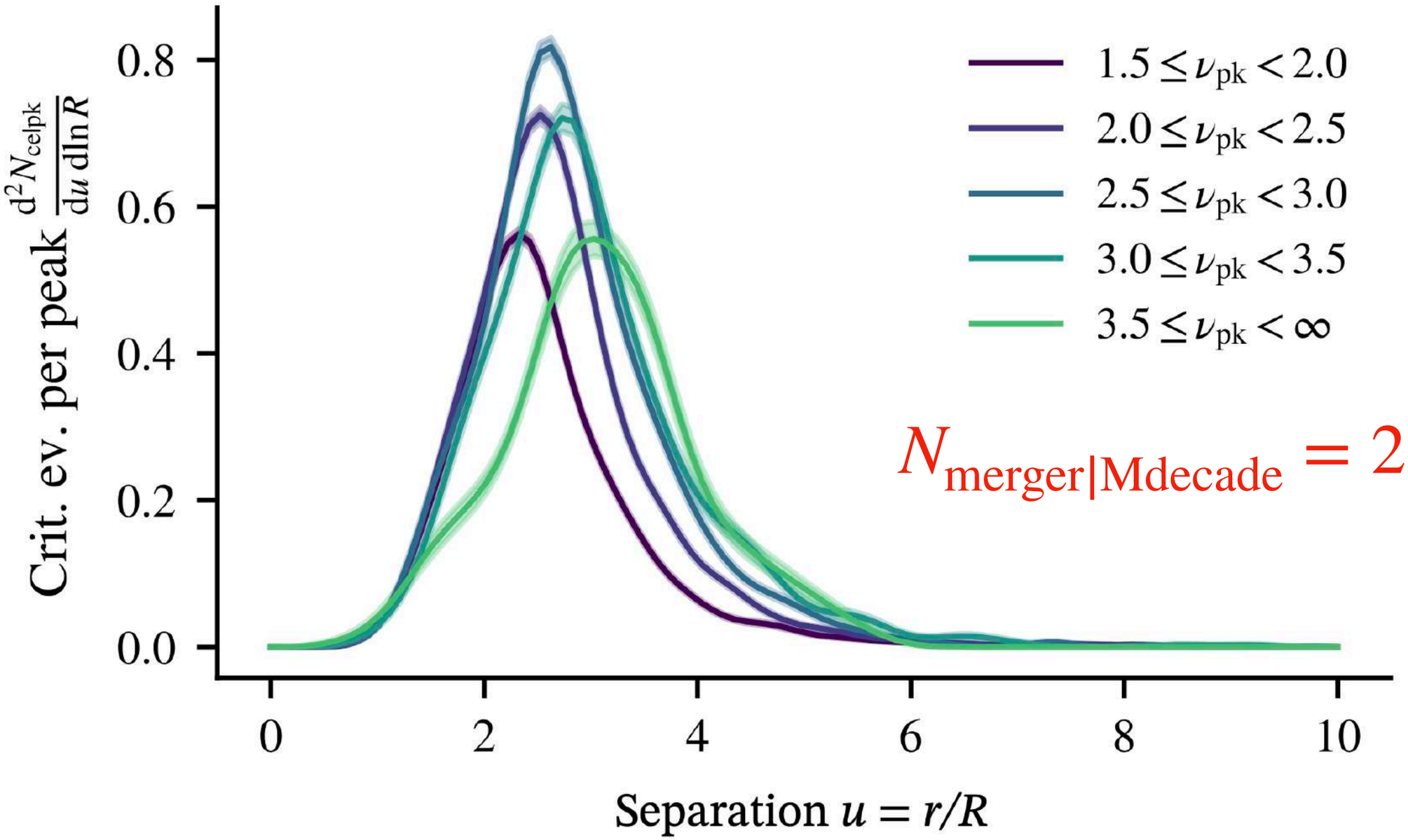
2.3 Solution: merging along filaments

Skeleton & critical points manifolds





2.3 cloud in cloud problem solved !



$N_{\mathrm{merger|Mdecade}} = 2$

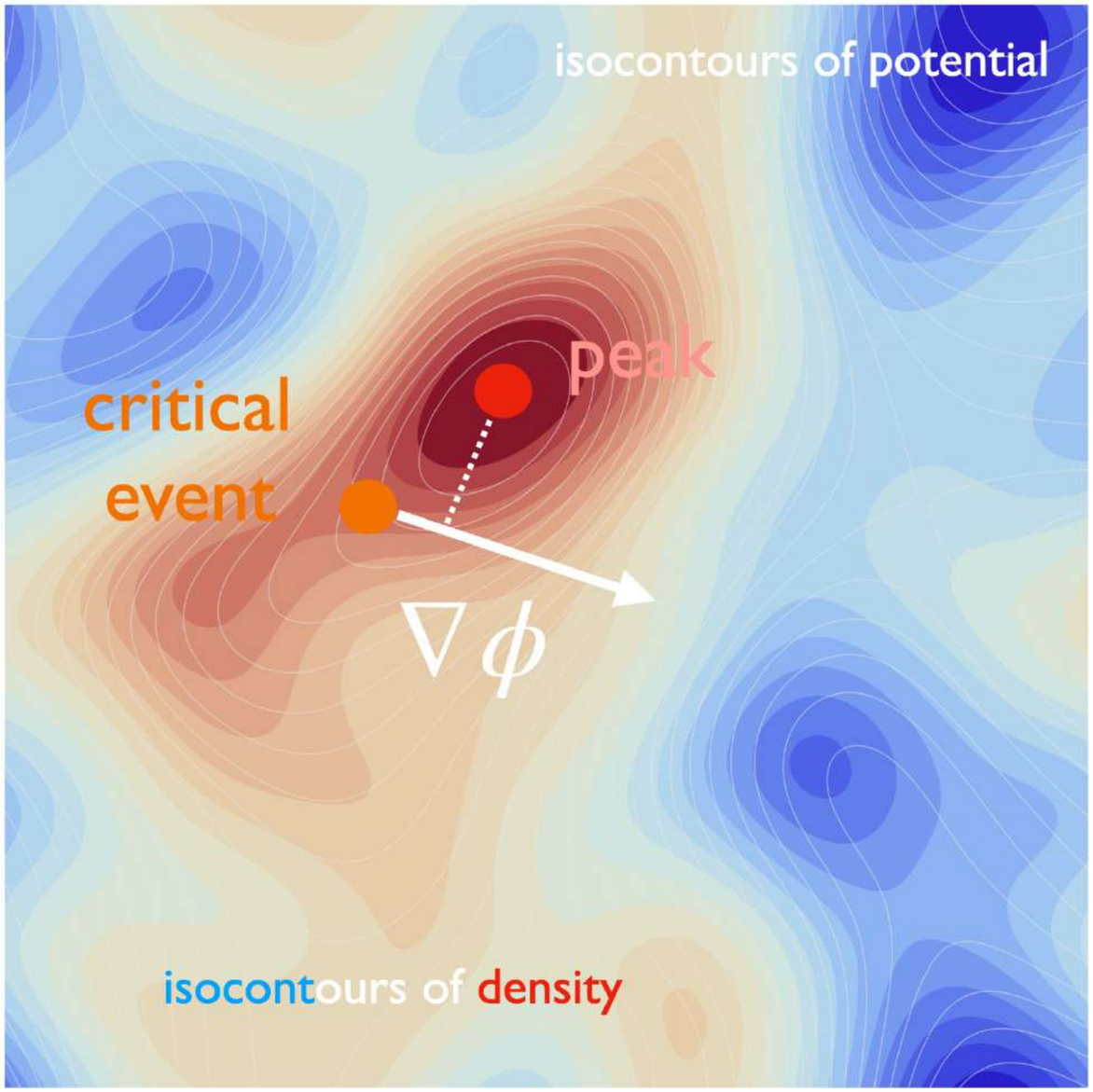
The radial number density of topologically linked critical event per peak as a function of distance to the peak on a slice of constant R and per logarithmic bin in smoothing scale

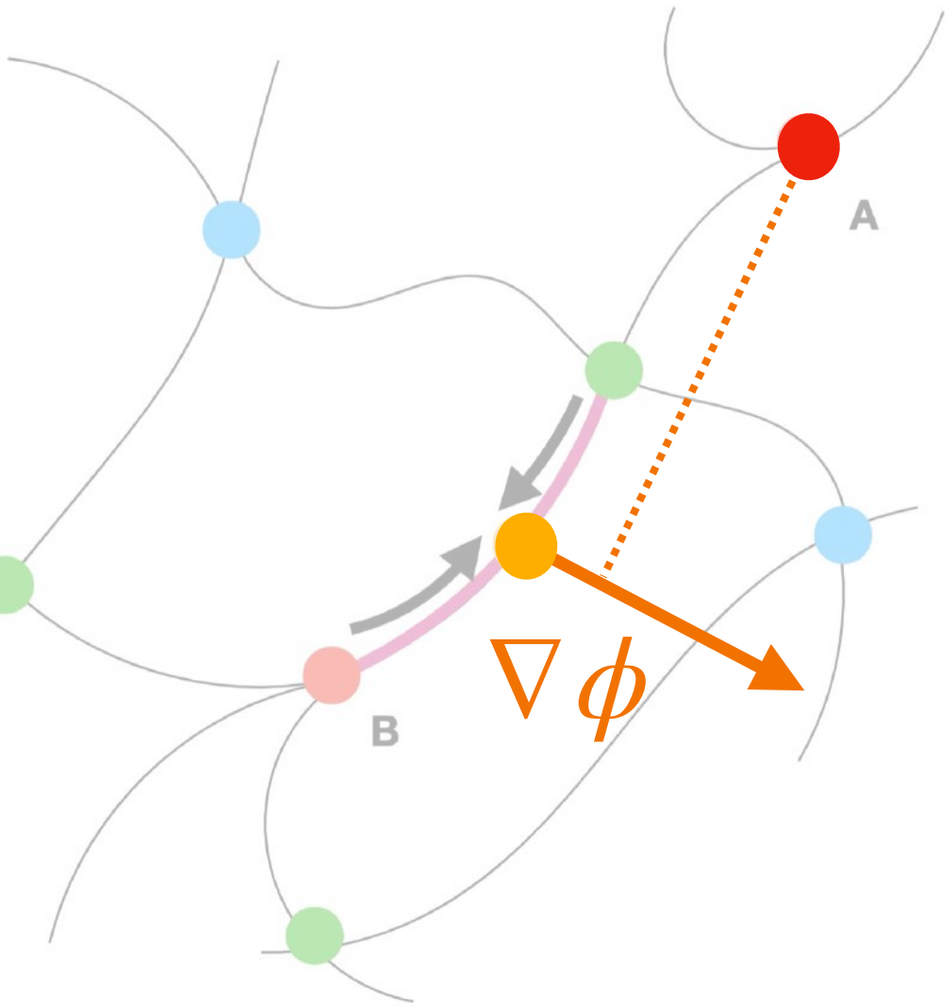
Fakhouri+ 2010

2.3 Spin of merger event

Let us revisit spin acquisition during merger

Topological mergers in 2D (and 3D), using the skeleton

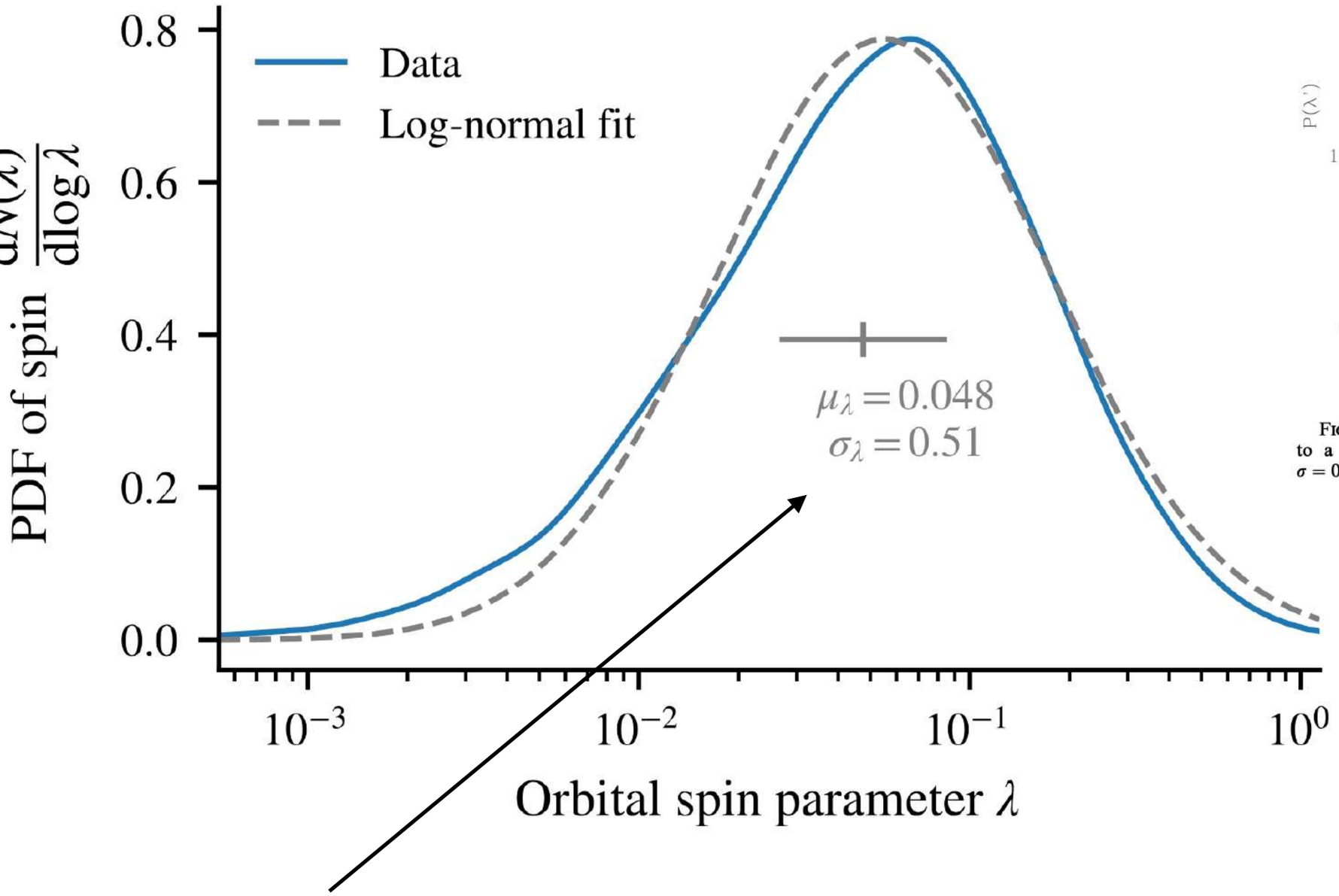




compute gradient of potential @ scale of event & assume only last event counts

2.3 Spin parameter prediction : geometry of PS theory

$$J/(\sqrt{2}MVR_{200})$$



Bullock+2001

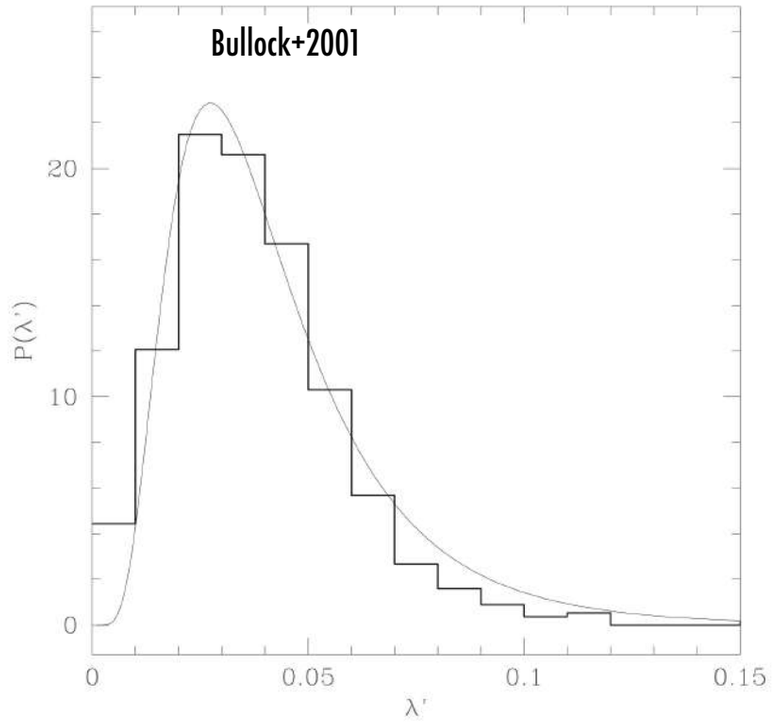


FIG. 1.—Distribution of halo spin parameter λ' (*histogram*), compared to a log-normal distribution about $\lambda'_0 = 0.035 \pm 0.005$ with a width $\sigma = 0.50 \pm 0.03$.

$$\lambda_0 = 0.042 \pm 0.006 \text{ and } \sigma = 0.5 \pm 0.04.$$

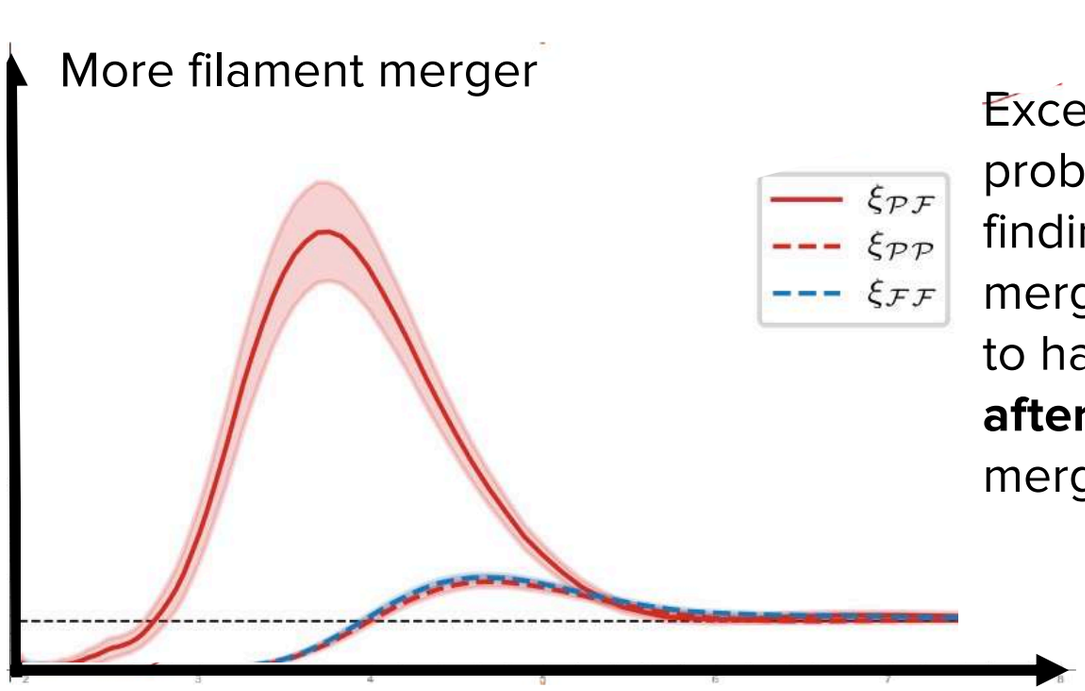
Barnes & Efstathiou 1987

NEW

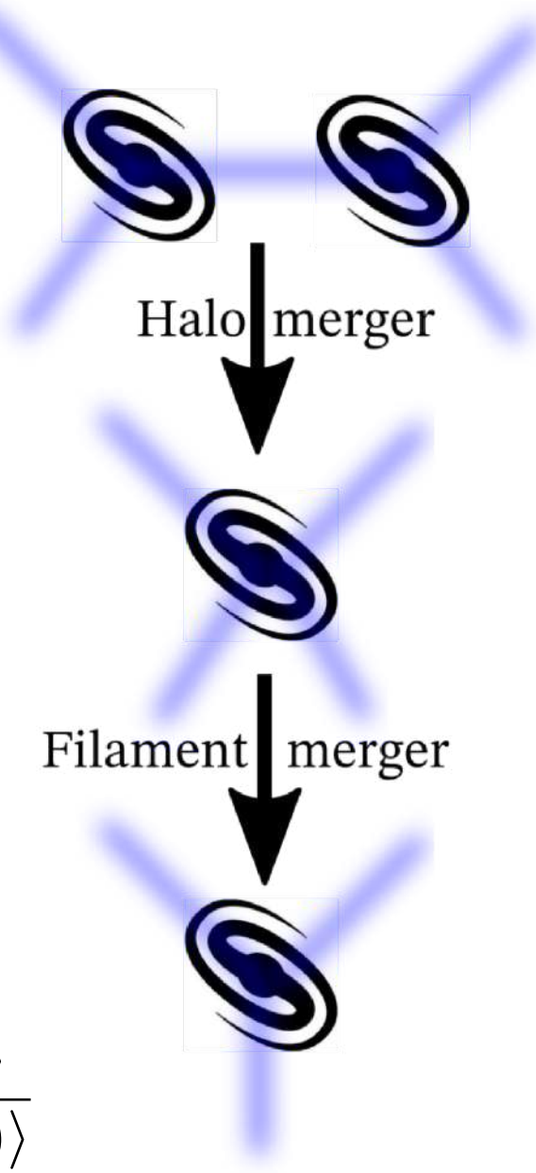
Cadiou+ (24)

On the connectivity of halos

Compute frequency of filament merger compared to halo merger in the vicinity of a halo merger event $\xi_{\mathrm{hf}}(r)$ $\xi_{\mathrm{hh}}(r)$.



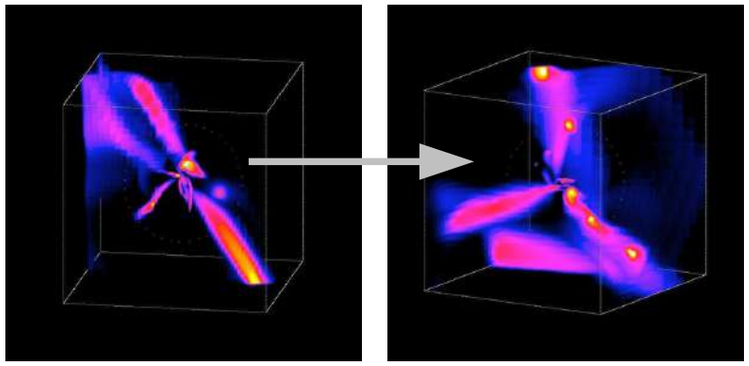
Excess of probability of finding filament merger compared to halo merger **after** a halo merger



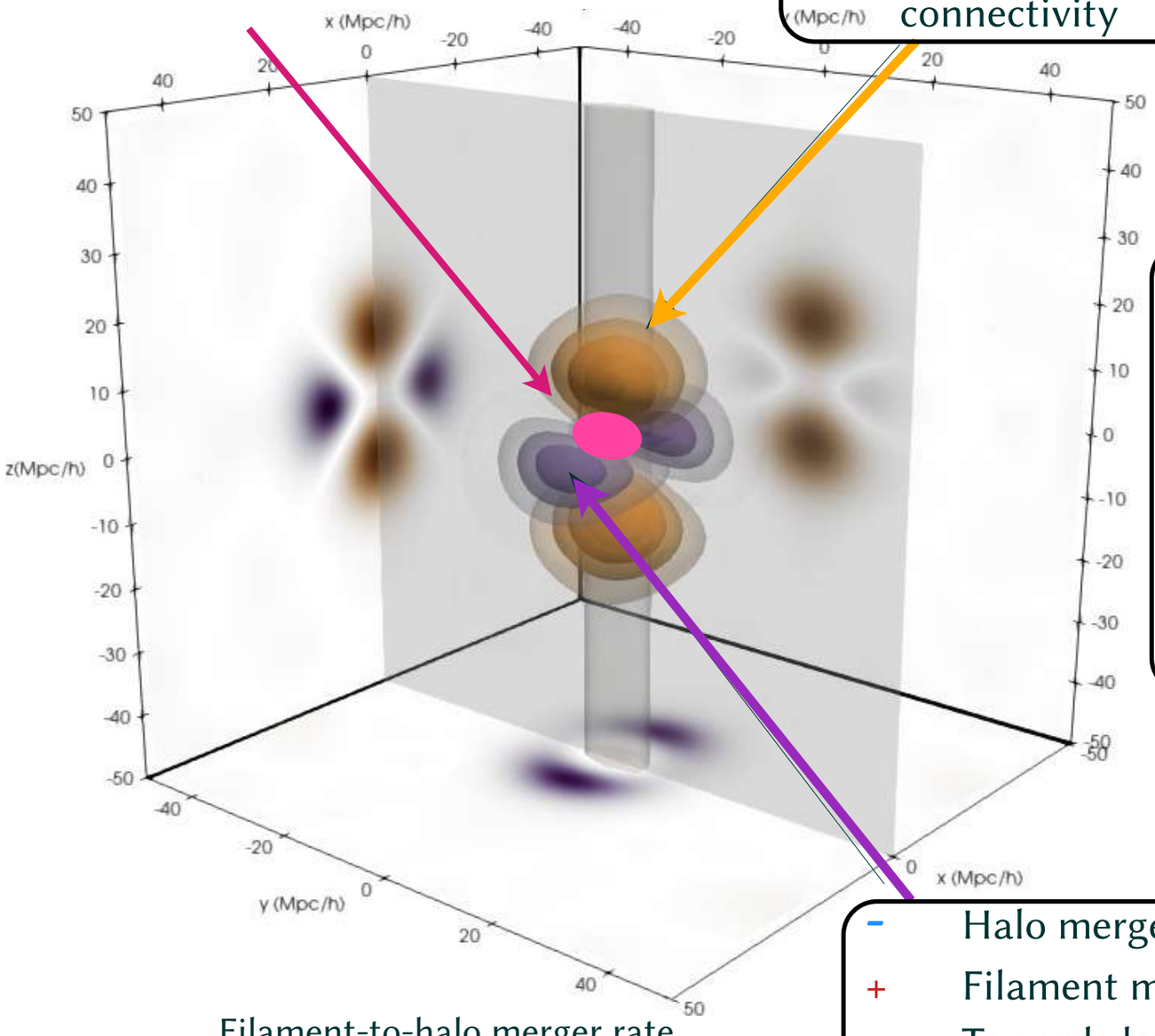
$$1+\xi_p = \frac{\langle \mathrm{cond}_p(\mathbf{x})\mathrm{cond}_p(\mathbf{y})\rangle}{\langle \mathrm{cond}_p(\mathbf{x})\rangle^2},\ 1+\xi_f = \frac{\langle \mathrm{cond}_f(\mathbf{x})\mathrm{cond}_p(\mathbf{y})\rangle}{\langle \mathrm{cond}_f(\mathbf{x})\rangle\langle \mathrm{cond}_p(\mathbf{x})\rangle}$$

Connectivity evolution across the cosmic web

+ Halo mergers
- Filament mergers
→ Towards higher connectivity

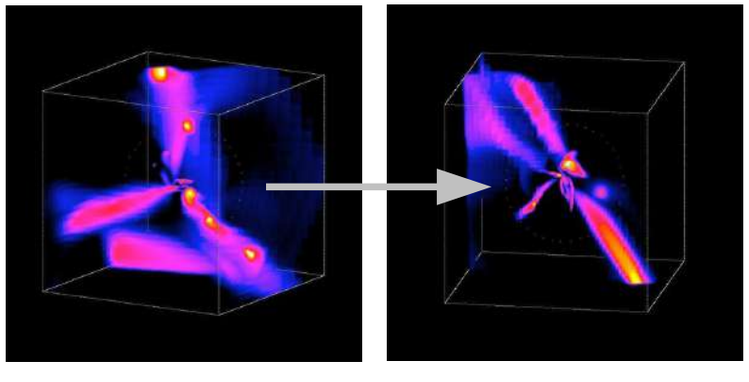


saddle point



Filament-to-halo merger rate, computed numerically.
Cadiou+in prep

- Halo mergers
+ Filament mergers
→ Towards lower connectivity

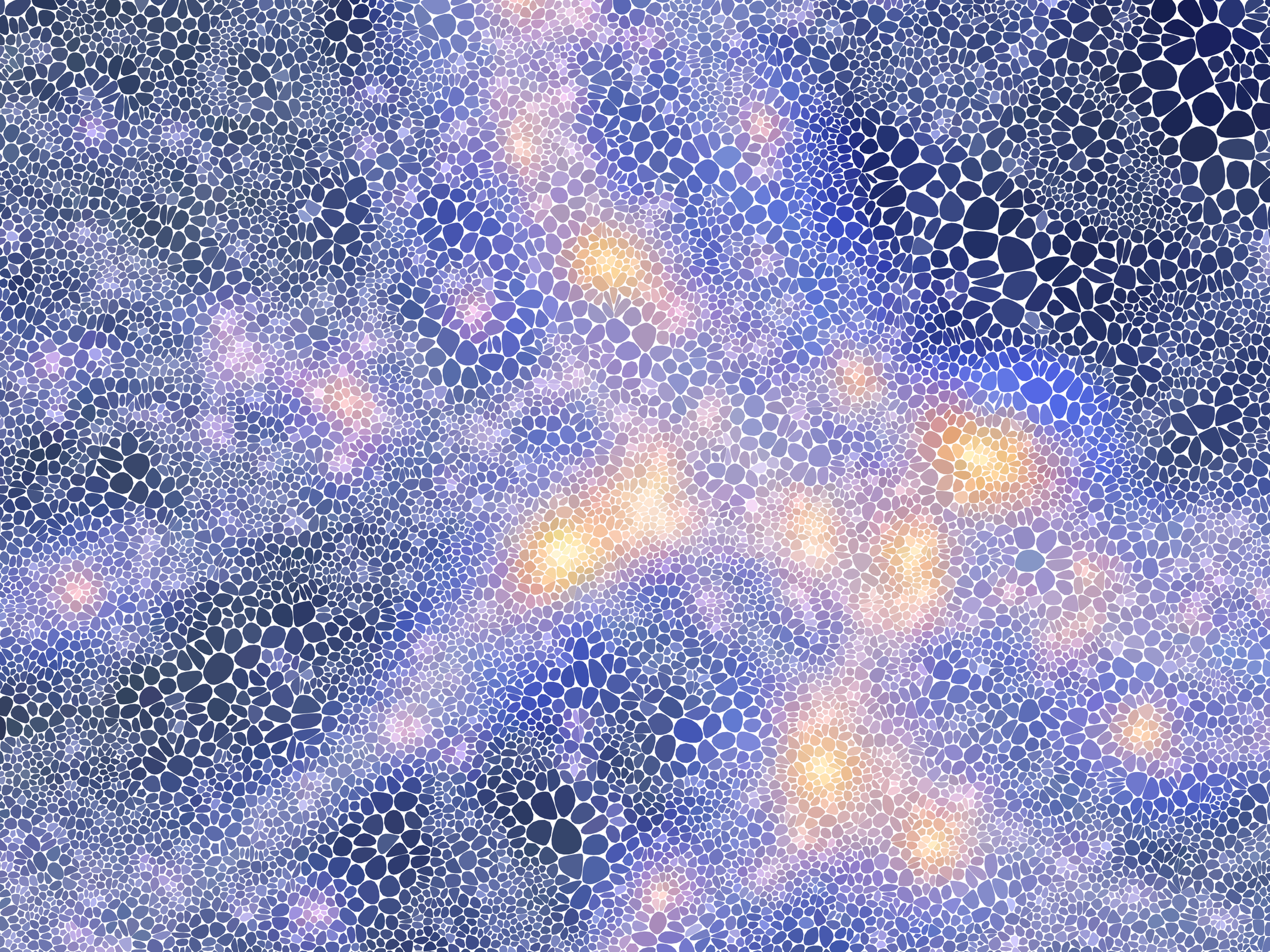


At **fixed smoothing scale**, in nodes
- more halo mergers,
- less filament mergers,
- growing towards higher connectivity,

than in voids.

Differential impact of LSS on halo connectivity evolution

[Connectivity: Codis+18]



revisit

- tidal torque theory
- excursion set theory
- critical event theory
- morphology theory

- CW metric changes (=biases) anisotropically the mean and variance of infall+fluctuations = specific signature of CW
- implication for Galaxy morphology/metallicity?



Morphology = orbital structure of the stellar component

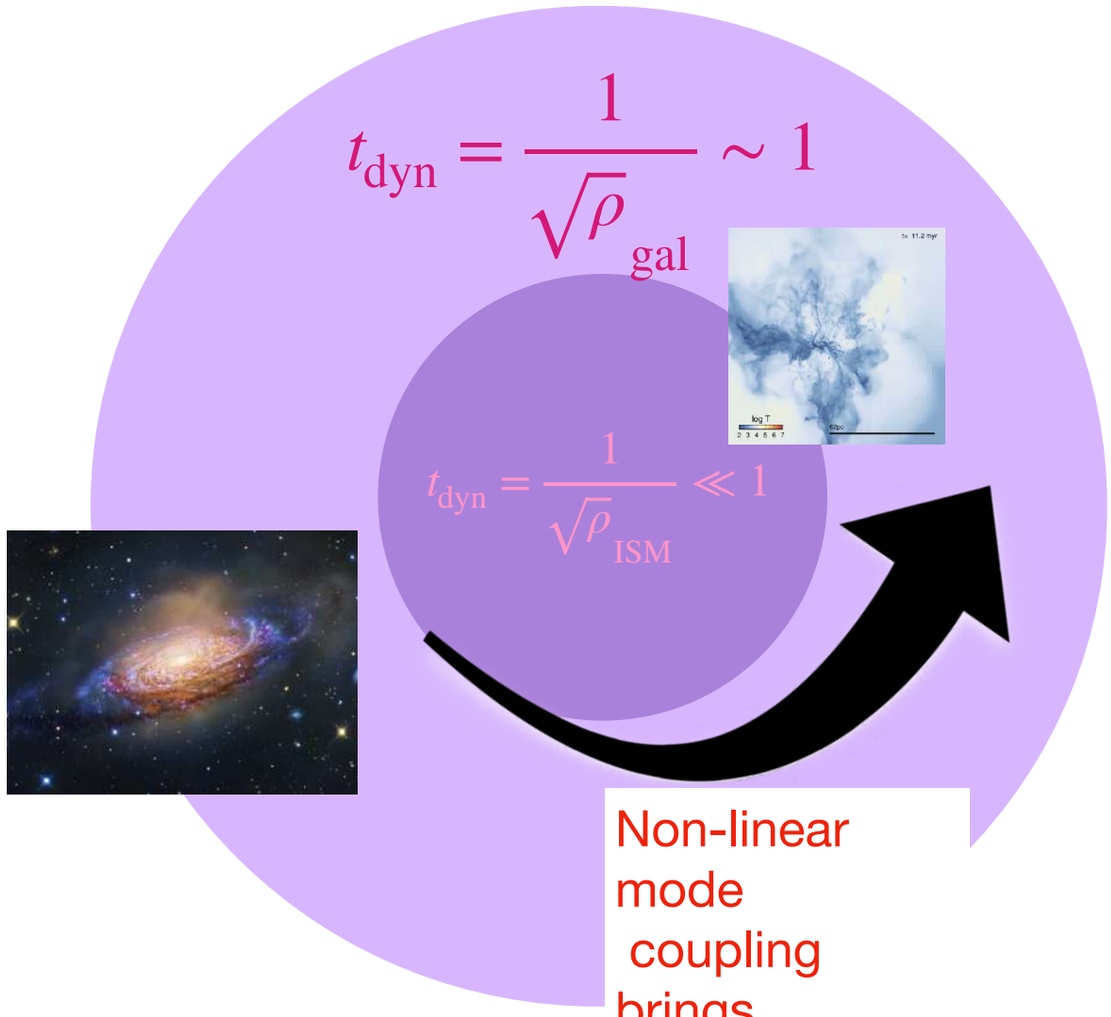
On secular timescales, morphological transformation = change of orbital structure

2.4 Galaxies are multi-scale non-linearly asymmetrically coupled systems

revisit

- tidal torque theory
- excursion set theory
- critical event theory
- morphology theory

- CW metric changes (=biases) anisotropically the mean and variance of infall+fluctuations = specific signature of CW
- implication for Galaxy morphology/metallicity?



Focussing on the impact of coupling to **smaller scales**

2.4 impact of coupling to small scale on orbital structure: heating +cooling

revisit

- tidal torque theory
- excursion set theory
- critical event theory
- morphology theory

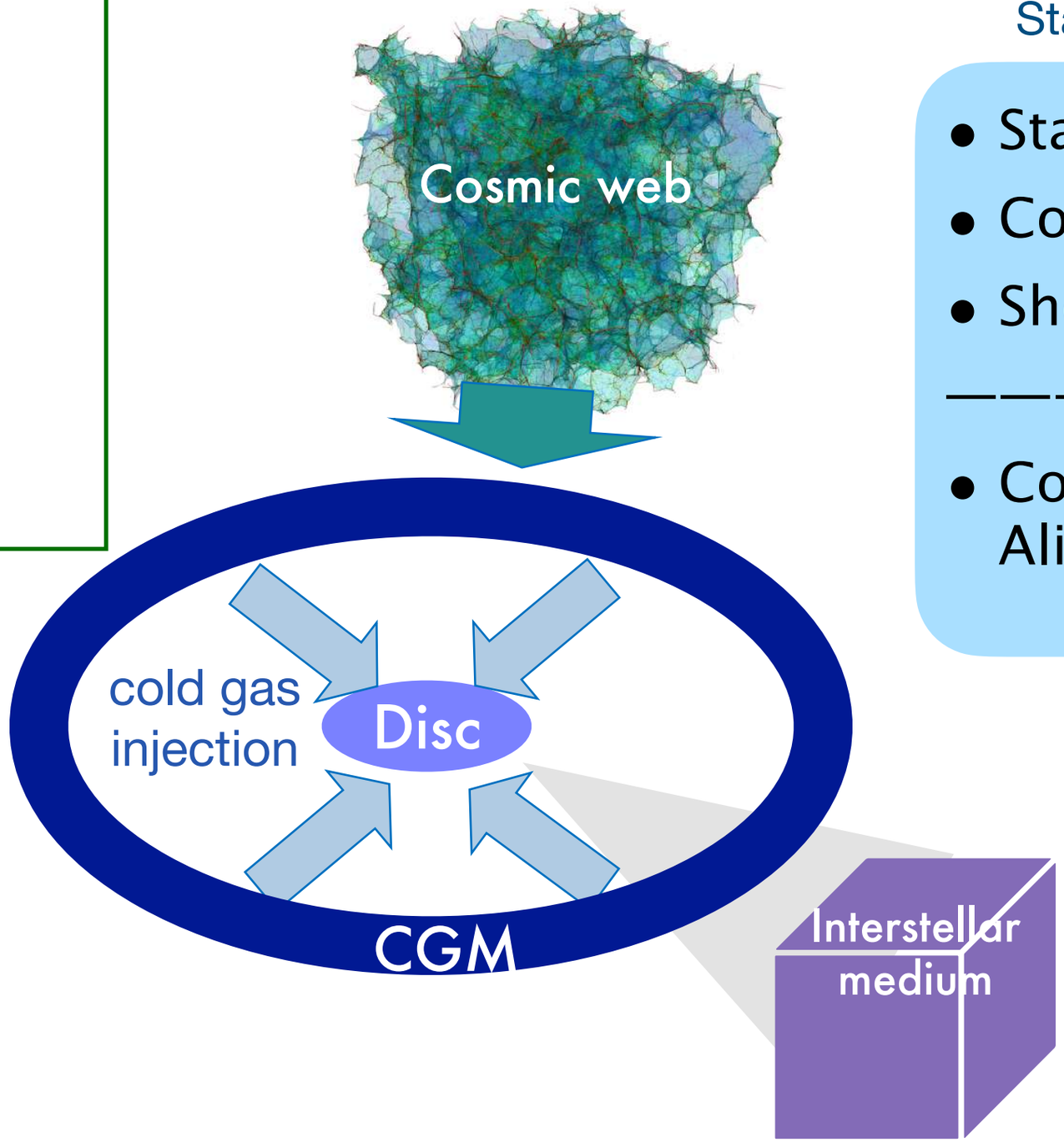
Heating

Destabilising effects

- SN1a
- Turbulence

———

- Minor Mergers
- Misaligned infall
- FlyBys



Stabilising effects

- Star formation
- Cooling
- Shocks

———

- Co-rotating
  Aligned infall

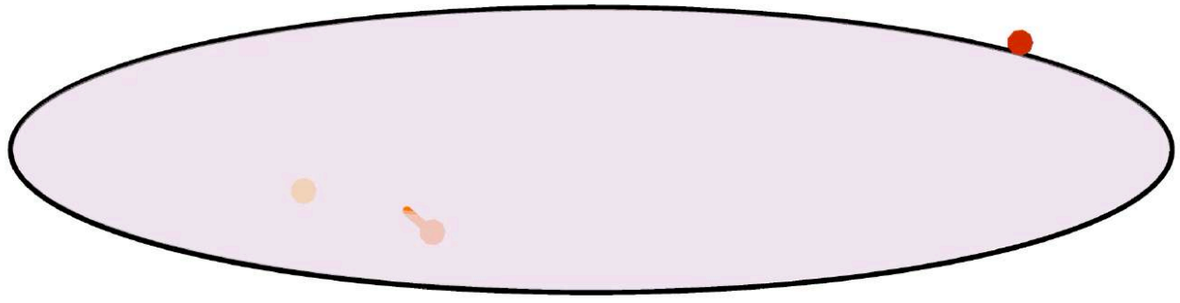
Cooling

Three components system coupled by gravitation.

Let us focus on heating…

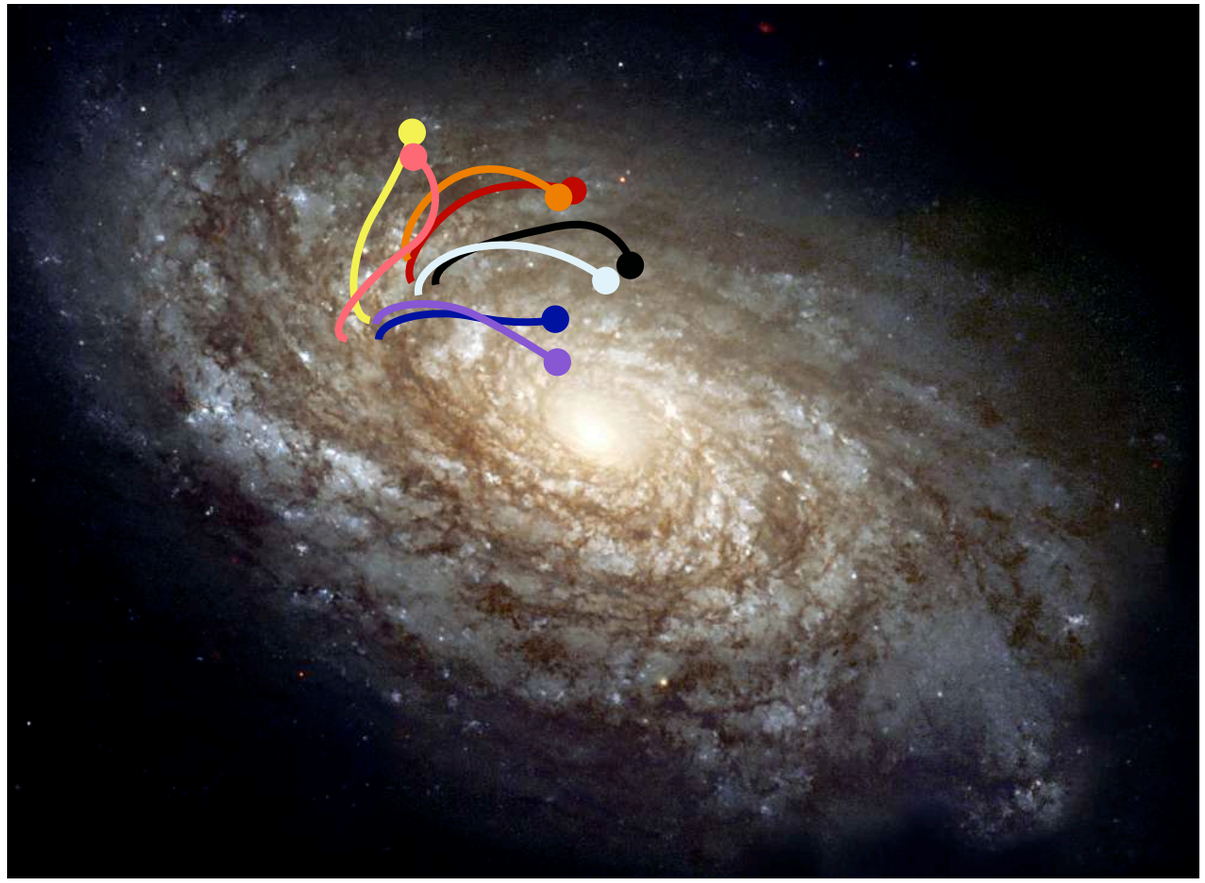
revisit

- tidal torque theory
- excursion set theory
- critical event theory
- morphology theory



Fluctuation-Dissipation Theorem

Diffusion rate ⟷ **Power-spectrum of the fluctuations**



≈



Orbits in a galaxy

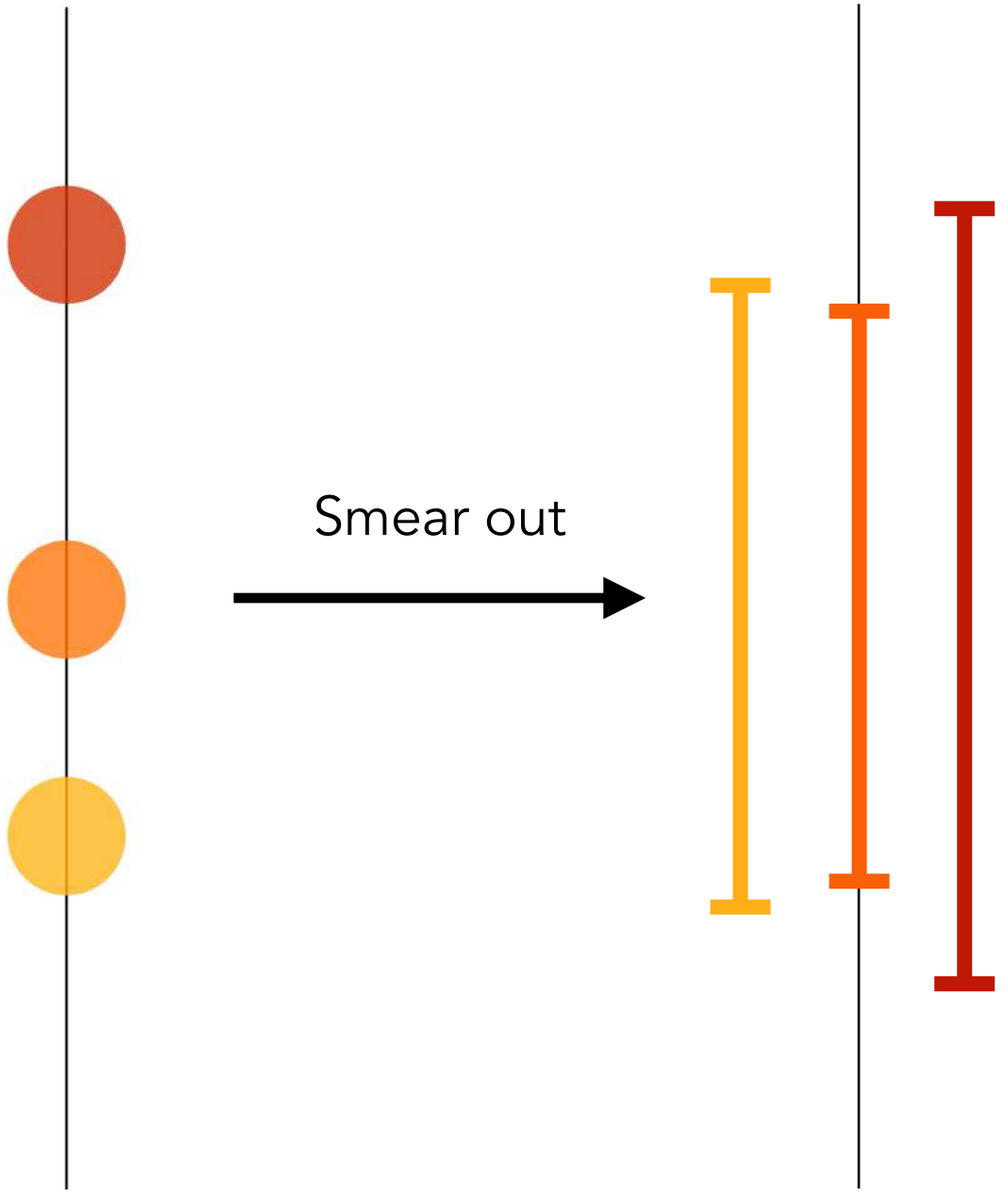
But temperature-driven wake : the colder the faster!

Ink in water



Vertical motion





One-dimensional

Orbits

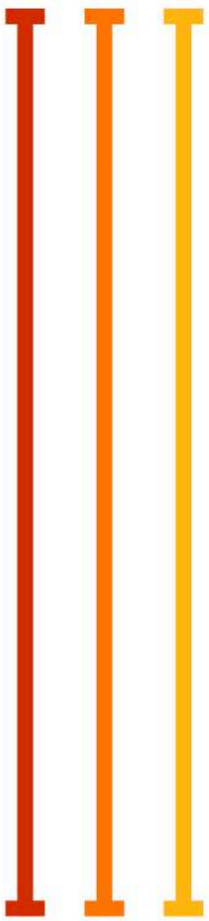
See also Joyce+(2010)

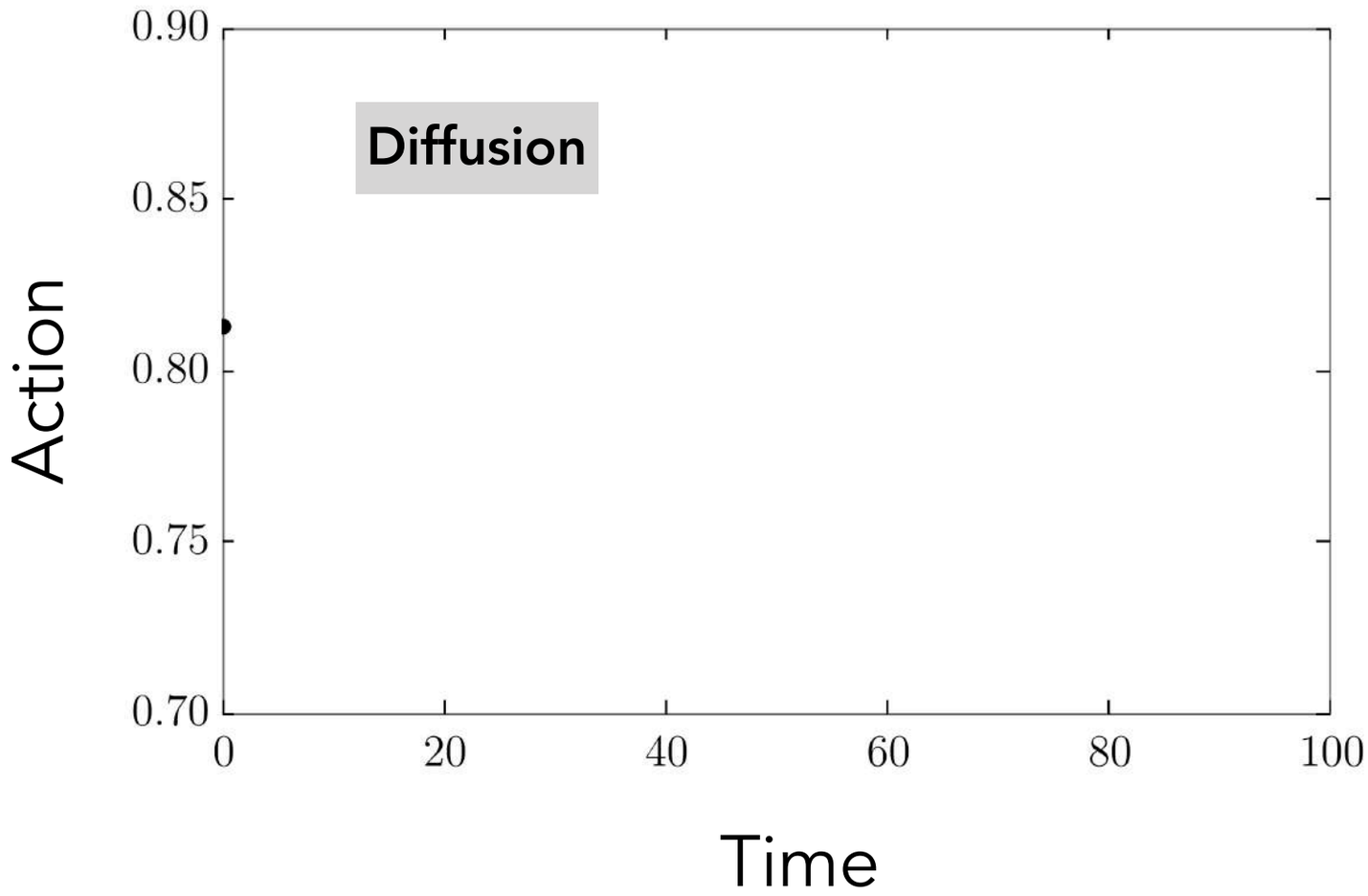




Orbital diffusion





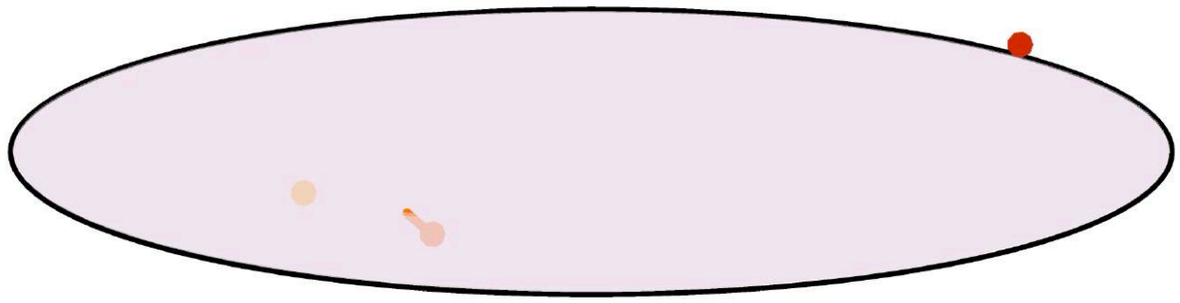


Perturbed

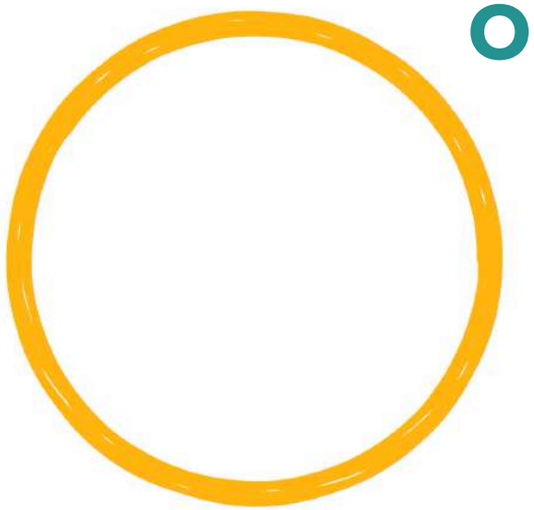
Disc **thickening**

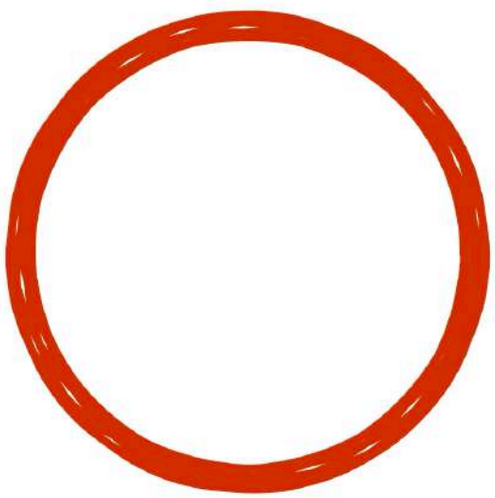






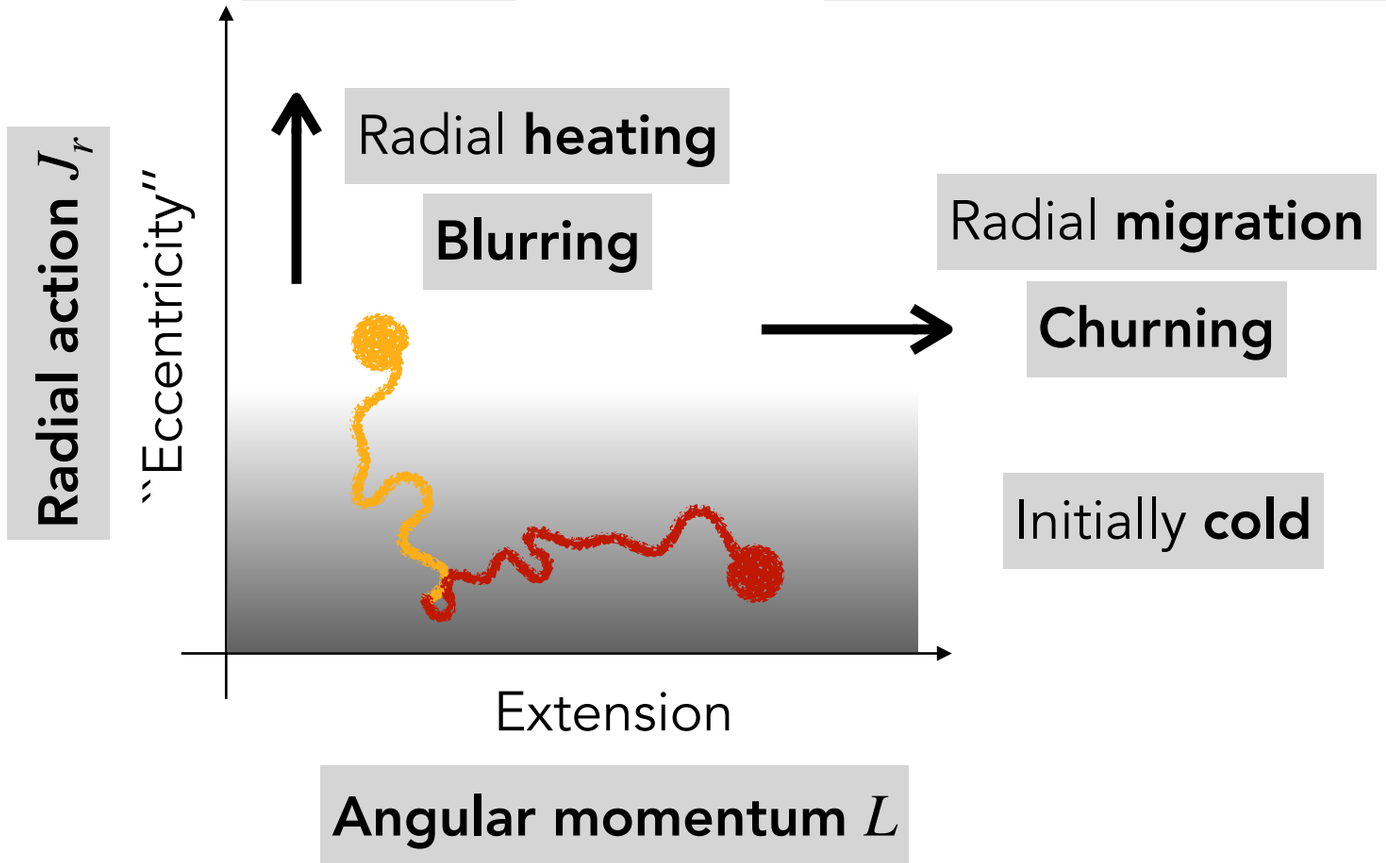
Orbital diffusion





Fluctuation-Dissipation Theorem

Diffusion rate ⟷ **Power-spectrum of the fluctuations**

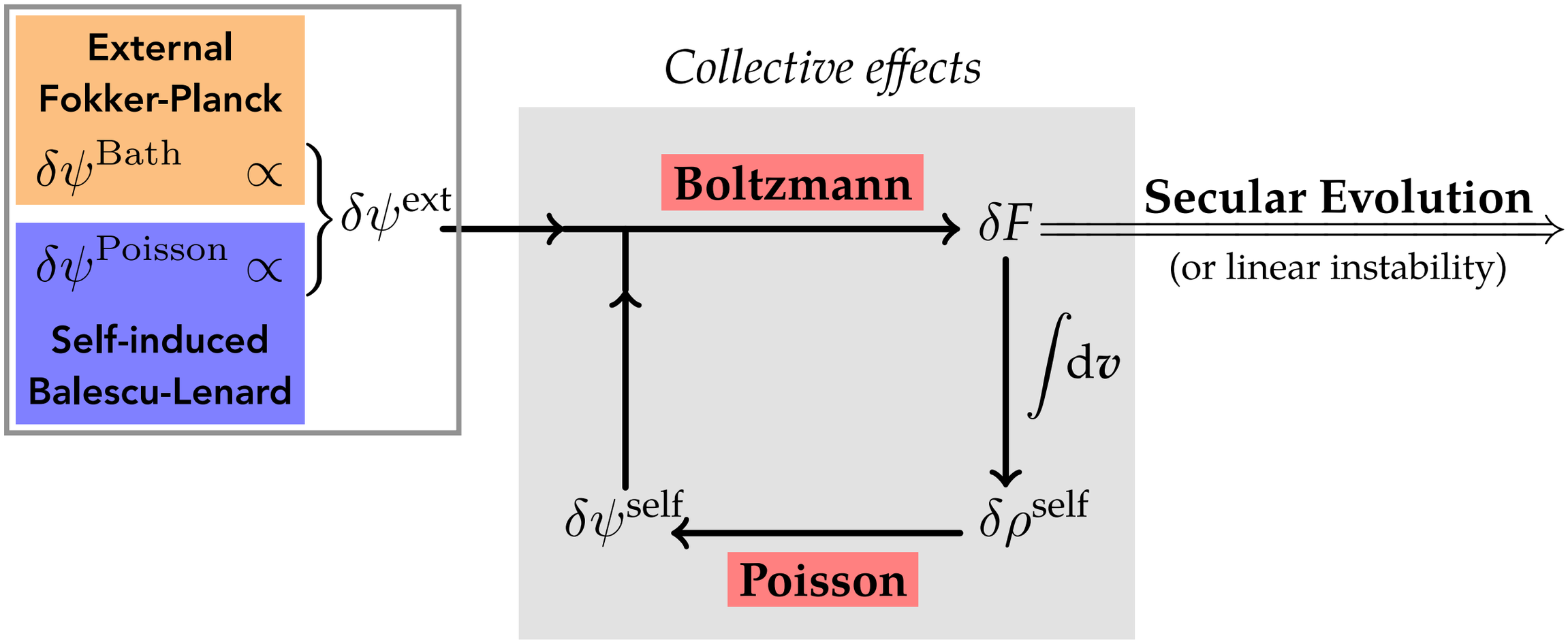


Fluctuations heat and statistically increase **radial oscillations**

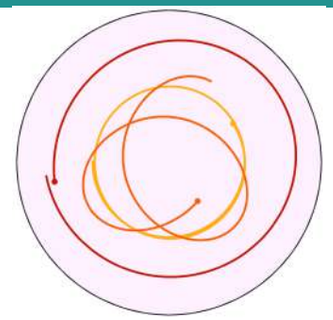
See also Sellwood+(2002)

Collective effects and perturbations

Self-gravitating amplification

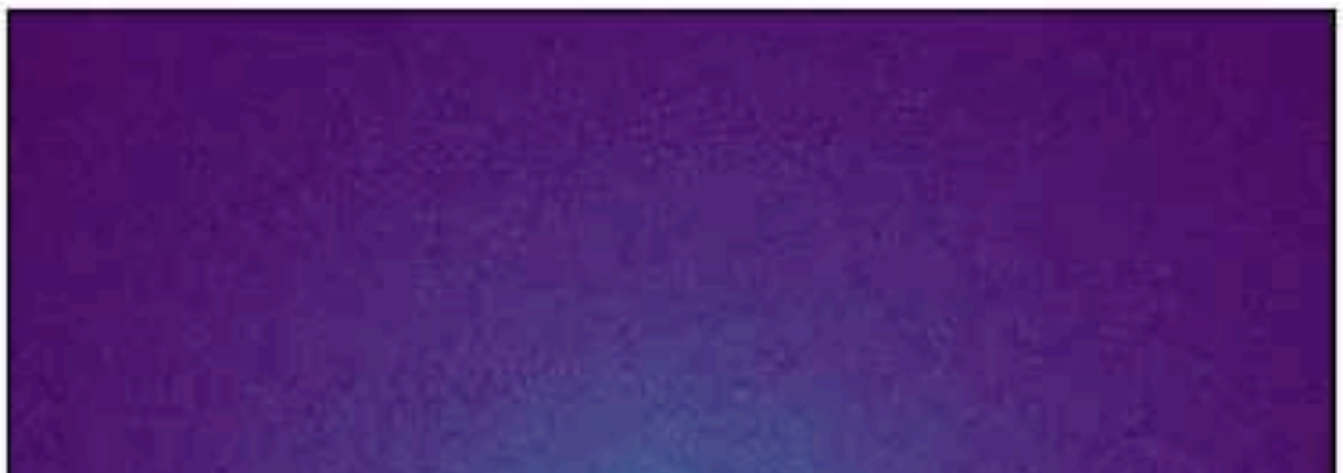


Collective effects drastically **amplify** wakes, in particular on **large scales**



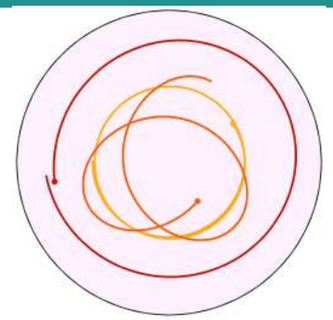
Linear instabilities

Massive cold disc

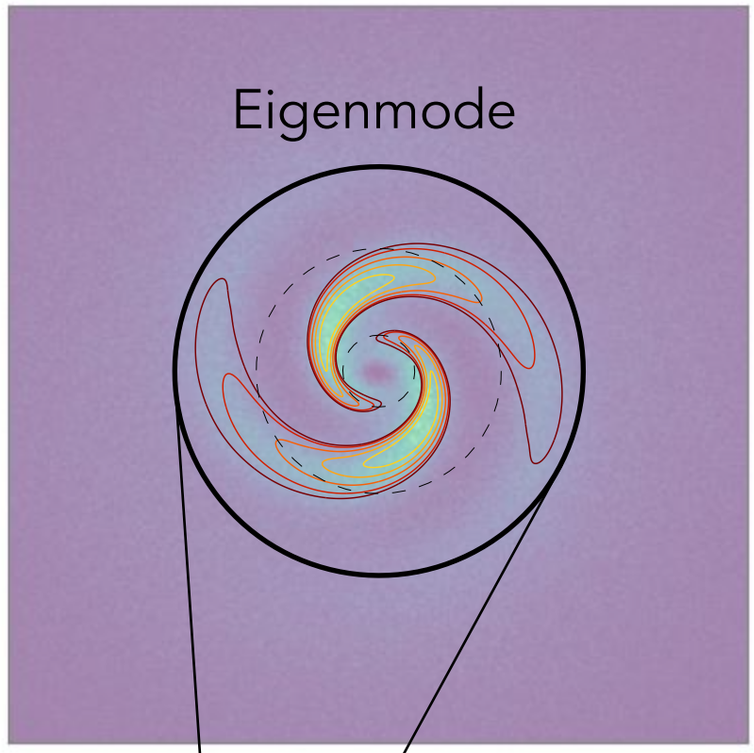


See also Zang (1976), Evans+(1998)

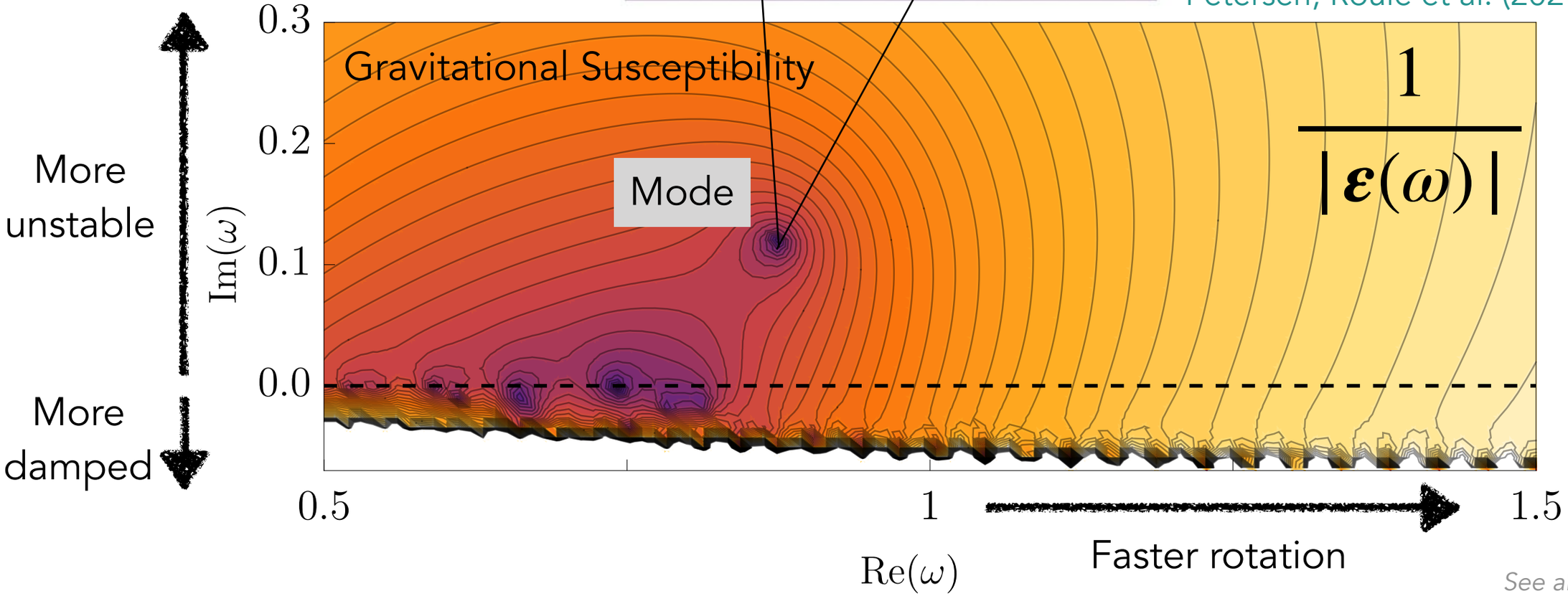
Collective effects drastically **amplify** wakes, in particular on **large scales**



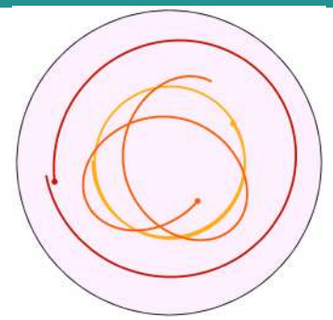
Linear instabilities



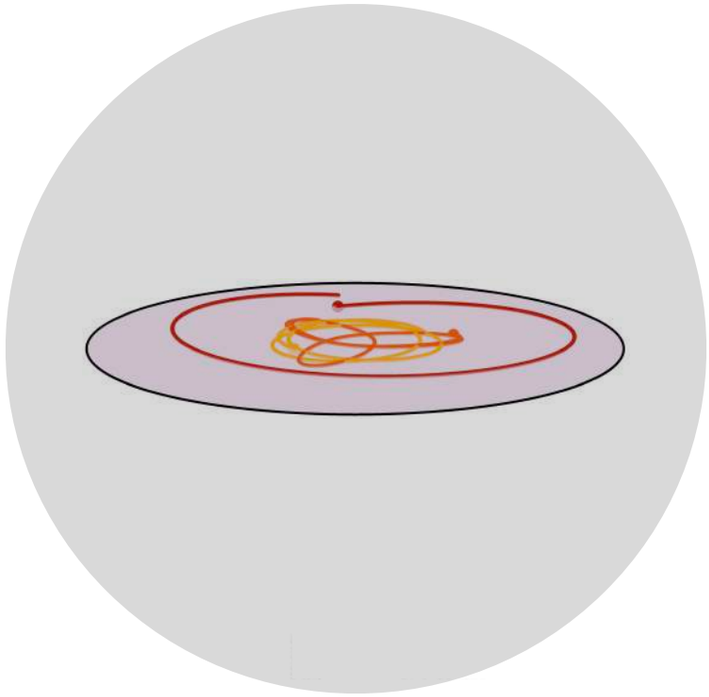
Petersen, Roule et al. (2024) - MNRAS

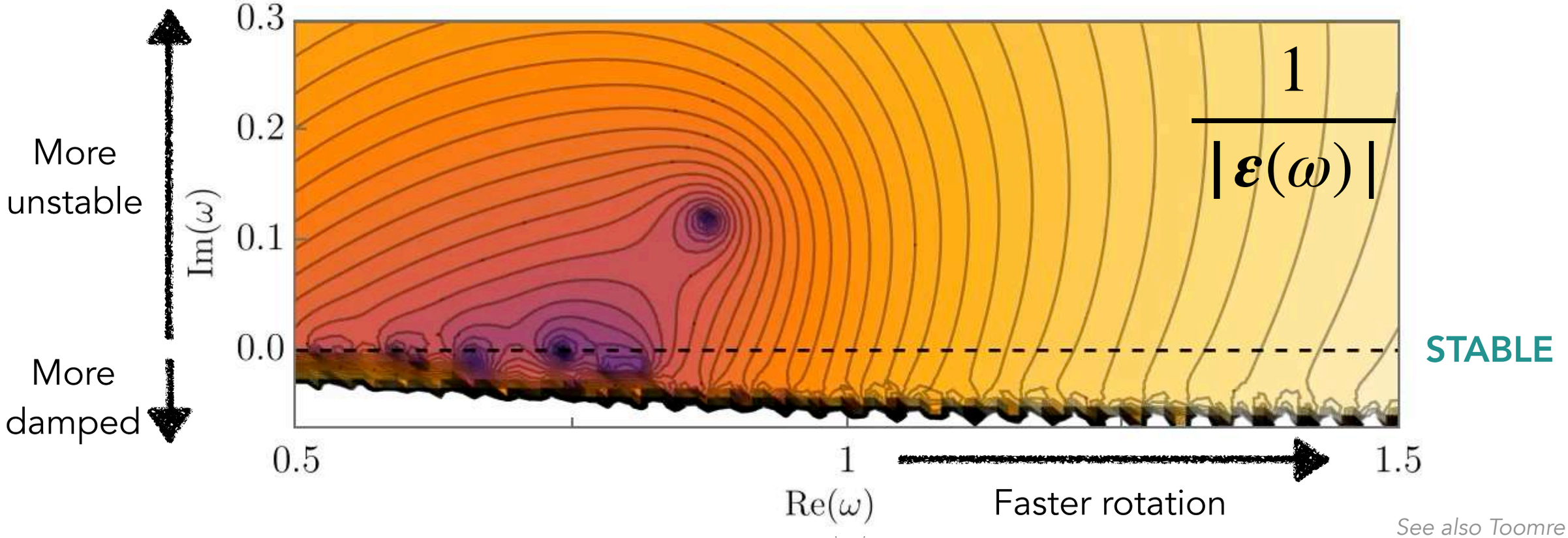


See also De Rijcke+(2016)



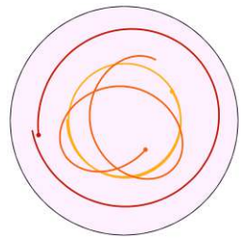
Halo and stability





See also Toomre (1981)

Collective amplification



Secular evolution equation

ensemble average

$$\frac{\partial F}{\partial t} = -\left\langle \left[\delta F, \delta H\right] \right\rangle$$

Dressing comes twice

$$M_{\text{wake}} = \frac{M_{\text{perturb}}}{|\,\varepsilon(\omega)\,|} \quad \longrightarrow \quad \frac{\partial F}{\partial t} \simeq \frac{M_{\text{perturb}}^2}{|\,\varepsilon(\omega)\,|^2}$$

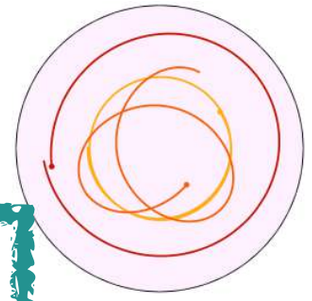
$$\frac{1}{|\,\varepsilon(\omega)\,|} \sim 30 \quad \longrightarrow \quad \frac{1}{|\,\varepsilon(\omega)\,|^2} \sim 1000$$

Toomre (1981)

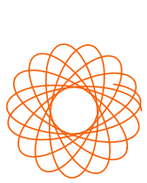
Collective effects drastically **accelerate** orbital **heating**, in particular on **large scales**



Self induced secular prediction



$N\partial F/\partial t$

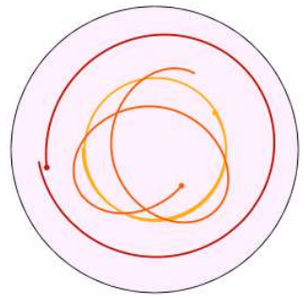


NEW

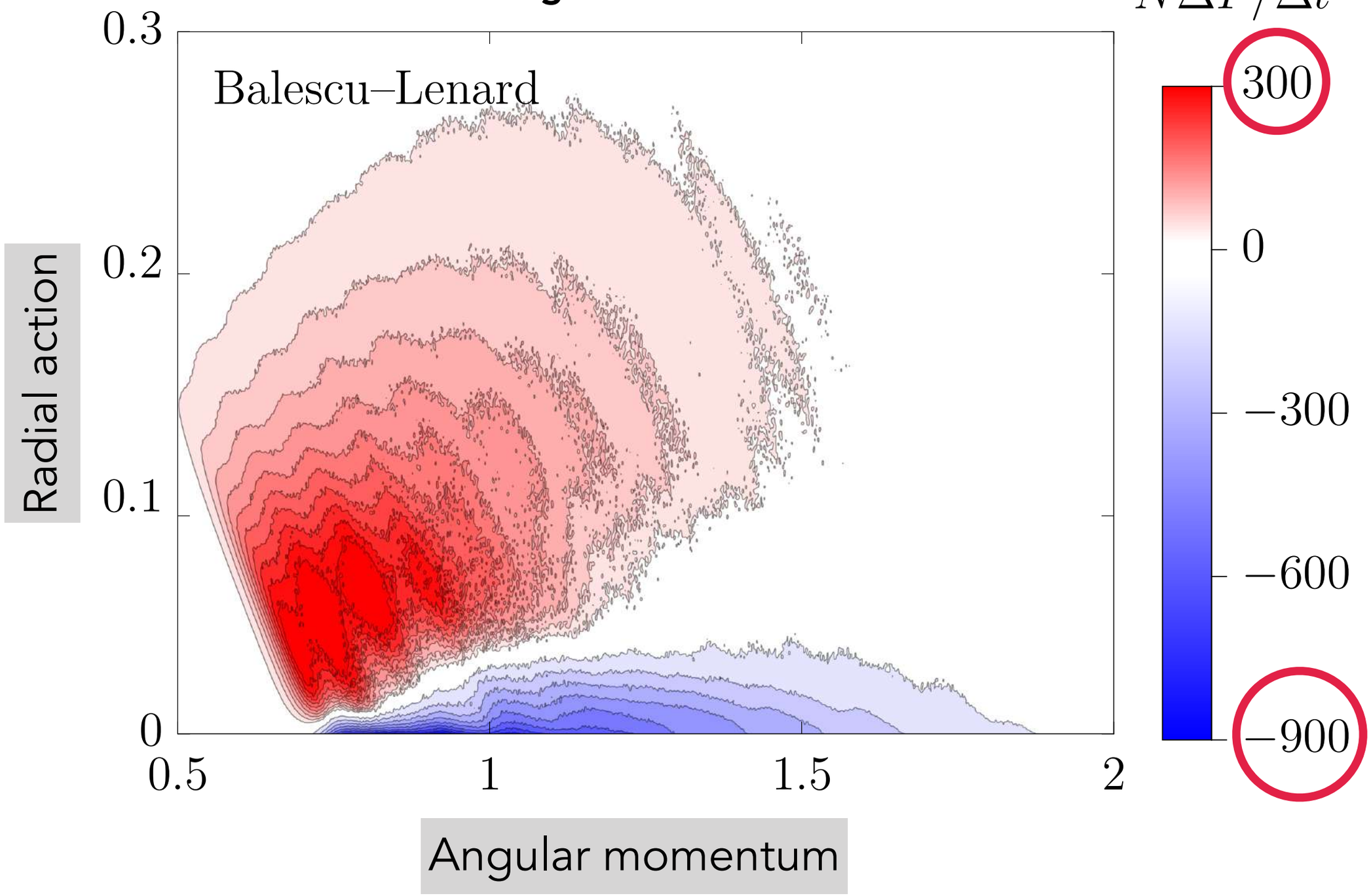
Roule+(in prep)



N–body measurements



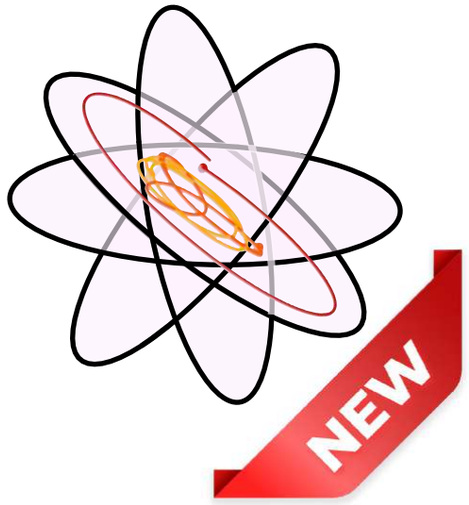
average over 100 simulations



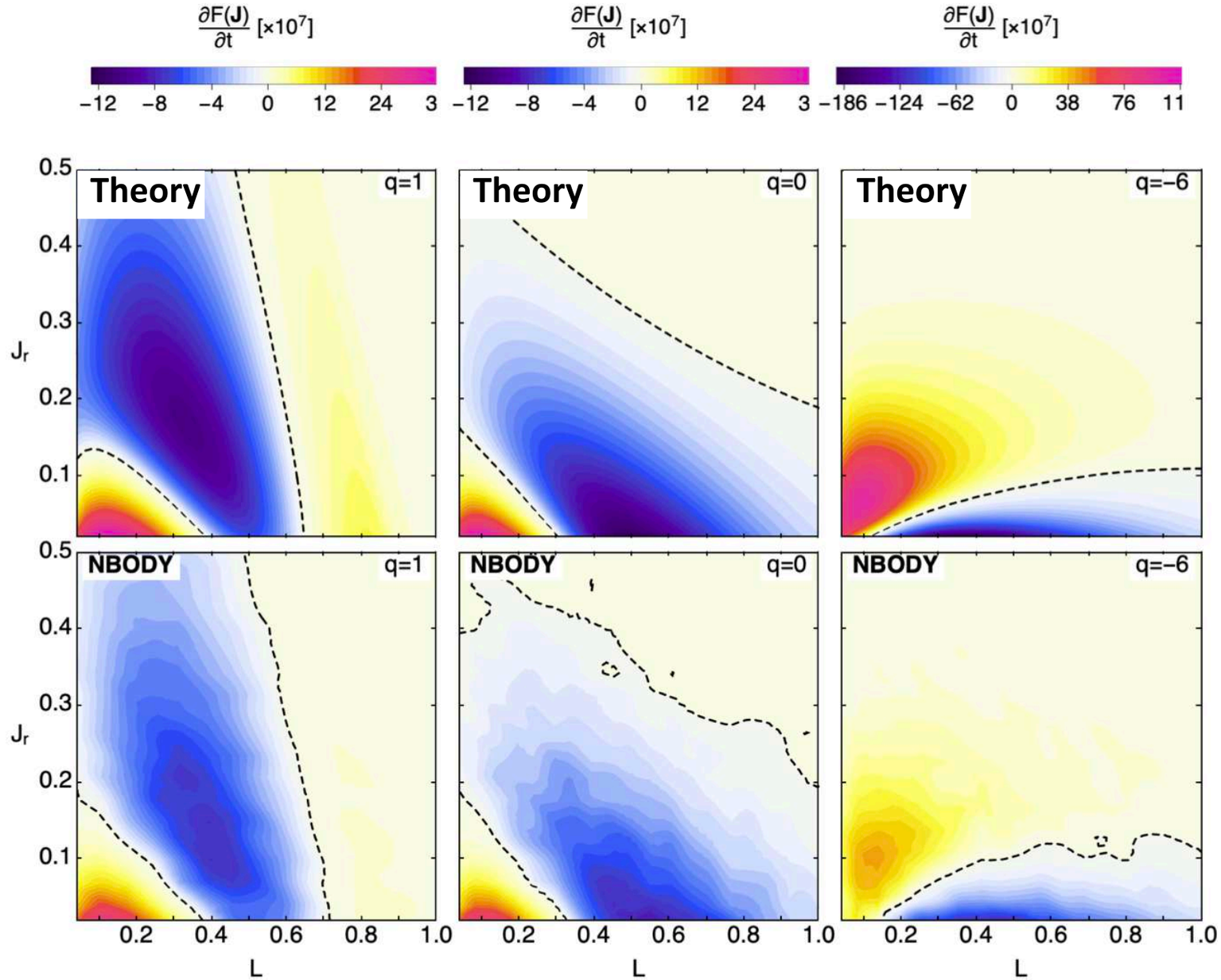
Kinetic theory satisfactorily captures the long-term heating of isolated cold discs

Dynamically hot systems : impact of **anisotropy**



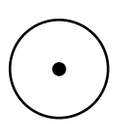


$\partial F / \partial t$



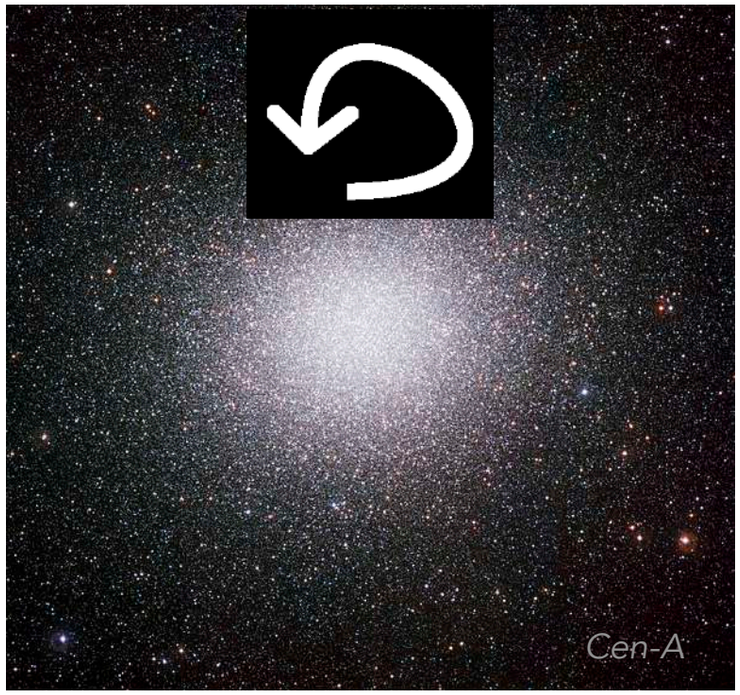
average over 100 simulations

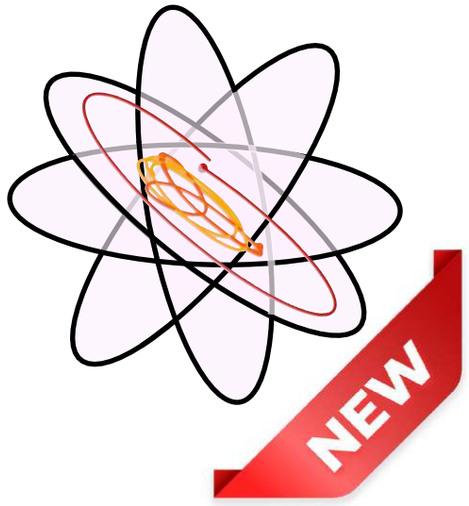




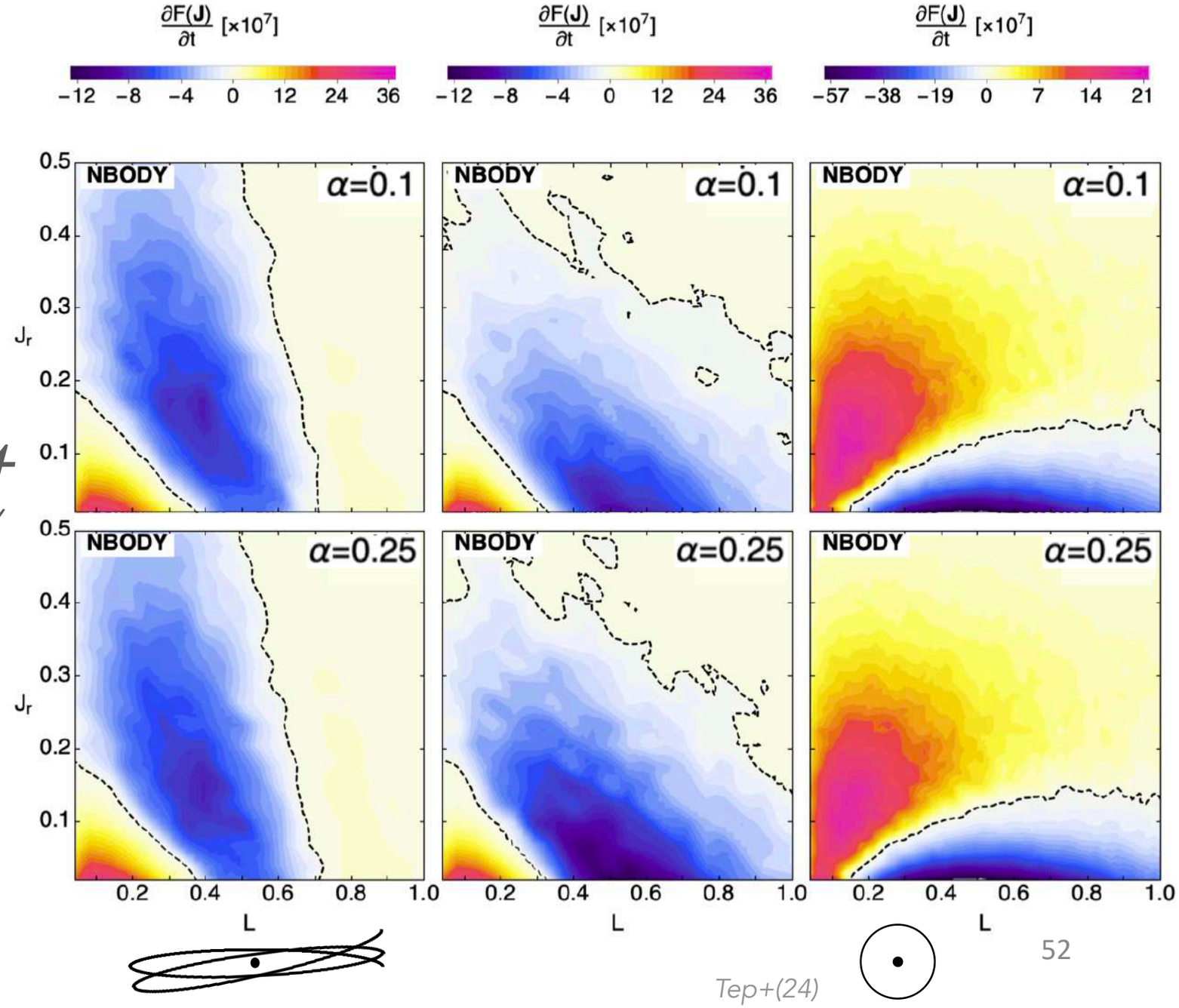
Tep+(24)

Dynamically hot systems : impact of **rotation**





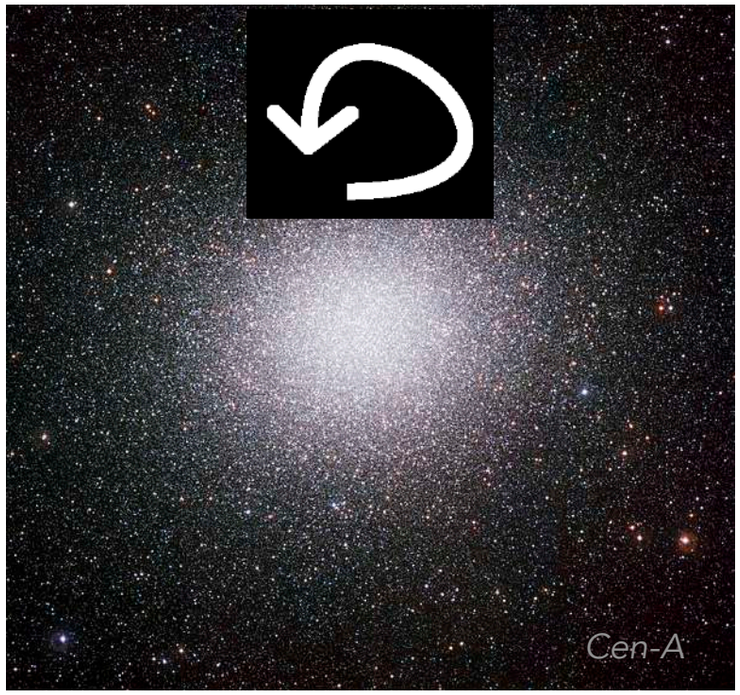
$\partial F / \partial t$

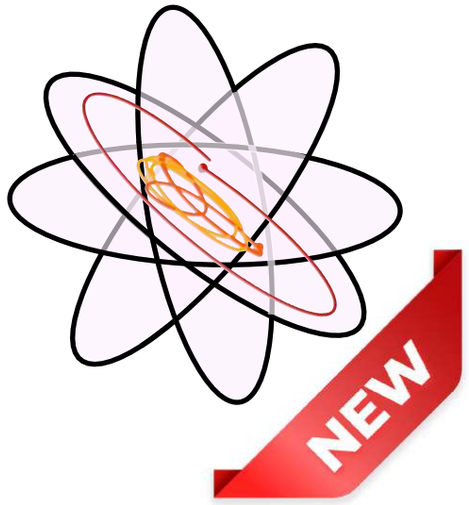


average over 100 simulations

Tep+(24)

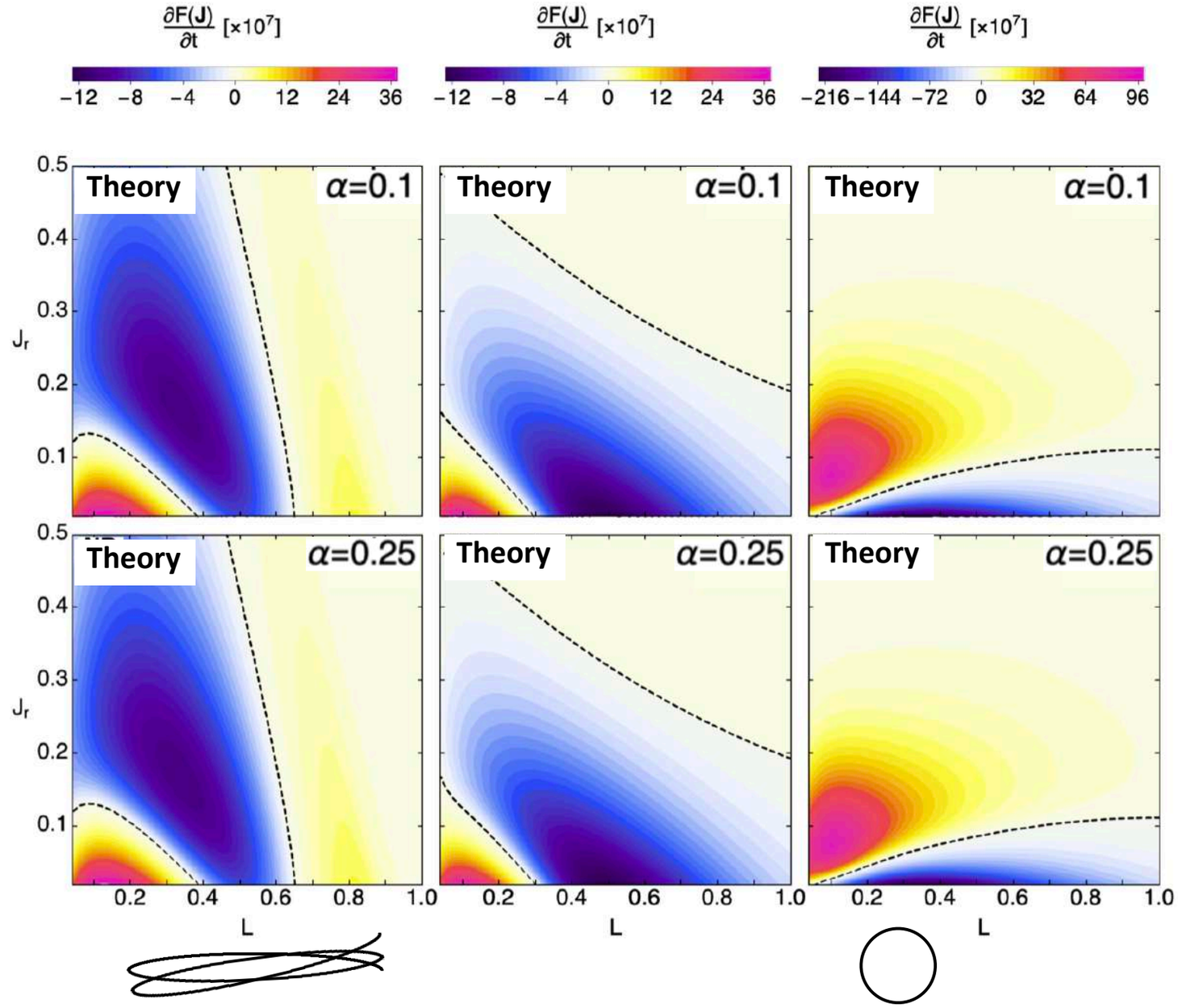
Dynamically hot systems : impact of **rotation**





$\partial F / \partial t$

average over 100 simulations



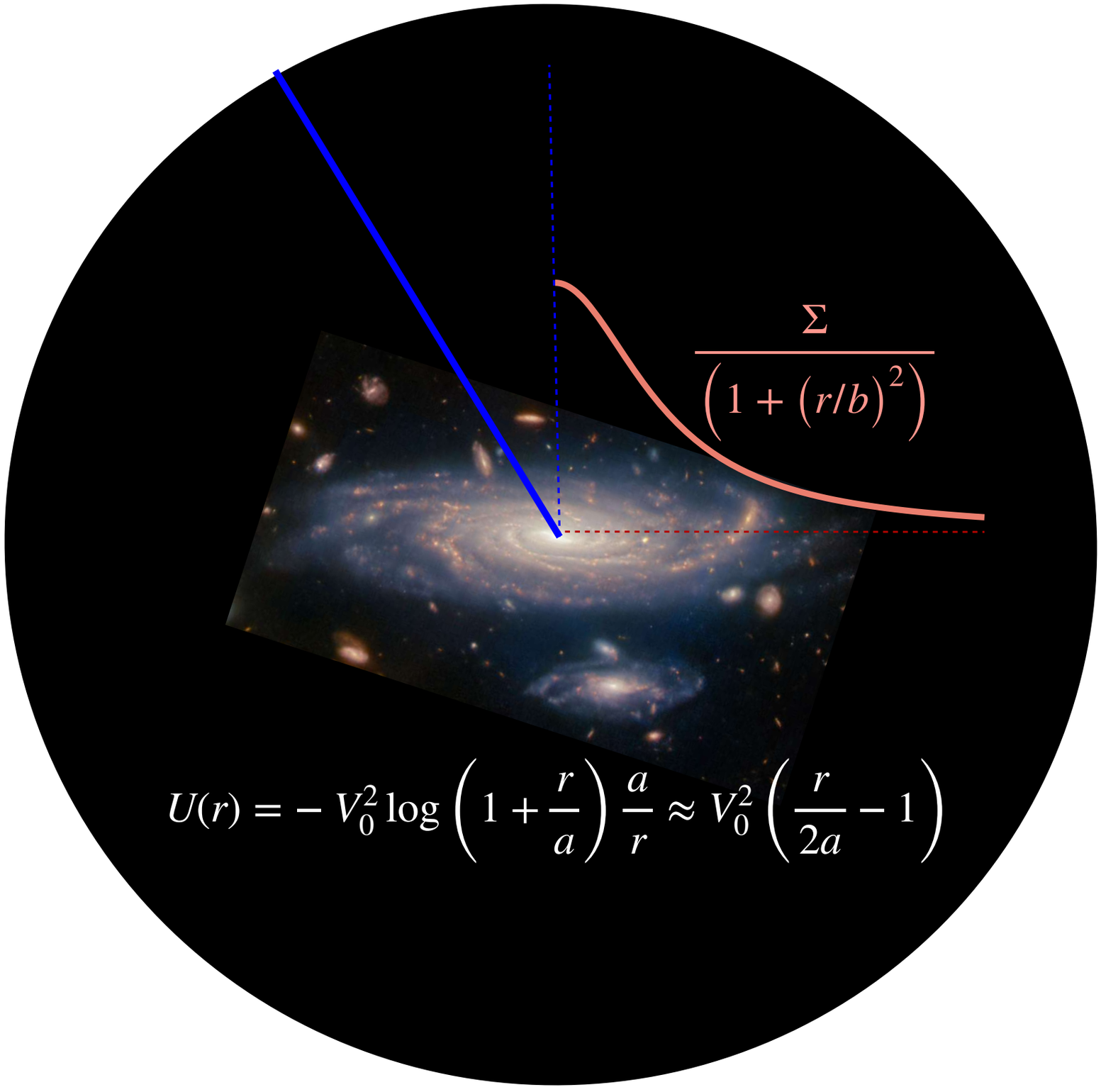
2.4 Example of external fluctuations: SN driven bubbles disolve DM cusps?

$R_{\rm bub}, T_{\rm bub}$



Toy model accounting for impact of baryons on orbital structure

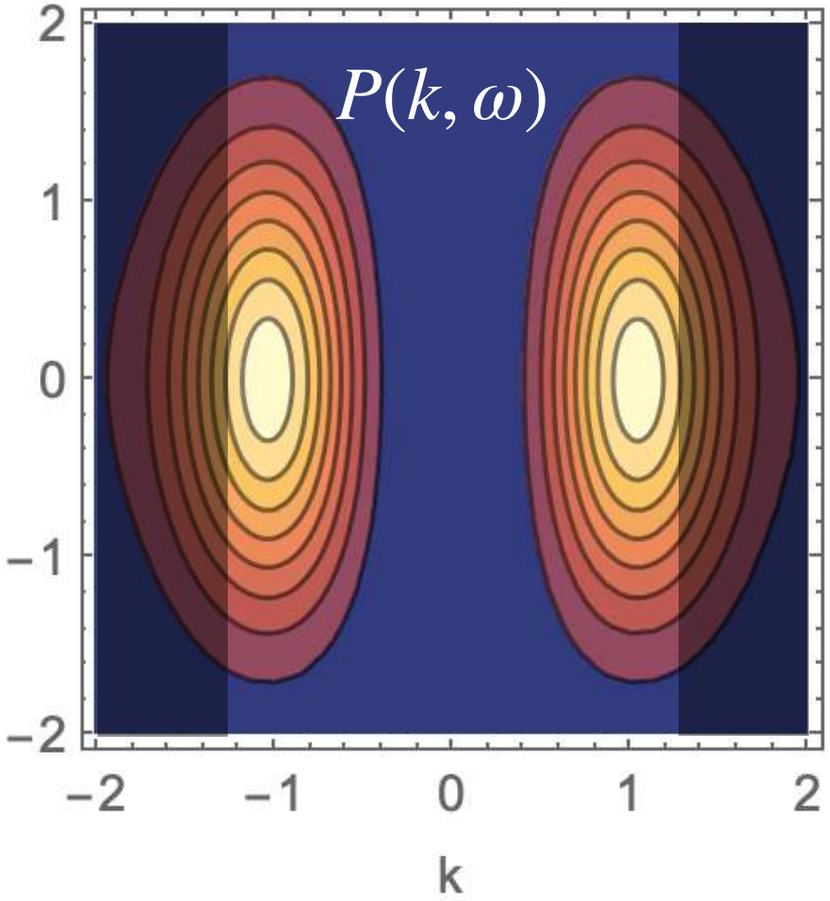
2.4 Toy model for NFW halo+ disc with SN bubbles

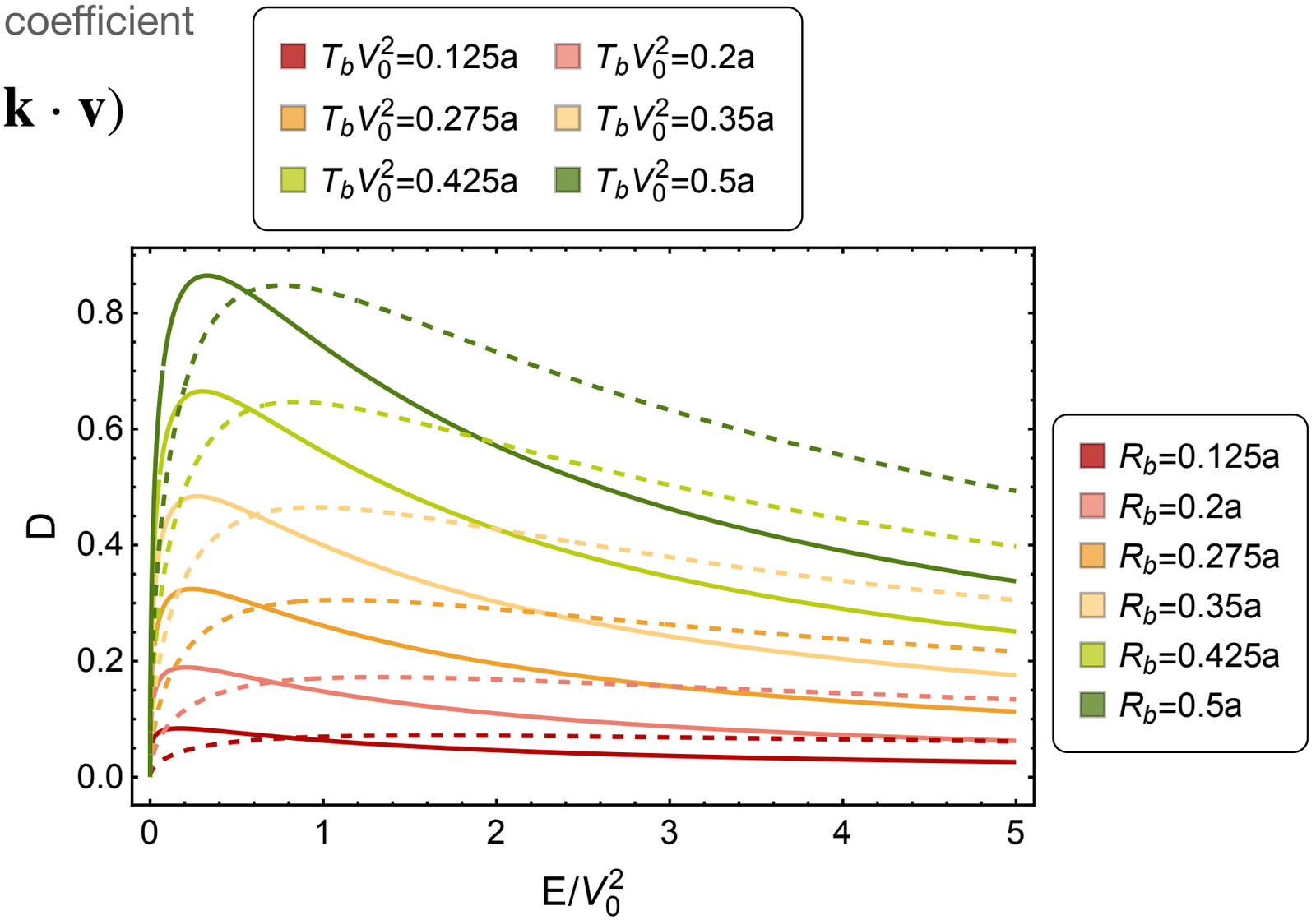


2.4 Bubble induced orbital diffusion

Chandrasekhar phase averaging Diffusion coefficient

$$D(\mathbf{J}) \propto \int \mathrm{d}\phi \int \mathrm{d}\mathbf{k}\mathbf{k} \otimes \mathbf{k} P_k(\omega = \mathbf{k}\cdot\mathbf{v})$$



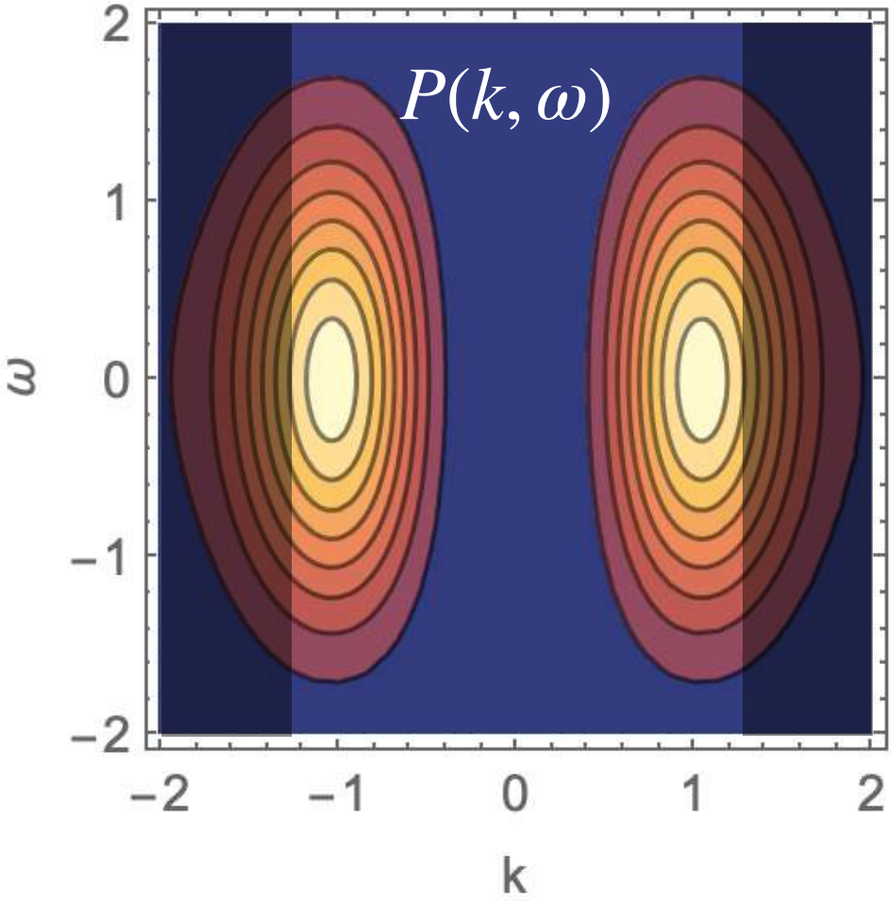


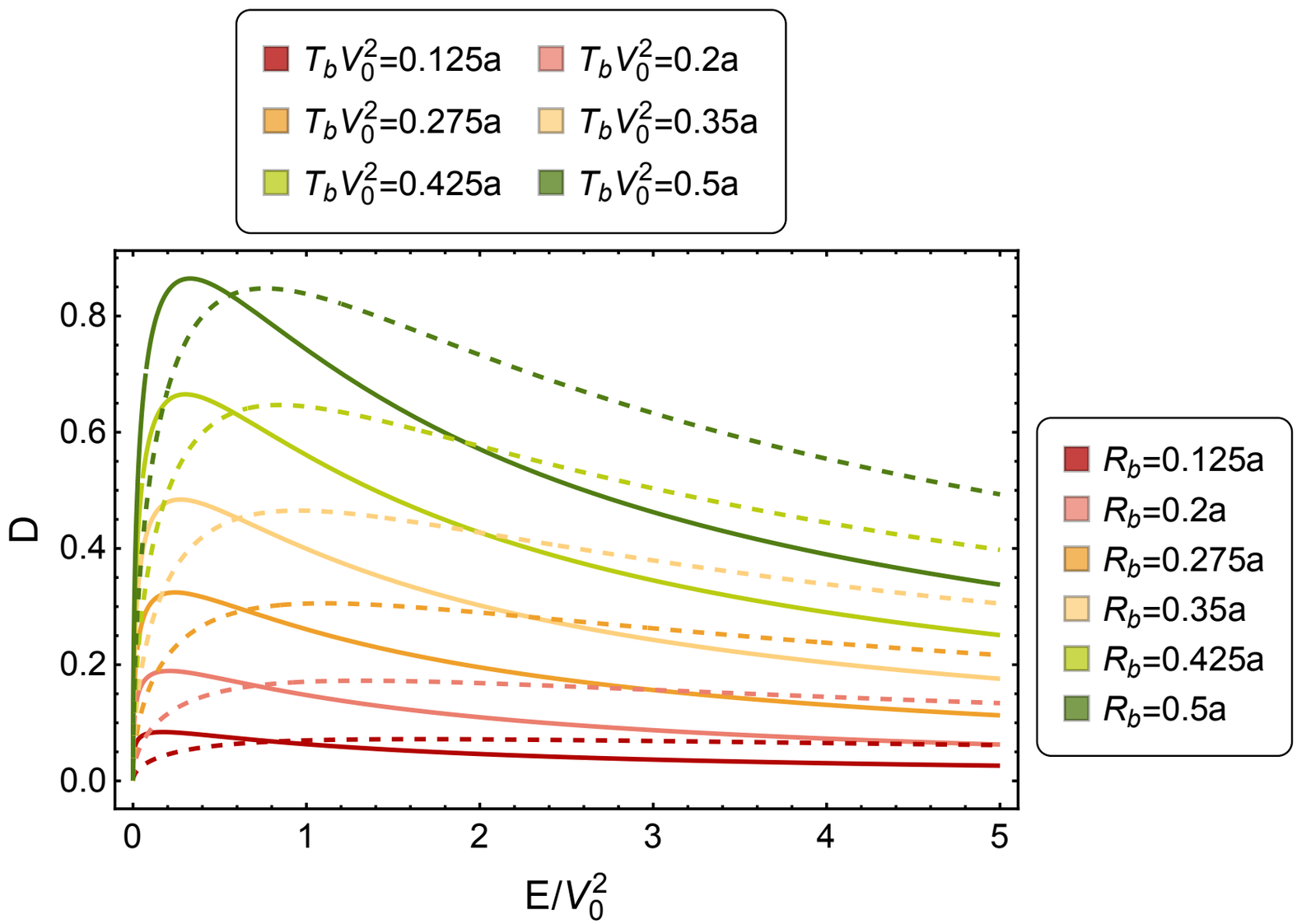
rate of bubble explosion; bubble lifespan; bubble size; energy

$$D_E = 5\sqrt{2}\pi\Sigma \frac{T_{\mathrm{bub}}^2 V_0^2 R_{\mathrm{bub}}}{3a^3} \sqrt{\frac{E}{V_0^2}} \left(1 - \frac{2}{3}\frac{a}{b}\frac{E}{V_0^2}\right)$$

potential scale length; potential amplitude; bubble distribution scale length

2.4 Orbital diffusion: implication for cusp core transformation

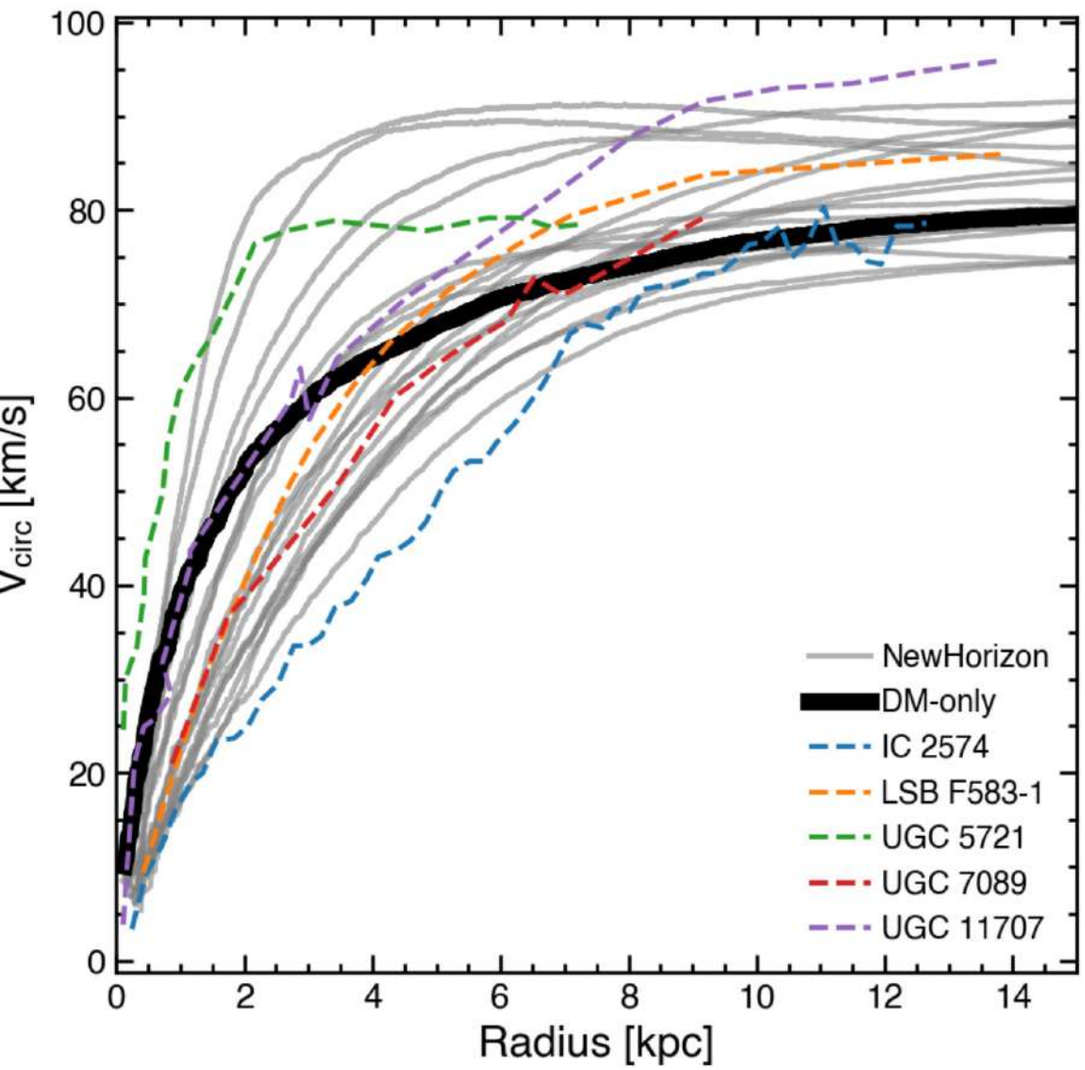


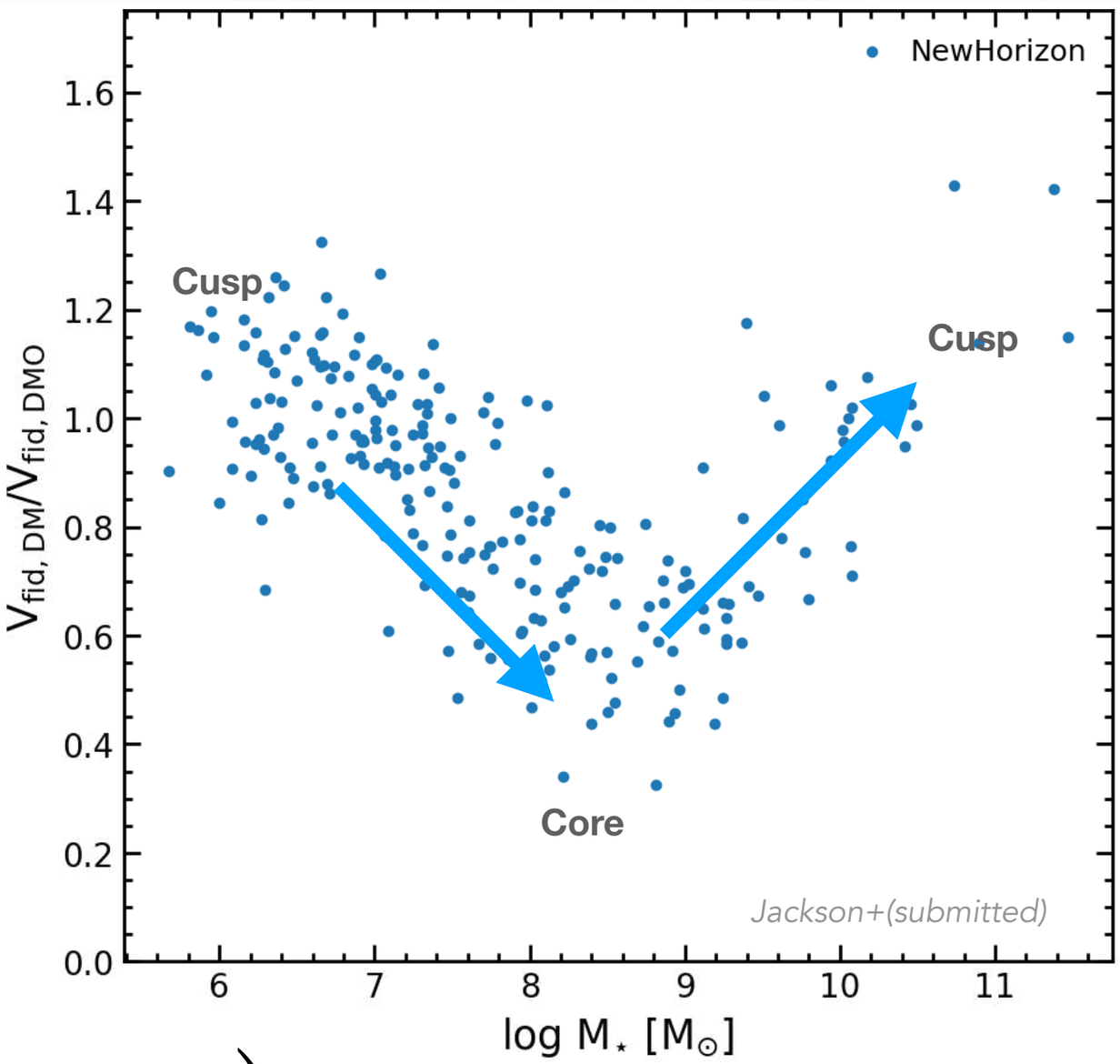


$$D_{\max} = \frac{10\pi}{9}\Sigma\sqrt{\frac{b}{a}}\frac{R_{\text{bub}}}{a}\frac{V_0^2 T_{\text{bub}}^2}{a^2}\left(1 - \frac{3}{5R_{\text{bub}}k_{\max}}\right) \quad \text{at} \quad E = V_0^2\frac{b}{3a}$$

- **Smaller scales** impact galactic scales through **fluctuations**
- Diffusion only effective when processes are commensurable

2.4 Application: rotational curve diversity+ Cusp-core-cusp transformation





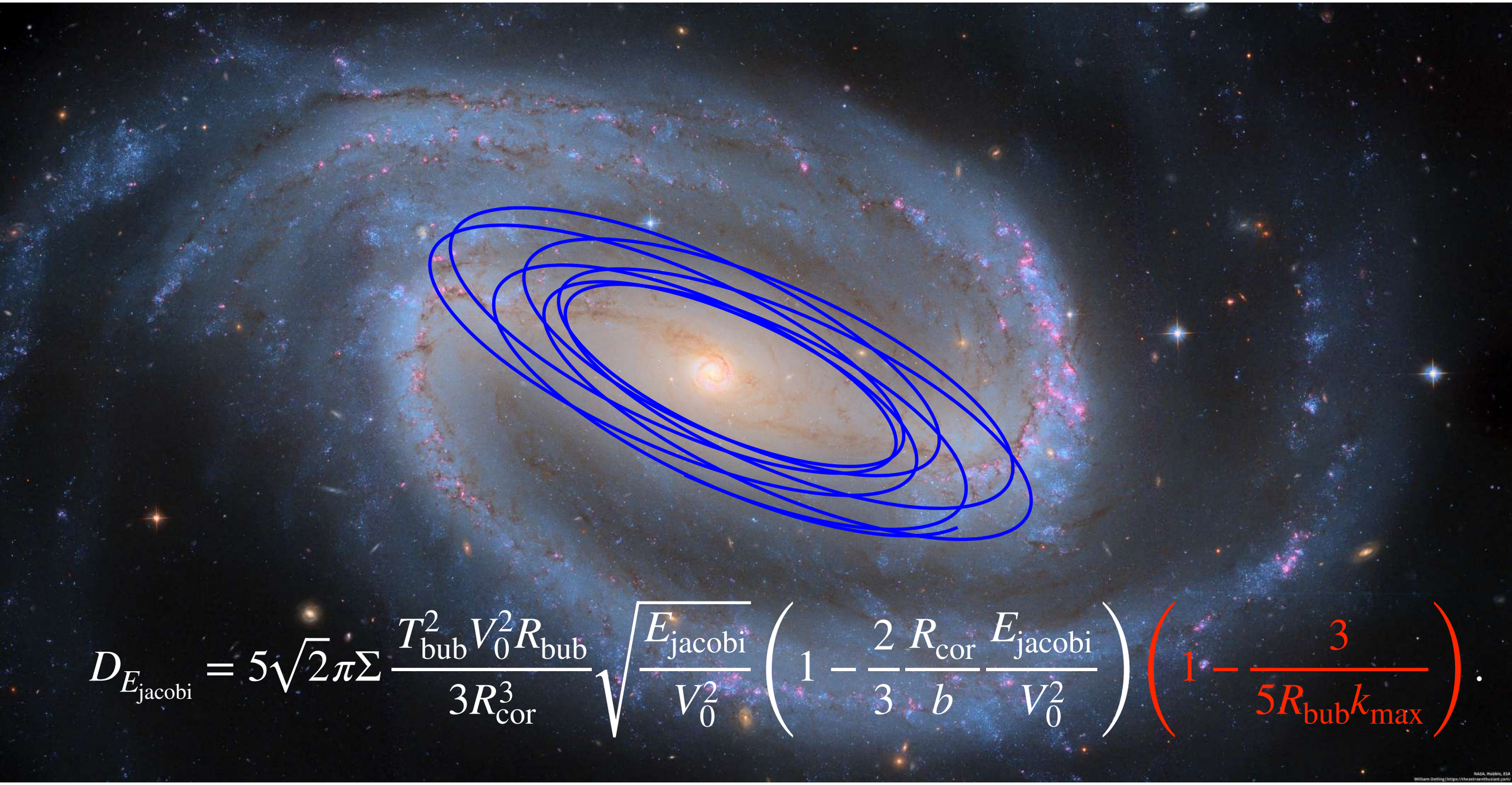
$$D_{\max} = \frac{10\pi}{9}\Sigma\sqrt{\frac{b}{a}}\frac{R_{\rm bub}}{a}\frac{V_0^2 T_{\rm bub}^2}{a^2}\left(1-\frac{3}{5R_{\rm bub}k_{\max}}\right) \quad \text{at} \quad E = V_0^2\frac{b}{3a}$$

- Why galaxy centres go through cusp → core → cusp

IN PROGRESS

2.4 Application: Bar orbits subject to deflection in co-rotating frame

Diffusion only effective when processes are commensurable



$$D_{E_{\mathrm{jacobi}}} = 5\sqrt{2}\pi\Sigma \frac{T_{\mathrm{bub}}^2 V_0^2 R_{\mathrm{bub}}}{3R_{\mathrm{cor}}^3} \sqrt{\frac{E_{\mathrm{jacobi}}}{V_0^2}} \left(1 - \frac{2}{3}\frac{R_{\mathrm{cor}}}{b}\frac{E_{\mathrm{jacobi}}}{V_0^2}\right) \left(1 - \frac{3}{5R_{\mathrm{bub}}k_{\mathrm{max}}}\right).$$

- Why resolution impacts bar resilience ($k_{\mathrm{max}}R_{\mathrm{bub}} \neq \infty$)

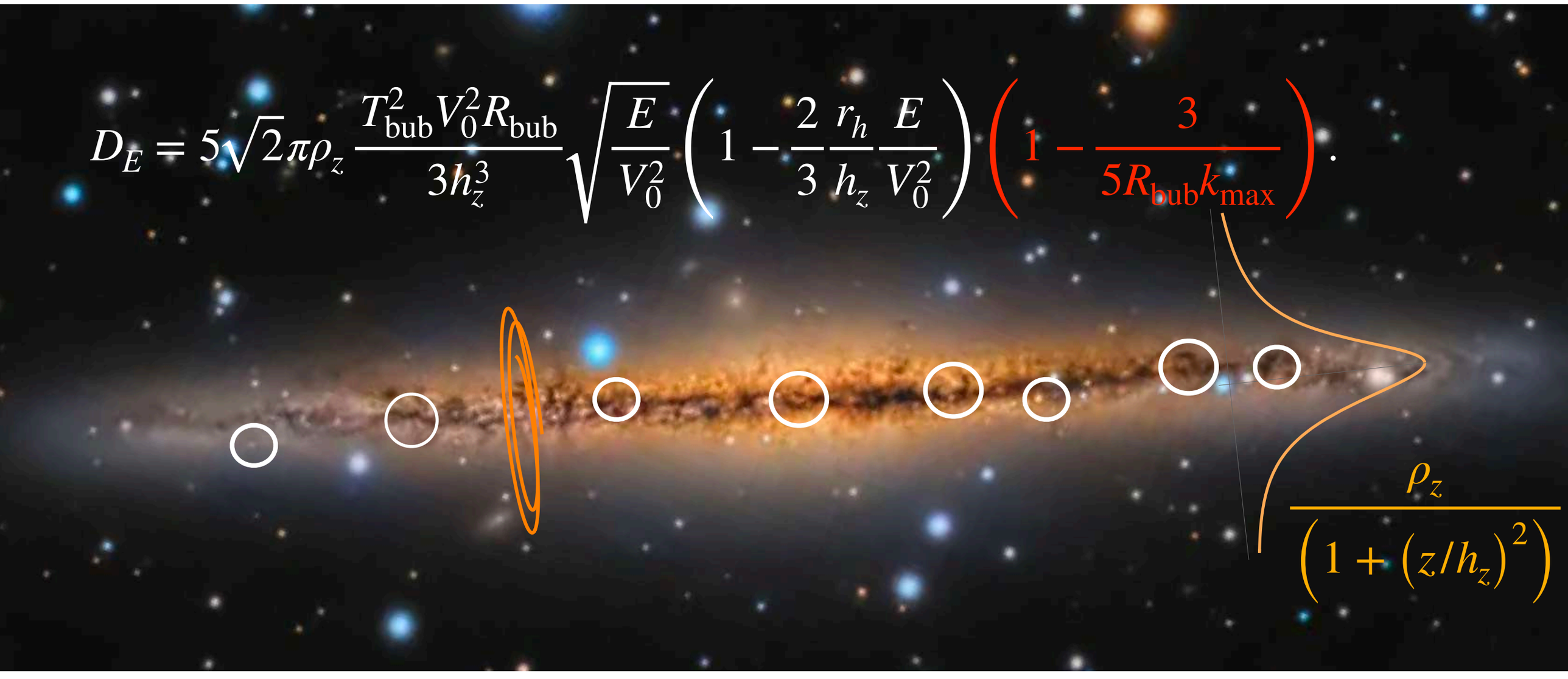
IN PROGRESS

2.4 Application: impact of SN bubbles on vertical disc heating

Diffusion only effective when processes are commensurable

IN PROGRESS

$$D_E = 5\sqrt{2}\pi\rho_z \frac{T_{\rm bub}^2 V_0^2 R_{\rm bub}}{3h_z^3}\sqrt{\frac{E}{V_0^2}}\left(1-\frac{2}{3}\frac{r_h}{h_z}\frac{E}{V_0^2}\right)\left(1-\frac{3}{5R_{\rm bub}k_{\rm max}}\right).$$



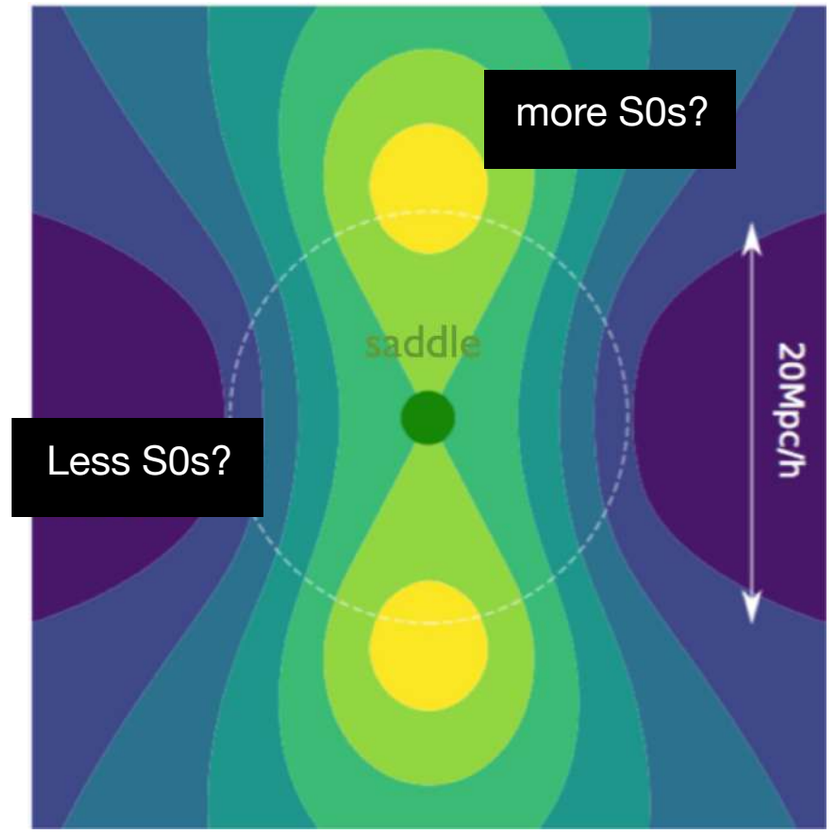
- Why resolution impacts vertical heating of disc ($k_{\rm max}R_{\rm bub} \neq \infty$)

2.4 Observable: morphology change within saddle frame

$$\frac{\partial f_R}{\partial t} = \frac{\partial}{\partial \mathbf{J}} \cdot \mathbf{D}_R \cdot \frac{\partial f_R}{\partial \mathbf{J}} + s_R(\mathbf{J}) \,,$$

$s_R(\mathbf{J})$: source of new stars

possibly subject to environmental variation labelled by R= $(r, \mathcal{Q})$



Green function

IN PROGRESS

$$G_R(\mathbf{J}, t \,|\, \mathbf{J}', t') \approx \frac{\exp\left(-\dfrac{(\mathbf{J}-\mathbf{J}')^{\mathrm{T}}\, \mathbf{D}_R^{-1}\, (\mathbf{J}-\mathbf{J}')}{4\,(t-t')} \right)}{\sqrt{(4\pi\,(t-t'))^3 \det(\mathbf{D}_R)}} \,.$$

predict **morphology** (subject to R?)

$$f_R(\mathbf{J}, t) = \int \mathrm{d}\mathbf{J}' \mathrm{d}t' G_R(\mathbf{J}, t \,|\, \mathbf{J}', t') s_R(\mathbf{J}', t') \,,$$

Differential competition between

- Heating by orbital diffusion
- Cooling by star formation on circular orbits
- Quenching of gas inflow

Diffusion coefficient

$$\mathbf{D}_R \propto \left\langle \left| \delta\psi(k, \omega) \right|^2 \right\rangle_R$$

$\delta\psi(k, \omega)$: potential fluctuations

$$\Sigma_R(r, t) = \int \mathrm{d}^3 v \, \mathrm{d}z f_R(\mathbf{J}, t) \,,$$

Surface density

$$1/n_s(R, t) \equiv - \left\langle \mathrm{d} \log \Sigma_R / \mathrm{d} \log r \right\rangle_r$$

n_s: Sérsic index

2.4 Observable: chemical change within saddle frame?

$$\frac{\partial f_R}{\partial t} = \frac{\partial}{\partial \mathbf{J}} \cdot \mathbf{D}_R \cdot \frac{\partial f_R}{\partial \mathbf{J}} + s_R(\mathbf{J}),$$

$s_R(\mathbf{J})$: source of new stars

possibly subject to environmental variation labelled by R= $(r, \mathcal{Q})$

Green function

IN PROGRESS

$$G_R(\mathbf{J}, t \,|\, \mathbf{J}', t') \approx \frac{\exp\left(-\dfrac{(\mathbf{J}-\mathbf{J}')^{\mathrm{T}}\,\mathbf{D}_R^{-1}\,(\mathbf{J}-\mathbf{J}')}{4\,(t-t')}\right)}{\sqrt{(4\pi\,(t-t'))^3\,\det(\mathbf{D}_R)}}.$$

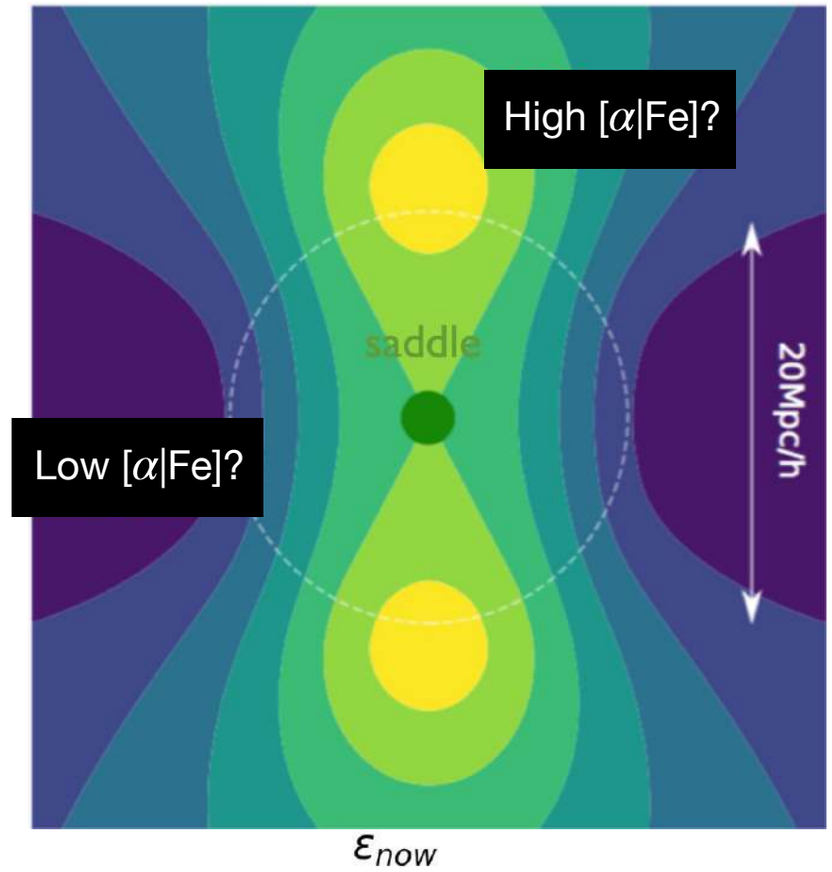
Diffusion coefficient

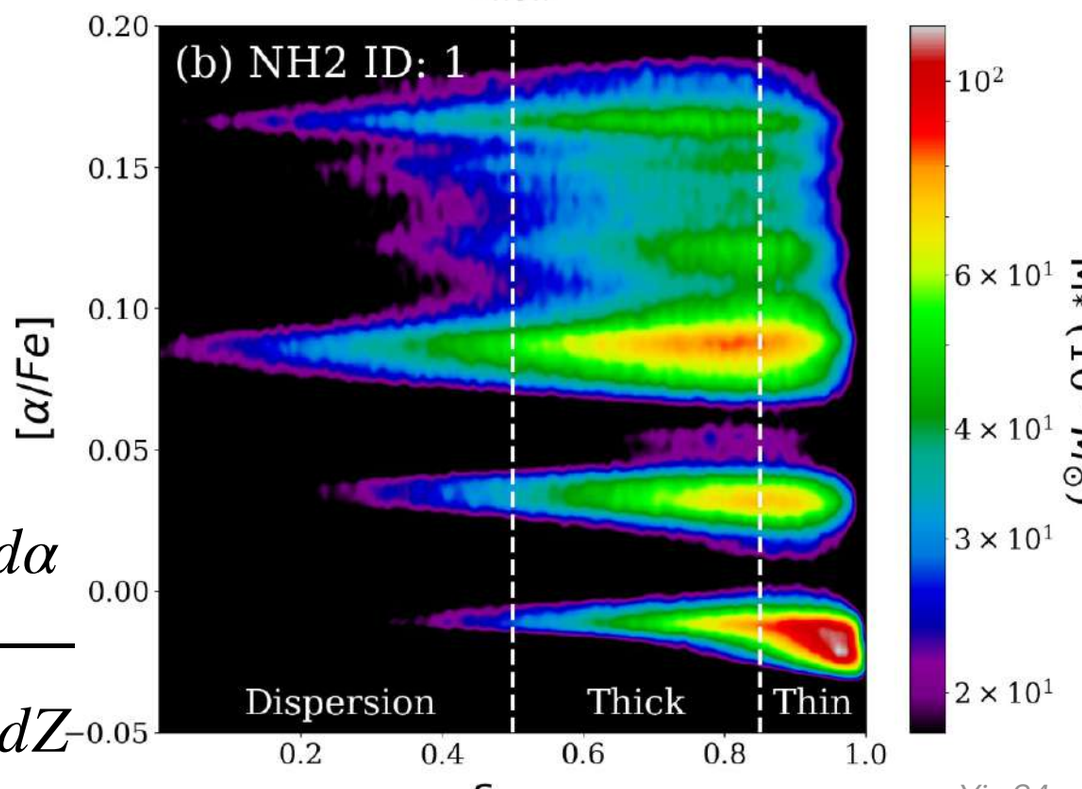
$$\mathbf{D}_R \propto \left\langle \left| \delta\psi(k,\omega) \right|^2 \right\rangle_R$$

$\delta\psi(k,\omega)$: potential fluctuations

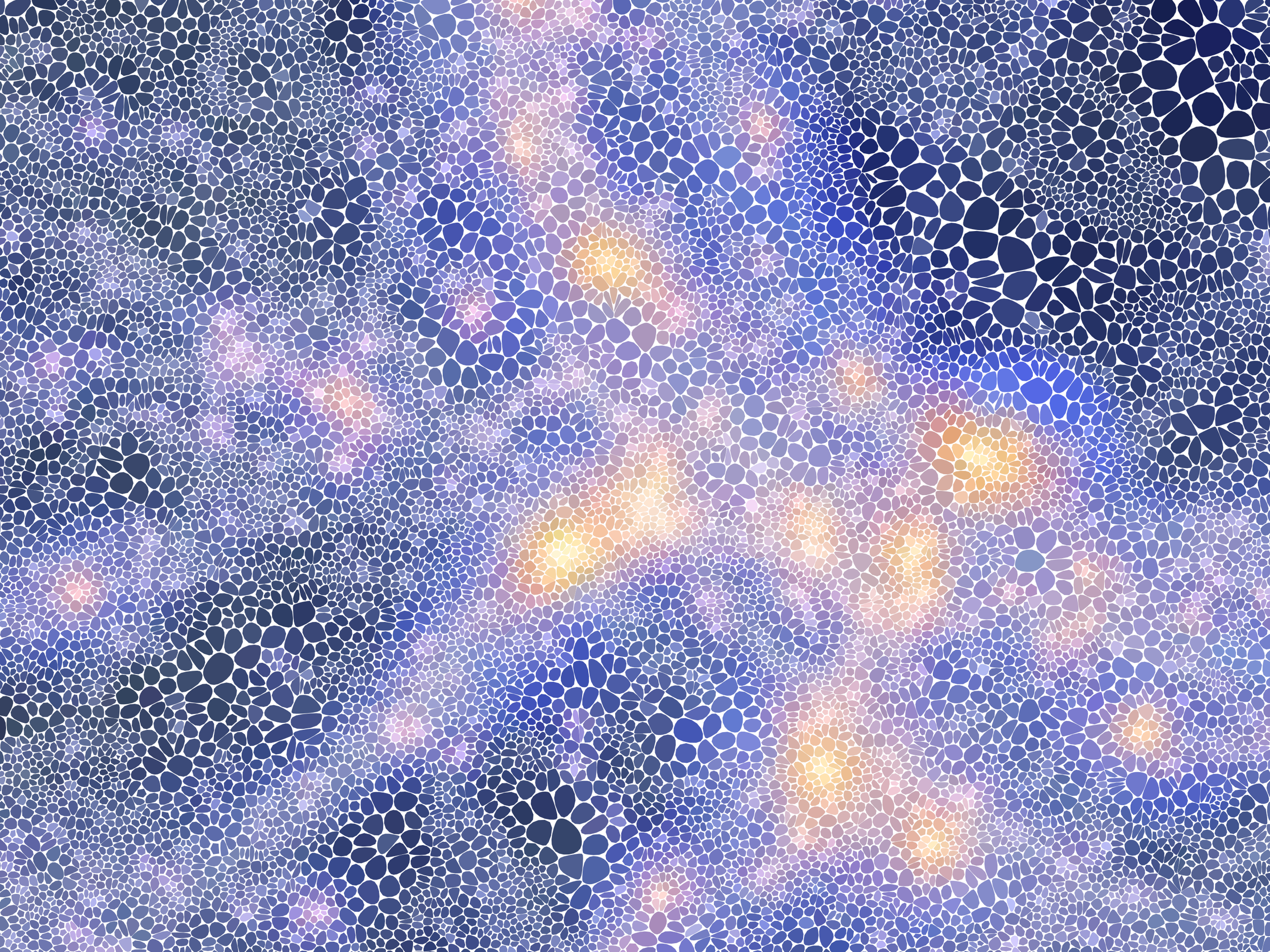
predict element abundance ratio (subject to R?)

$$[\alpha/Z](\mathbf{I}, t) = \log \frac{\int \dot{\Sigma}_*(t_0(\alpha)) G(\mathbf{I}, t \,|\, \mathbf{I}_0, t_0(\alpha)) \left| dt_0/d\alpha \right| \alpha d\alpha}{\int \dot{\Sigma}_*(t_0(Z)) G(\mathbf{I}, t \,|\, \mathbf{I}_0, t_0(Z)) \left| dt_0/dZ \right| Z dZ}$$





Yi+24



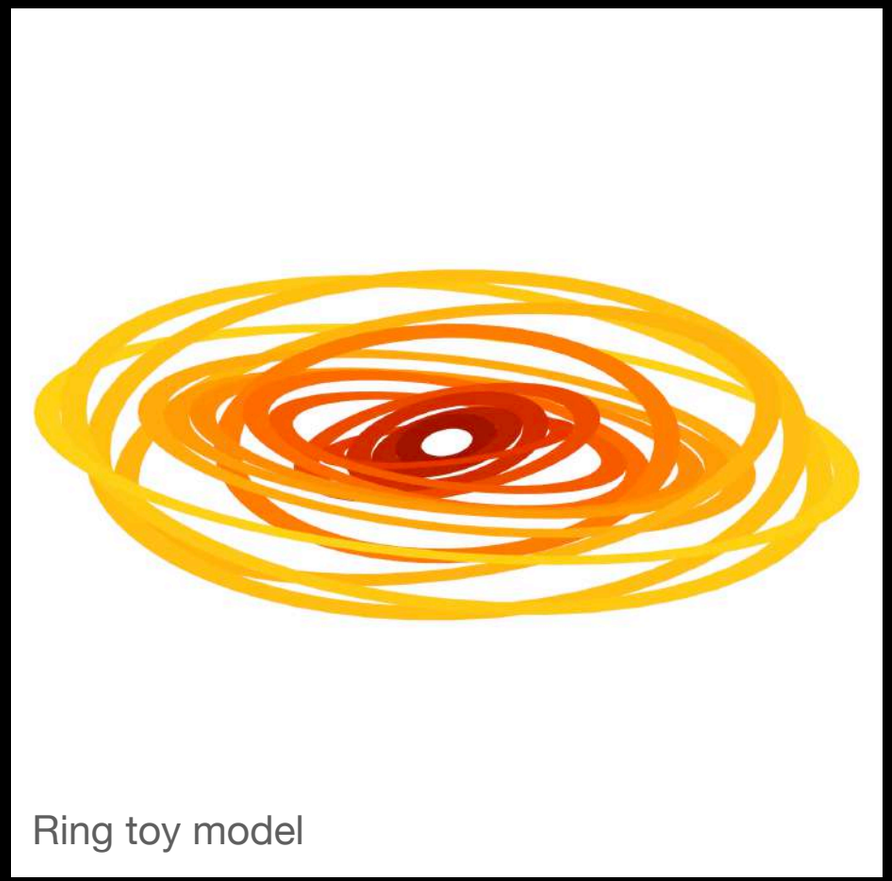
Synopsis of thin disc emergence:

- Why do disc settle ? Because Q→1
- But Why does Q→1? Because tighter control loop ($t_{\rm sec} \ll 1$) via wake
- But how does it impact settling? Because wake also stiffens coupling

The appearance of tight scaling laws (KS, bTF,RAR) likely follows

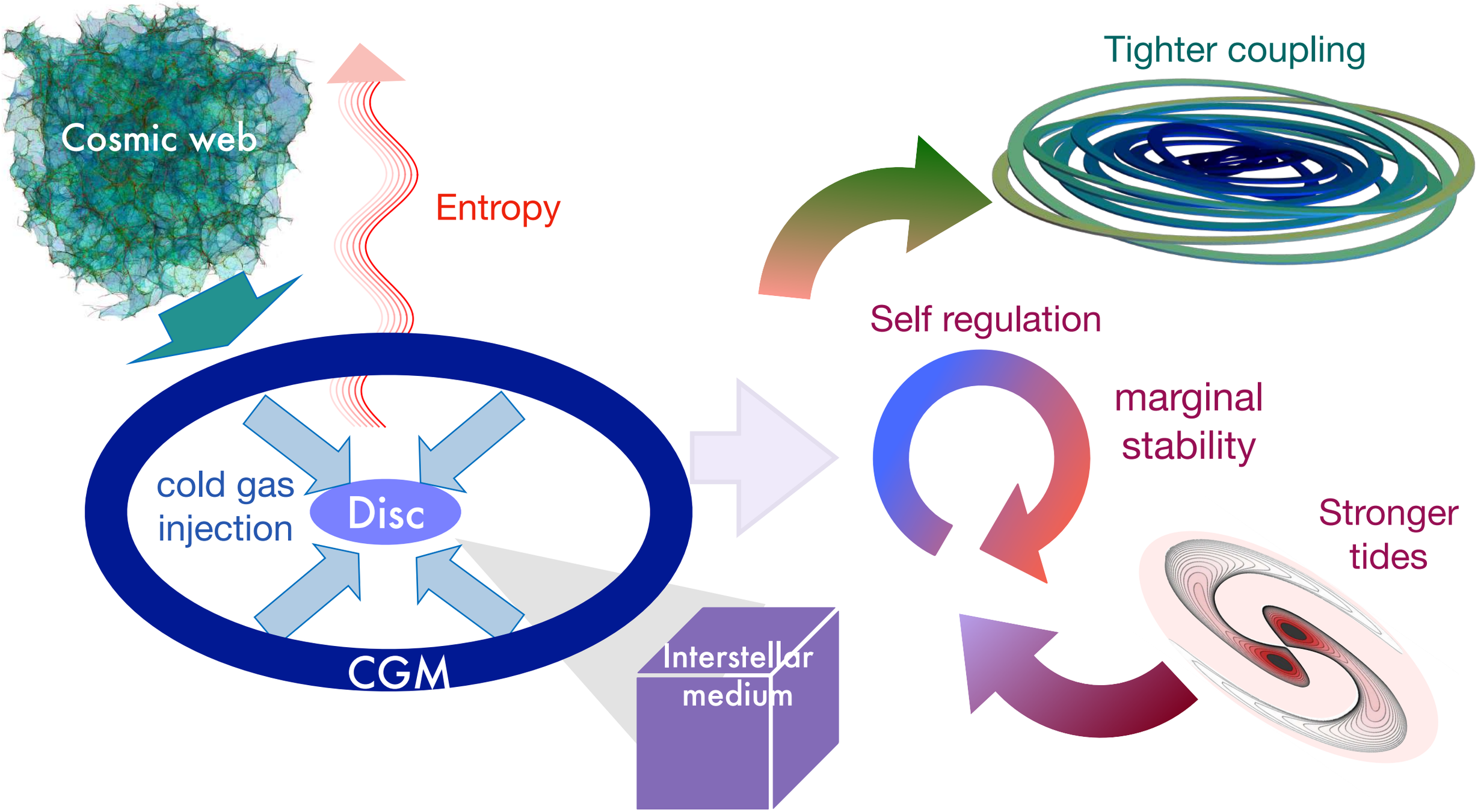


New Horizon



Ring toy model

2.4 Synopsis of thin disc emergence induced by CW



Three components system coupled by gravitation.

- A CGM reservoir fed by the CW (top down *causation*)
- Convergence towards marginal stability : acceleration of dynamical control-loop by wakes
- Tightening of stellar disc by boosting of torques, & increased dissipation.

Self regulating loop boosted by wake

Transition to secularly-driven morphology promoting self-regulation around an effective Toomre $Q \sim 1$.

$$T_{\rm dressed} \simeq |\varepsilon| \; T_{\rm bare}$$

so long as $T_{\rm dressed} > T_{\rm cool}$

Q ↗

Attraction point of feedback loop

$$Q_{\rm eff}^{-1} = Q_g^{-1} + Q_\star^{-1} = \frac{G\pi}{\kappa}\left(\frac{\Sigma_g}{\sigma_g} + \frac{\Sigma_\star}{\sigma_\star}\right)$$

Tighter loop

Destabilising effects

- SN1a
- Turbulence

———

- Minor Mergers
- Misaligned infall
- FlyBys

Stabilising effects

- Star formation
- Cooling
- Shocks

———

- Co-rotating Aligned infall

Gravitational Wake

Heating

Cooling

Q ↘

Cosmic perturbation

Free energy reservoir in CGM

Open system with control loop generates complexity through self-organisation

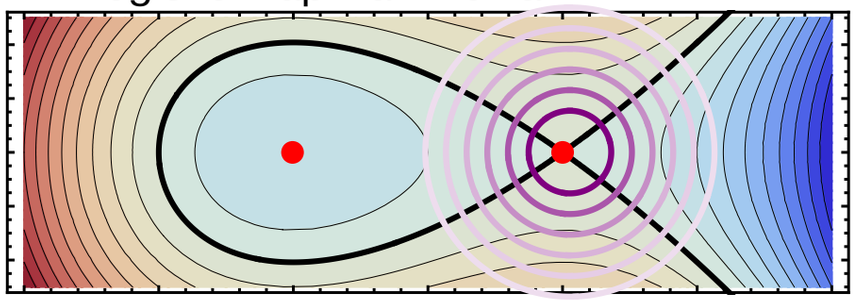
Chemistry of emergence... introduce tides

*Now let us take into account for the **vertical** secular diffusion of the cold component*

Dissipation converts kinetic instable point into an **attractor**.

Gravitational Wake

Logistic map Hamiltonian



Dressed Reaction-Diffusion equation (cf morphogenesis)

$$\frac{d}{dt}\bigstar = \frac{\delta_D}{\epsilon^2}\bigstar(1-\bigstar)+\frac{1}{\epsilon^2}\Delta\bigstar$$

wake driven $\varepsilon(z) \to 0$ as $Q \to 1$

SF efficiency

$$\left.\begin{aligned}\eta_{\text{dressed}} &\propto \eta_{\text{raw}}/\epsilon^2(Q)\\ D_{\text{dressed}} &\propto D_{\text{raw}}/\epsilon^2(Q)\end{aligned}\right\} \sim \text{quadratic in } \varepsilon \qquad \Longrightarrow \quad dt \to \frac{dt}{\epsilon^2}$$

Diffusion

Rapid correction

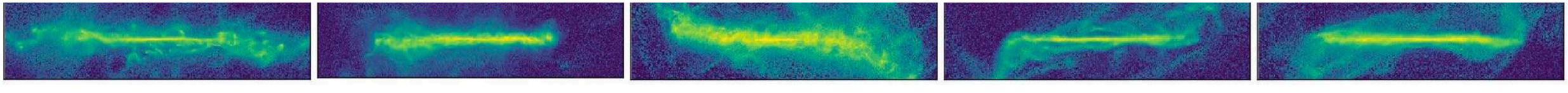
→ Cosmic **resilience** of thin disc **driven by CW**

→ Operates **swiftly** near self-organised **Criticality**

→ **Robustness** / feedback details

No fine tuning !

all discs are fairly thin whatever the feedback



Perturbative (quasi-linear) expansion

$$F_{\mathrm{d}} = F + \delta F$$

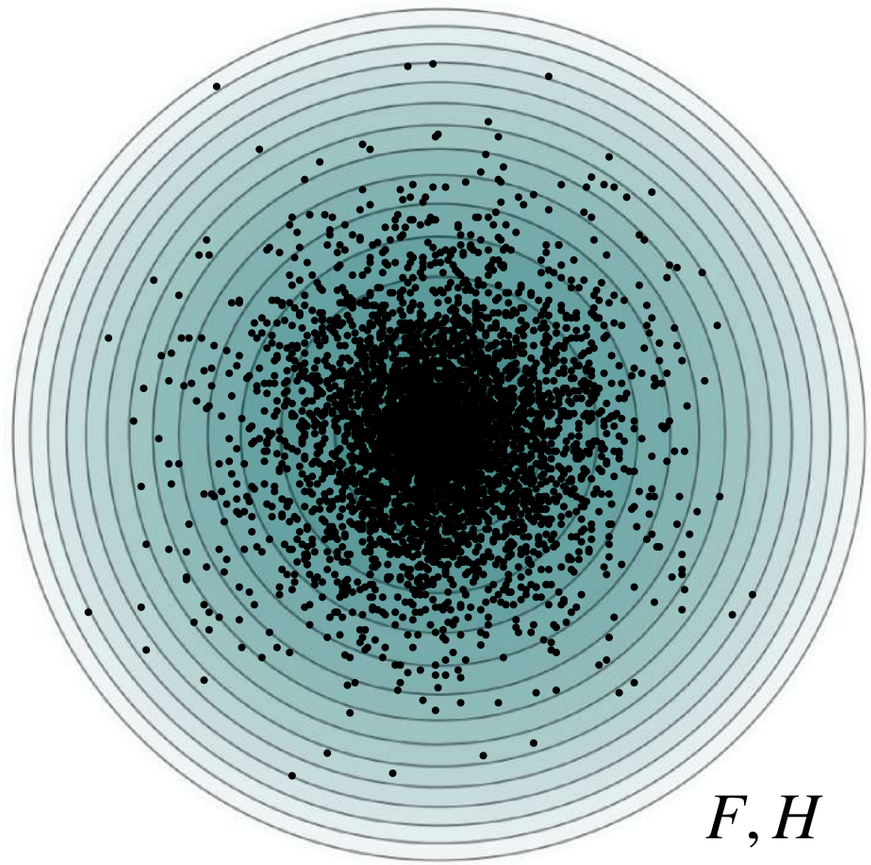
One galaxy

$$F = \langle F_{\mathrm{d}} \rangle$$

Smooth ensemble average

$$H_{\mathrm{d}} = H + \delta H$$

$$H = \langle H_{\mathrm{d}} \rangle$$



Predicts the secular evolution of the **mean** galaxy

Perturbative (quasi-linear) expansion

$$F_{\mathrm{d}} = F + \delta F$$

$$F = \langle F_{\mathrm{d}} \rangle$$

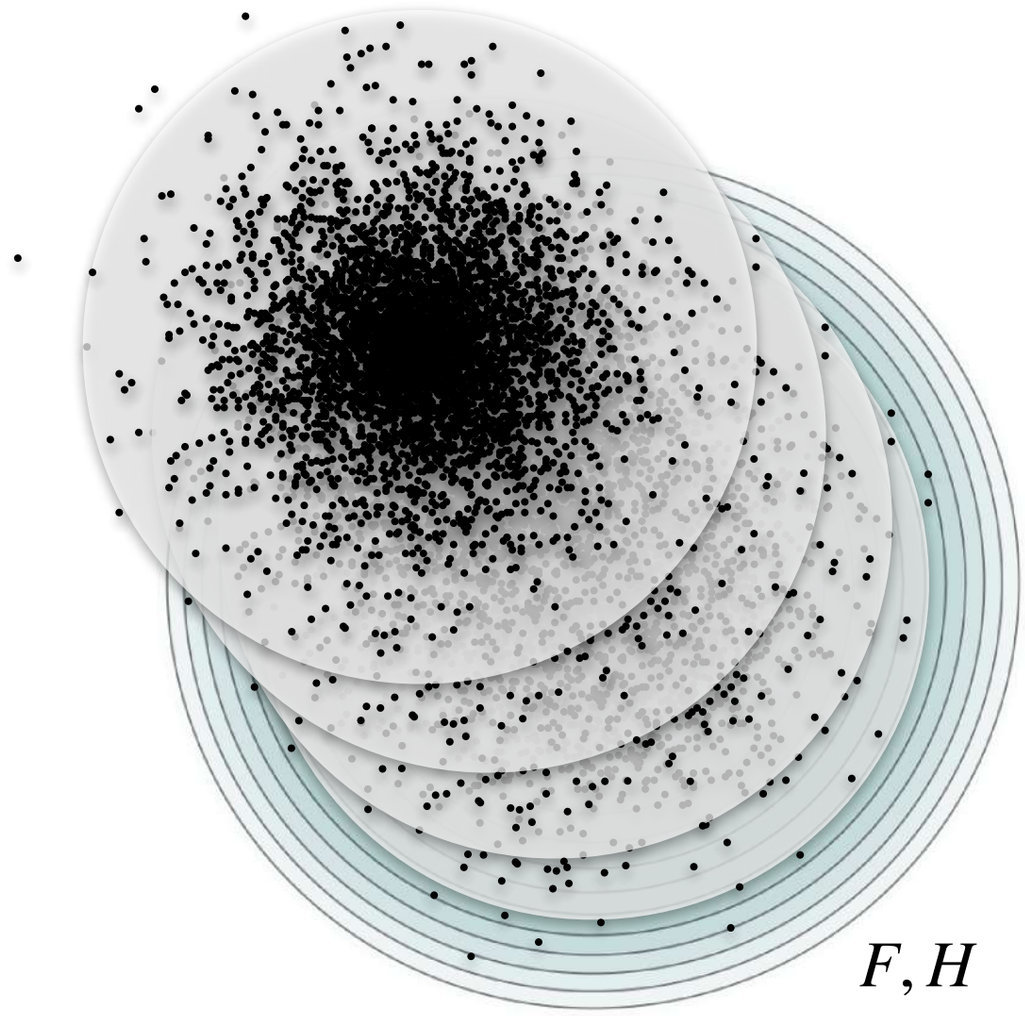
Smooth ensemble average

One galaxy

Stacking galaxies

$$H_{\mathrm{d}} = H + \delta H$$

$$H = \langle H_{\mathrm{d}} \rangle$$



Predicts the secular evolution of the **mean** galaxy

Perturbative (quasi-linear) expansion

$F_{\mathrm{d}} = F + \delta F$

$F = \langle F_{\mathrm{d}} \rangle$

What ensemble?

Smooth **ensemble** average

One galaxy

Stacking galaxies

Density fluctuations

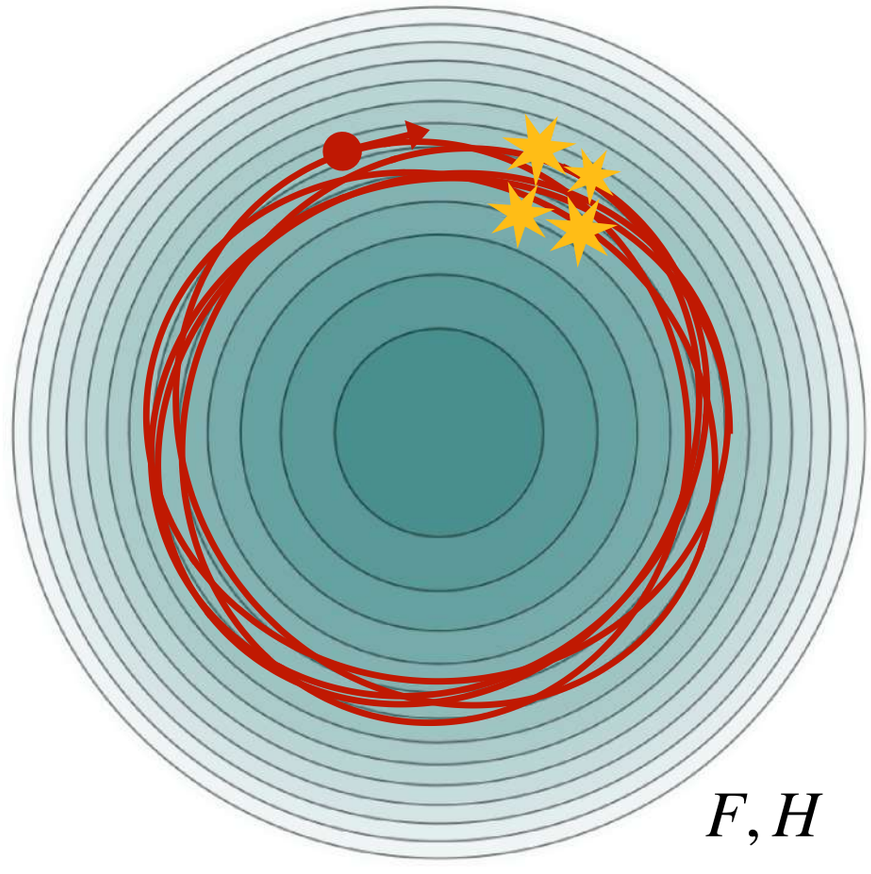
$H_{\mathrm{d}} = H + \delta H$

$H = \langle H_{\mathrm{d}} \rangle$

mean field **Orbits**

Potential fluctuations

Diffusion





Kinetic theory

Exact **Klimontovich** equation

Perturbative expansion

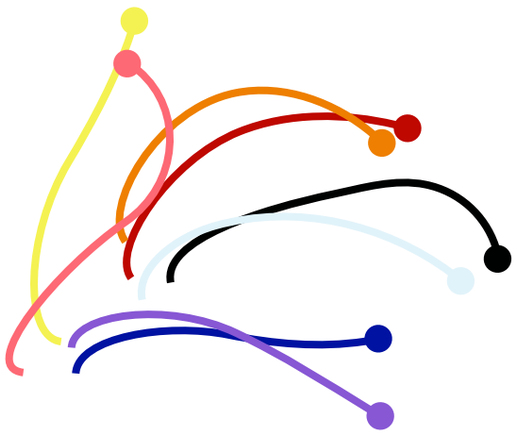
Quasi-stationary **equilibrium**

$$\left\langle \frac{\partial F_{\mathrm{d}}}{\partial t} + \left[F_{\mathrm{d}}, H_{\mathrm{d}}\right] = 0 \right\rangle + \begin{array}{l} F_{\mathrm{d}} = F + \delta F \\ H_{\mathrm{d}} = H + \delta H \end{array} + \left\langle \left[F, H\right] = 0 \right\rangle$$

$$=$$

$$\frac{\partial F}{\partial t} = -\left\langle \left[\delta F, \delta H\right] \right\rangle$$

Fluctuations **correlations**



Fluctuation-Dissipation Theorem

Diffusion rate ⟷ **Power-spectrum of the fluctuations**

Mean galaxy subject to **deterministic** orbital **diffusion**

Access **full statistics** in the (least rare) small perturbation regime through **cumulant generating function**

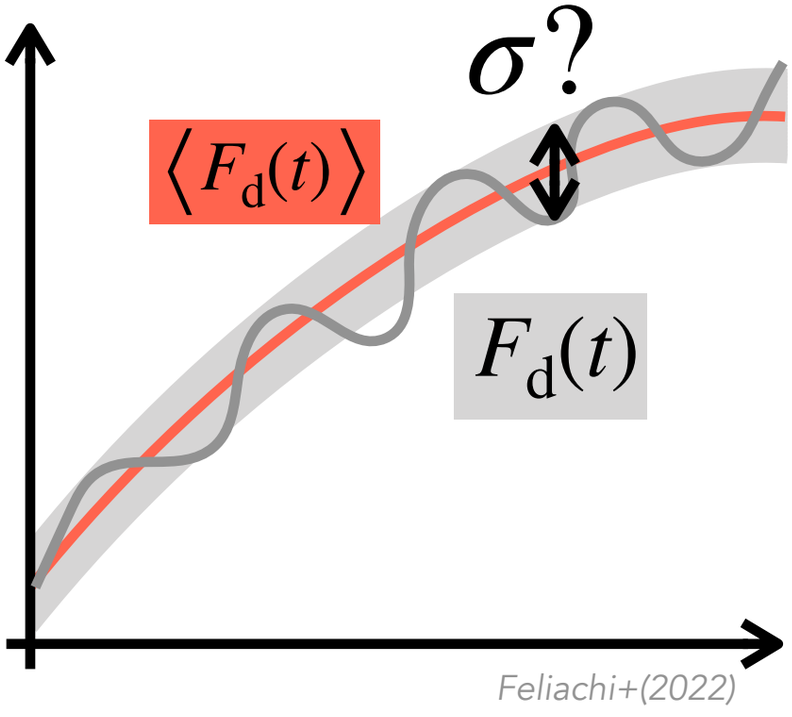
$$\Phi\left[H\right] = \log\Bigg($$

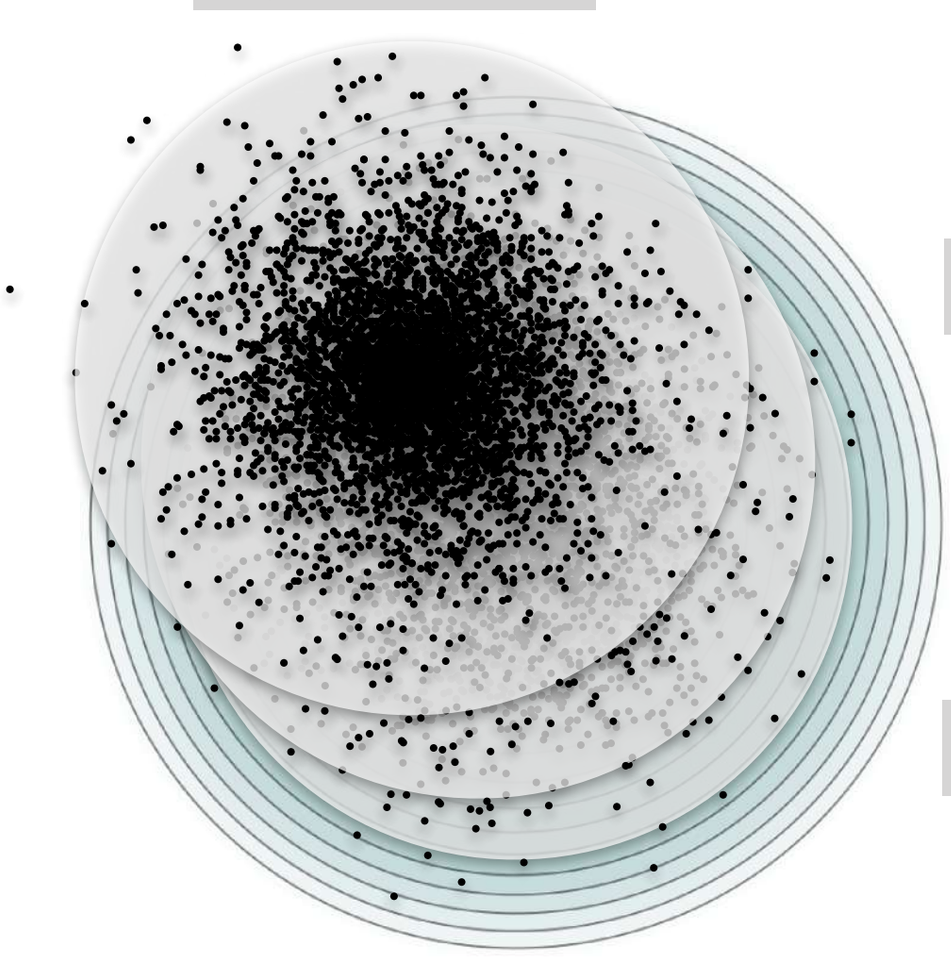
Exact **Klimontovich** equation

$$\Bigg\langle \quad \frac{\partial F_{\mathrm{d}}}{\partial t} + \left[F_{\mathrm{d}}, H_{\mathrm{d}}\right] = 0$$

Perturbative expansion

$$+ \quad \begin{aligned} F_{\mathrm{d}} &= F + \delta F \\ H_{\mathrm{d}} &= H + \delta H \end{aligned} \quad \times \quad \exp\left(\int \mathrm{d}t\, \dot{F}_d\, H\right) \Bigg\rangle \Bigg)$$





Weighted **Stack**

Predict **variance**

IN PROGRESS

Explain tightness of scaling relations?

Robust *gravity-driven* top-down causation : no fine tuning required



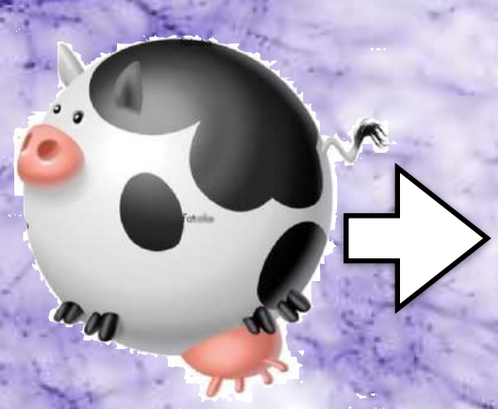
gravity-driven baryonic processes operate on **multiple anisotropic** scales, working to **spontaneously** set up a remarkably efficient level of **self-regulation**.

This regulation is responsible for disc resilience
& likely the **tightness** of observed scaling laws (KS,bTF,RAR).

Conclusion:

We should care about the cosmic web!

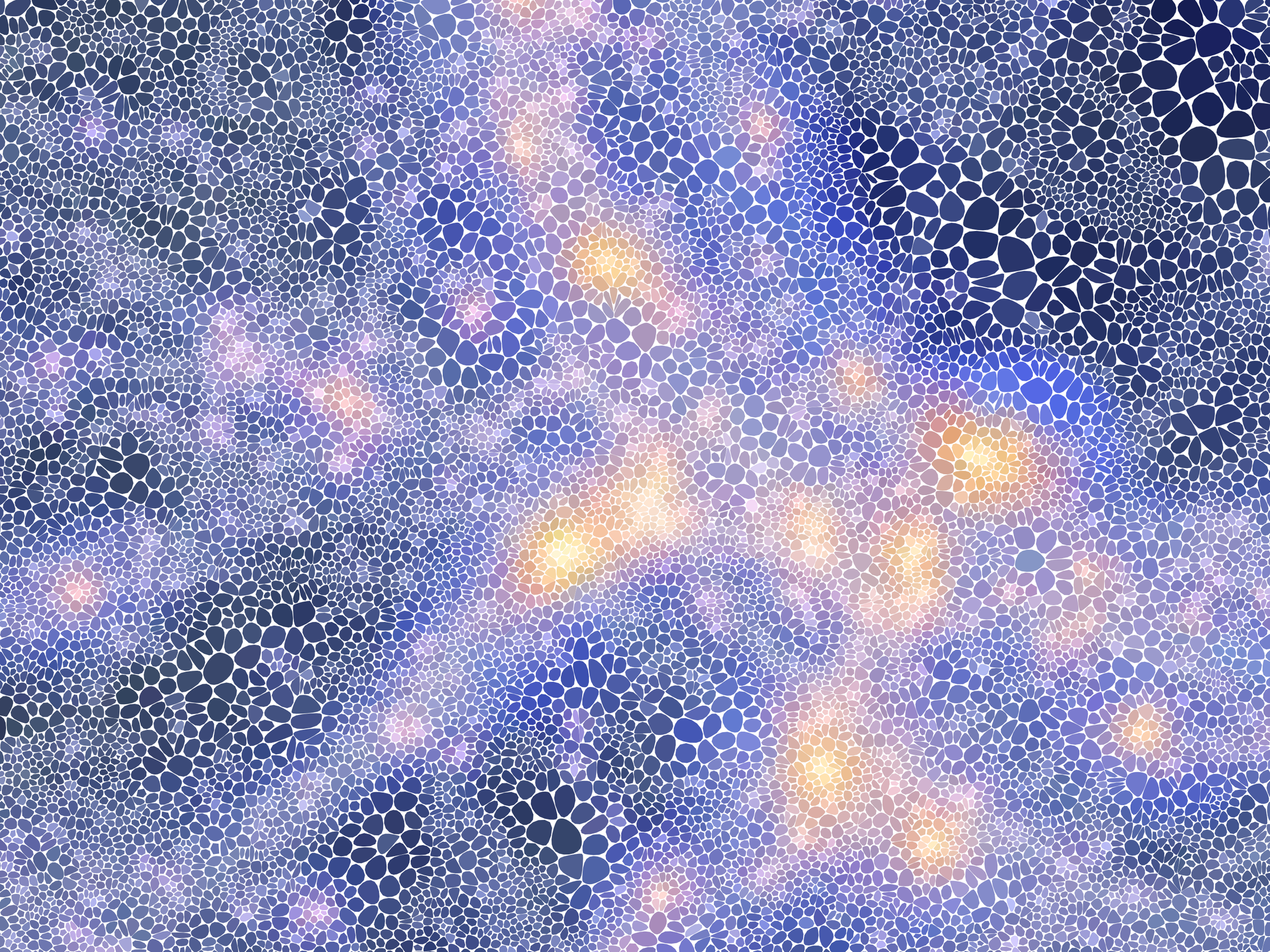
cosmic web = metric set by eigframe $\left[\frac{\partial^2 \rho}{\partial x_i \partial x_j}\right]_{\text{sad}}$

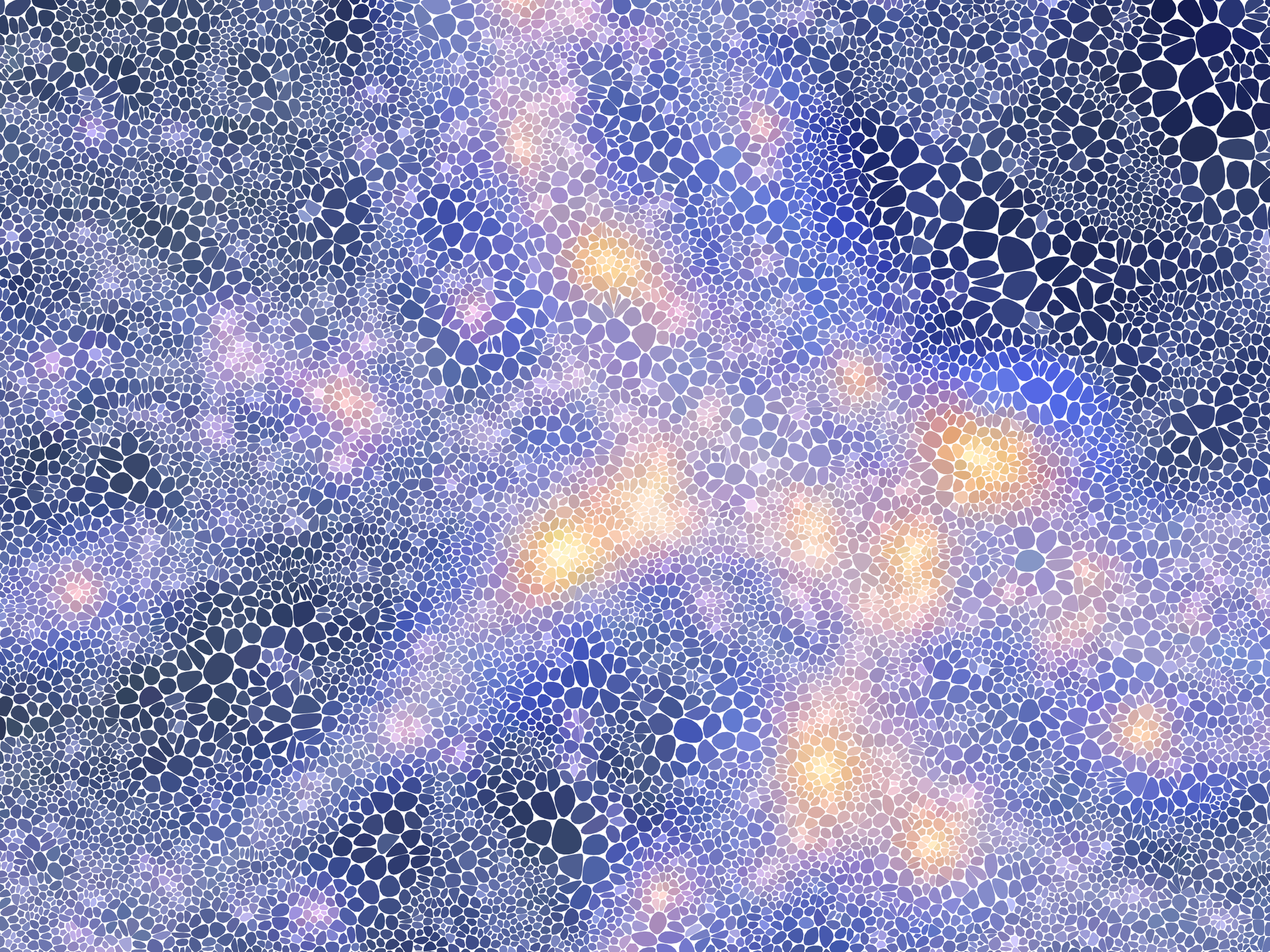




Merci !

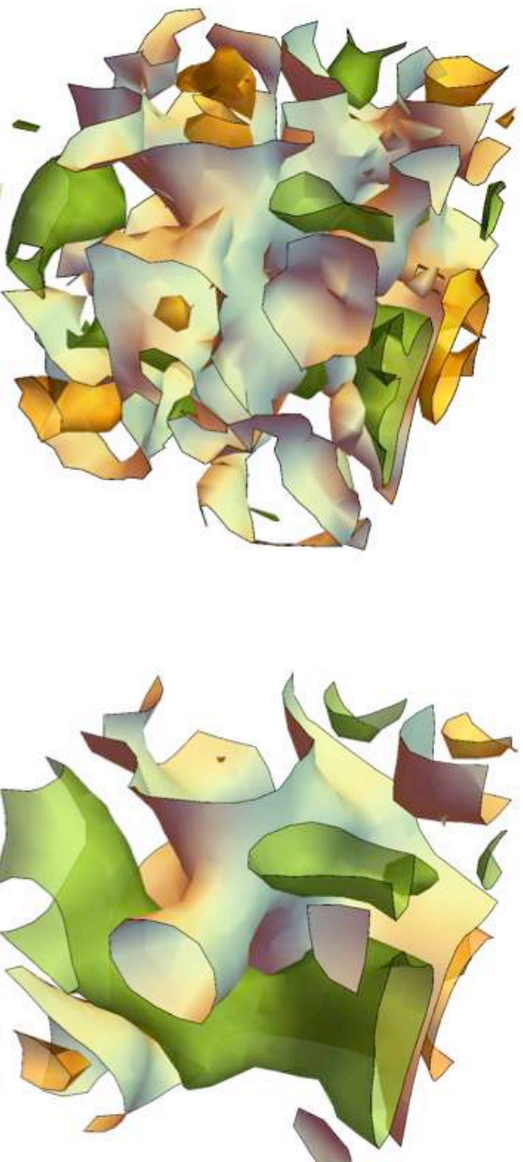


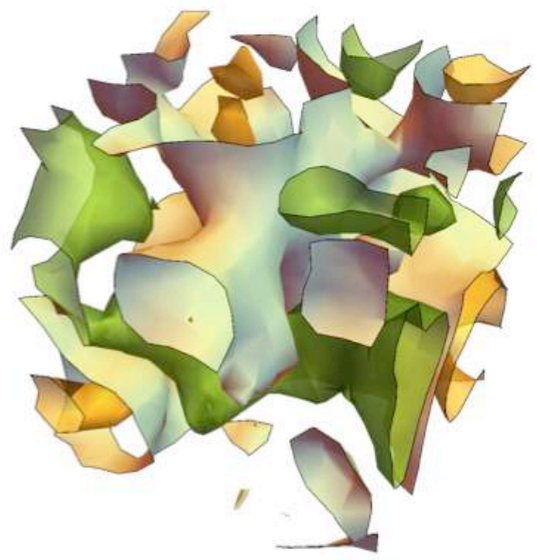


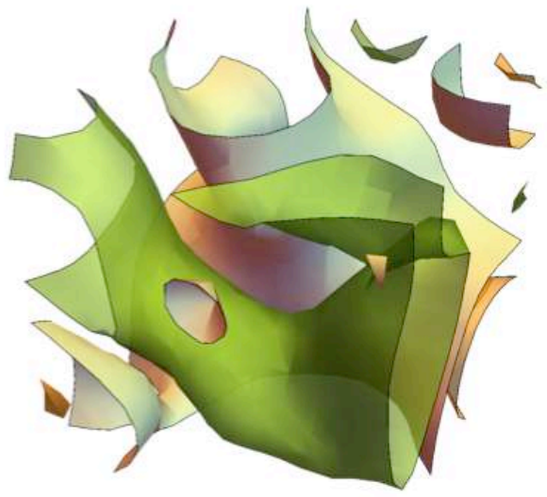


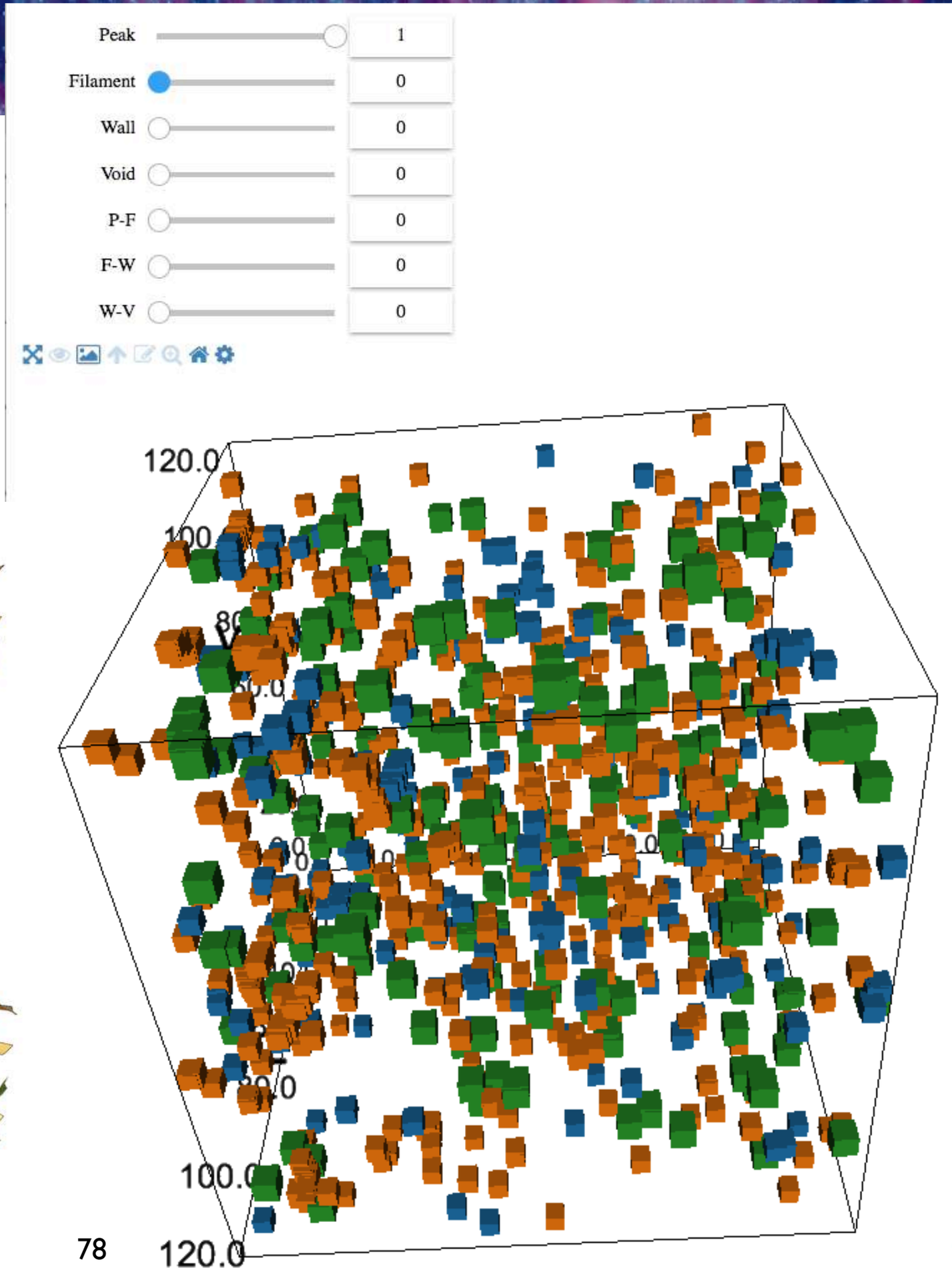
Critical event examples 3D

- GRF smoothed at ≠ scales
- identify critical points
- find critical events

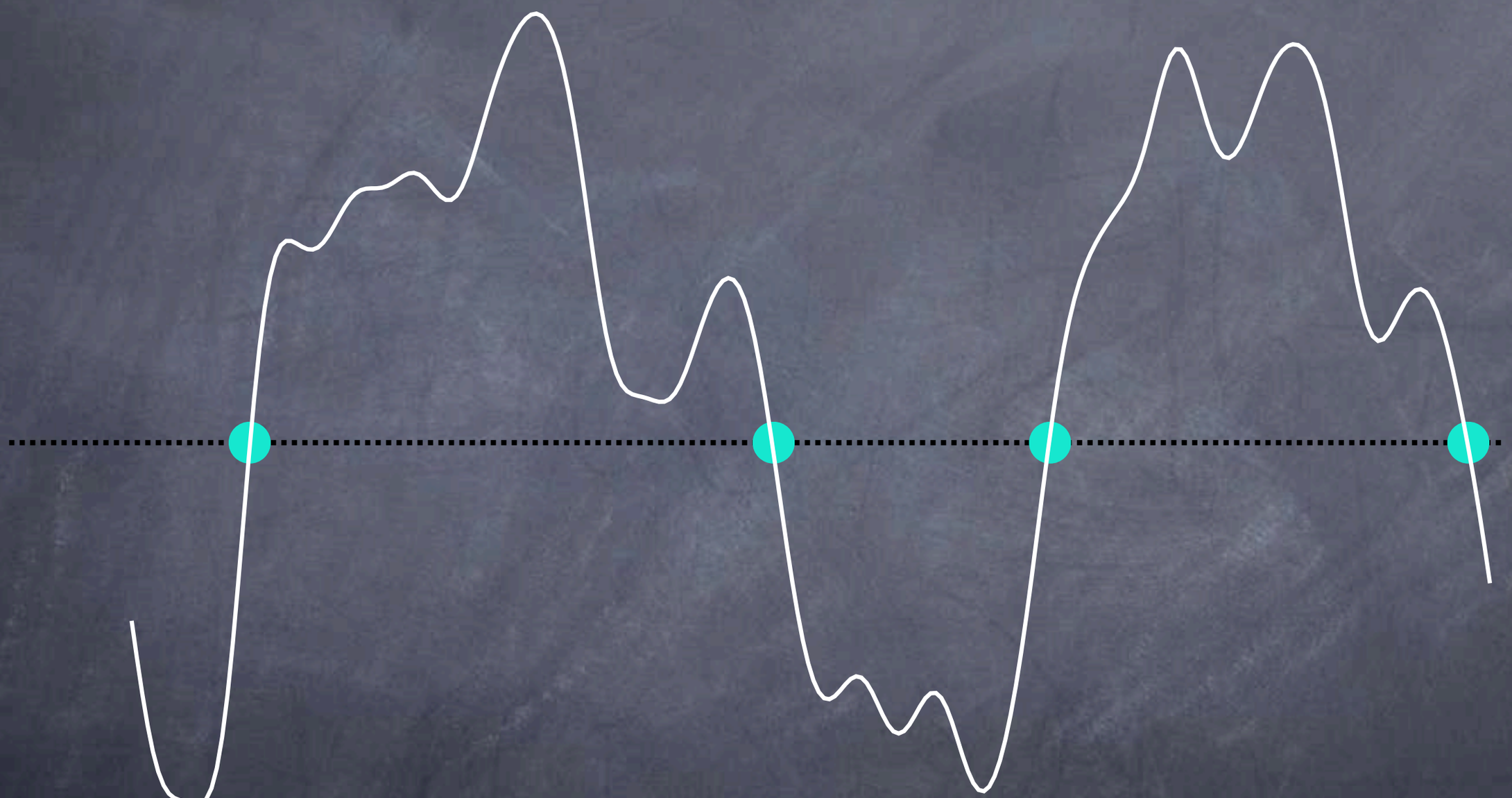






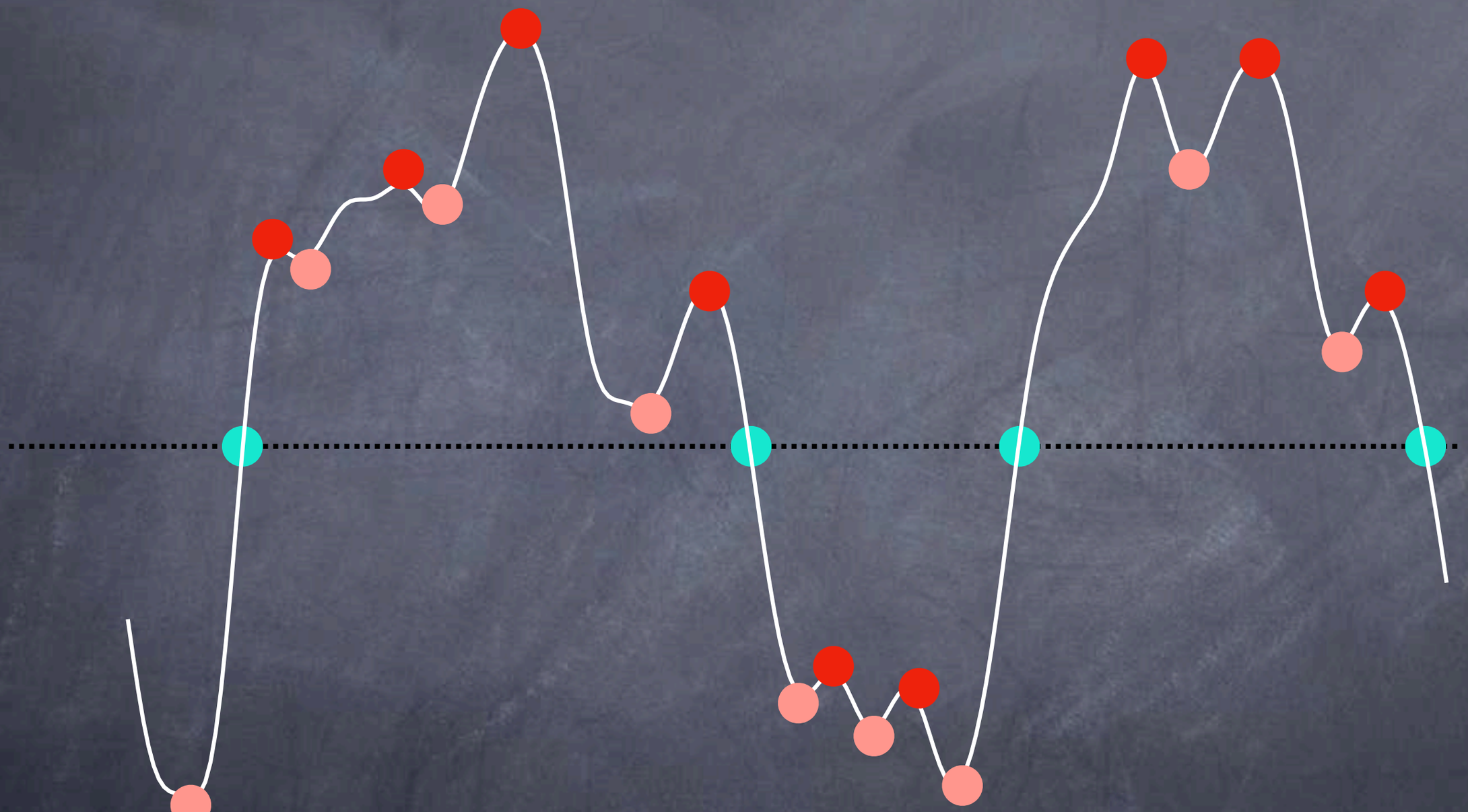


$R_0 = \sigma_0/\sigma_1$ distance between zero crossing



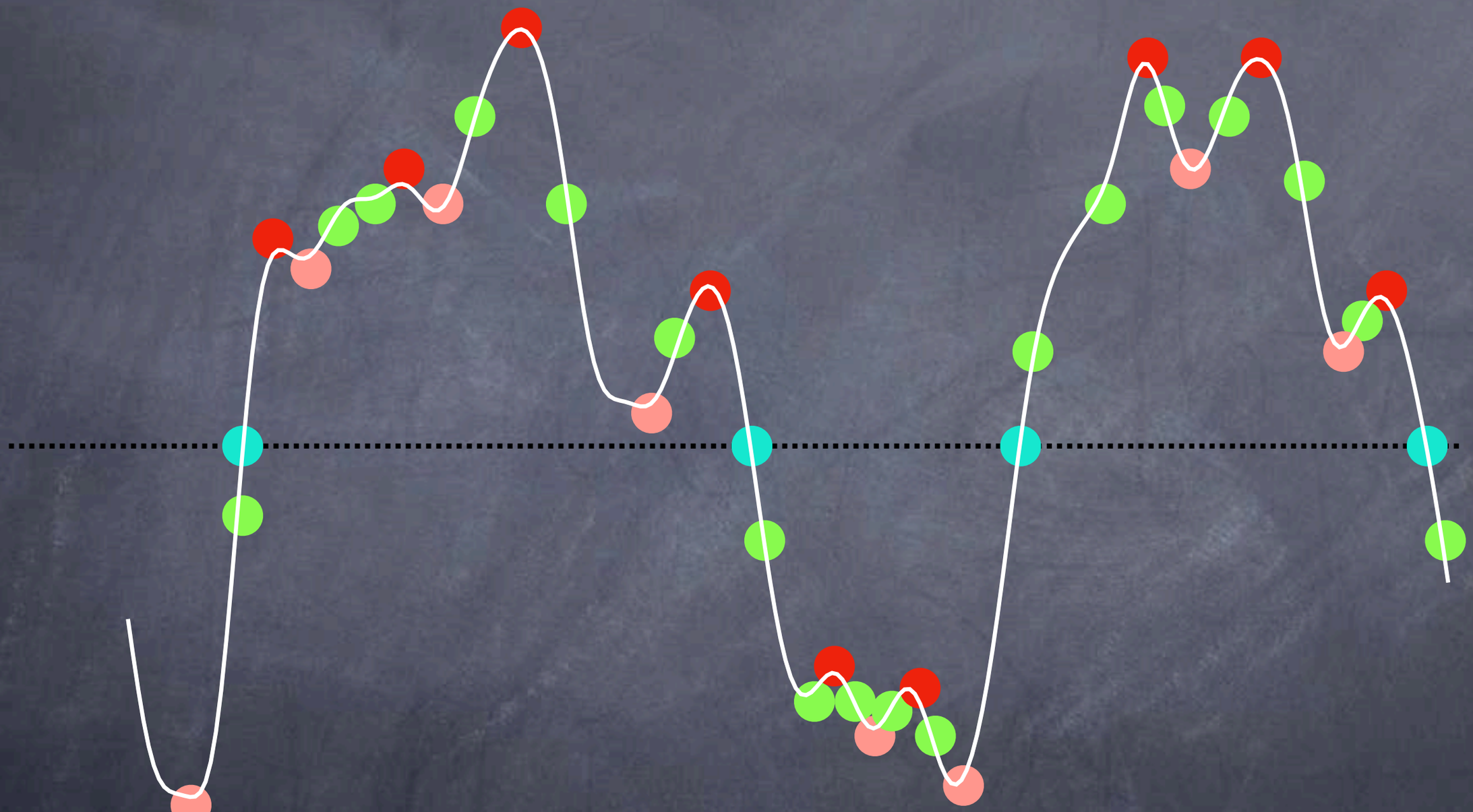
R_0

$R_\star = \sigma_1/\sigma_2$ distance between extrema



$R_0 > R_\star$

$\tilde{R} = \sigma_2/\sigma_3$ distance between inflection points



$$R_0 > R_\star > \tilde{R}$$

1D fields

$$x = \delta/\sigma_0, \quad x' = \delta'/\sigma_1, \quad \text{etc}$$

Zero crossing

$$\text{cond(zero)} = \frac{1}{R_0}\delta_{\rm D}(x)\,|x'|$$

$R_0 = \sigma_0/\sigma_1$ distance between zero crossing
$R_\star = \sigma_1/\sigma_2$ distance between extrema
$\tilde{R} = \sigma_2/\sigma_3$ distance between inflection points

Critical points

$$\text{cond(crit)} = \frac{1}{R_\star}\delta_{\rm D}(x')\,|x''|$$

Inflection points

$$\text{cond(inf)} = \frac{1}{\tilde{R}}\delta_{\rm D}(x'')\,|x'''|$$

Critical events

$$\text{cond(event)} = \frac{R}{R_\star \tilde{R}^2}\delta_{\rm D}(x')\delta_{\rm D}(x'')\,|x'''|^2$$

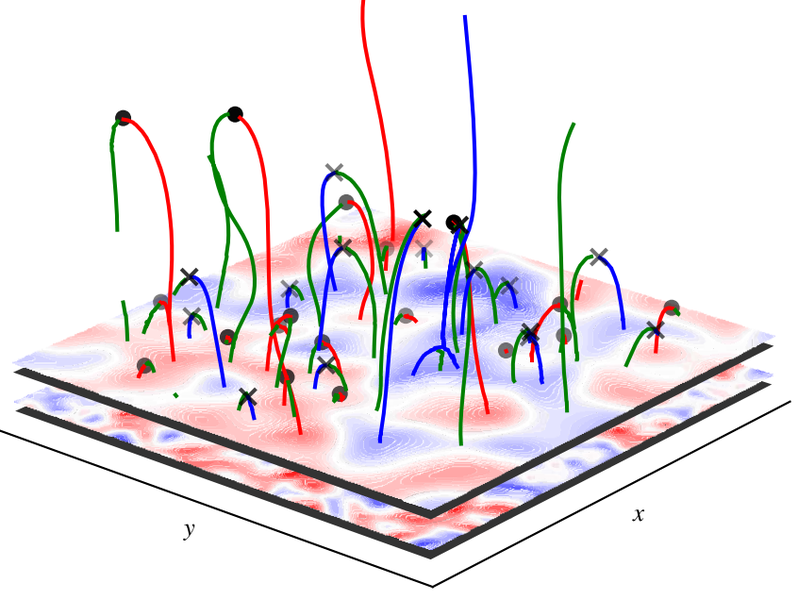
Number counts

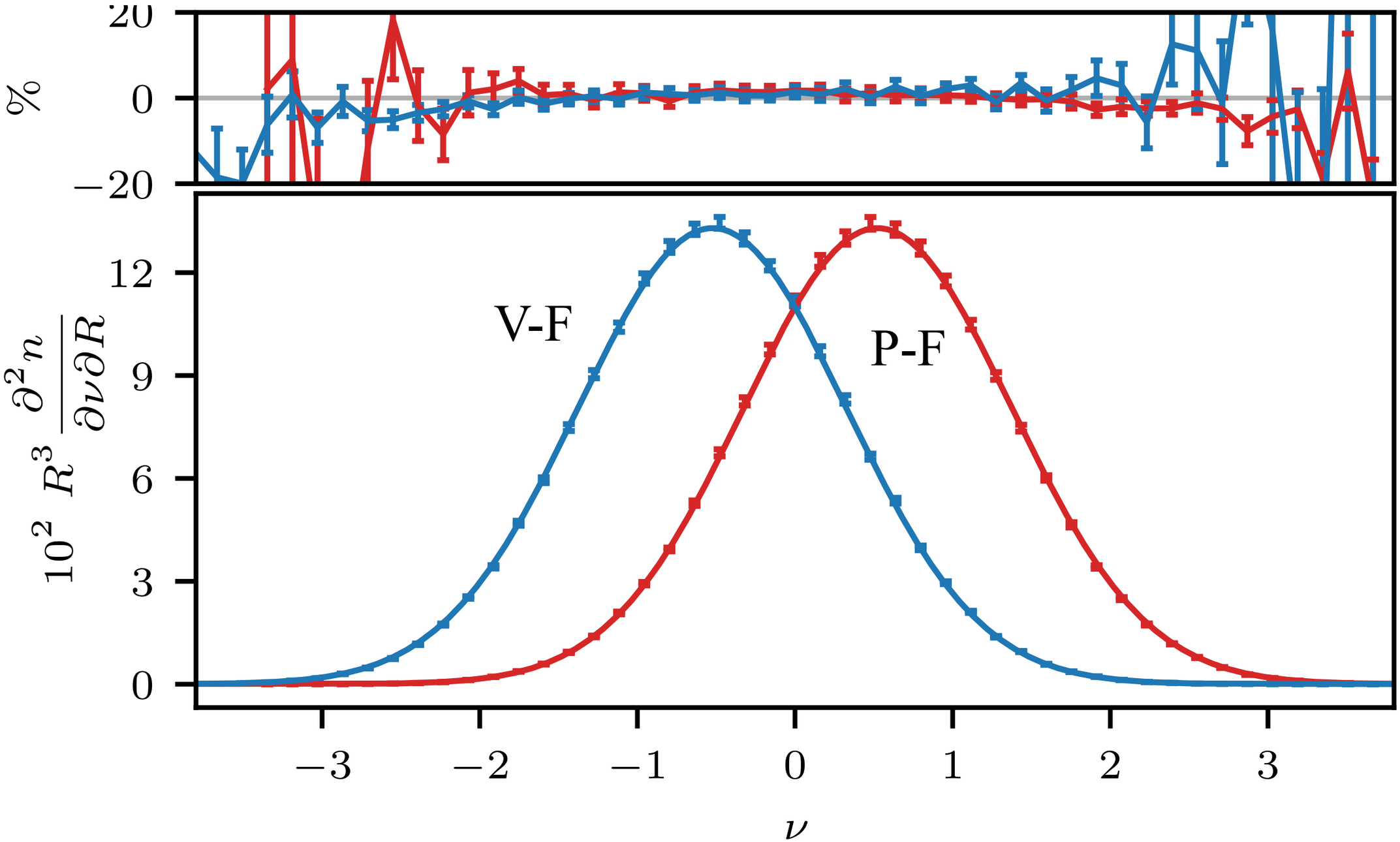
$$n_{\rm i} = \langle \text{cond}_{\rm i} \rangle$$

Clustering

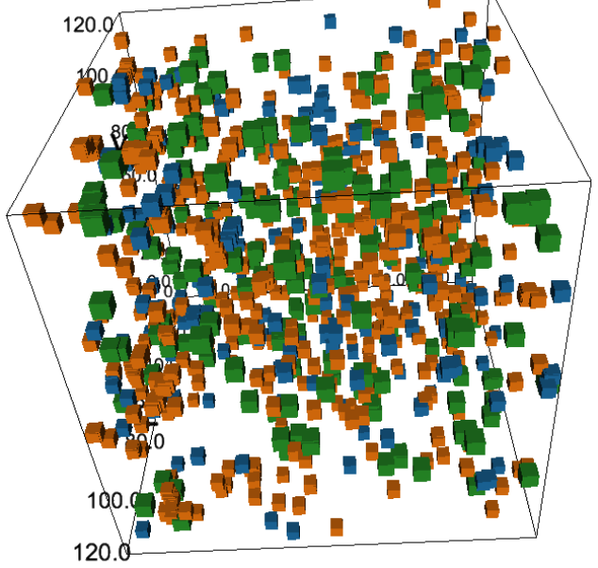
$$1 + \xi_{\rm ij} = \frac{\langle \text{cond}_{\rm i}\text{cond}_{\rm j} \rangle}{\langle \text{cond}_{\rm i} \rangle \langle \text{cond}_{\rm j} \rangle}$$

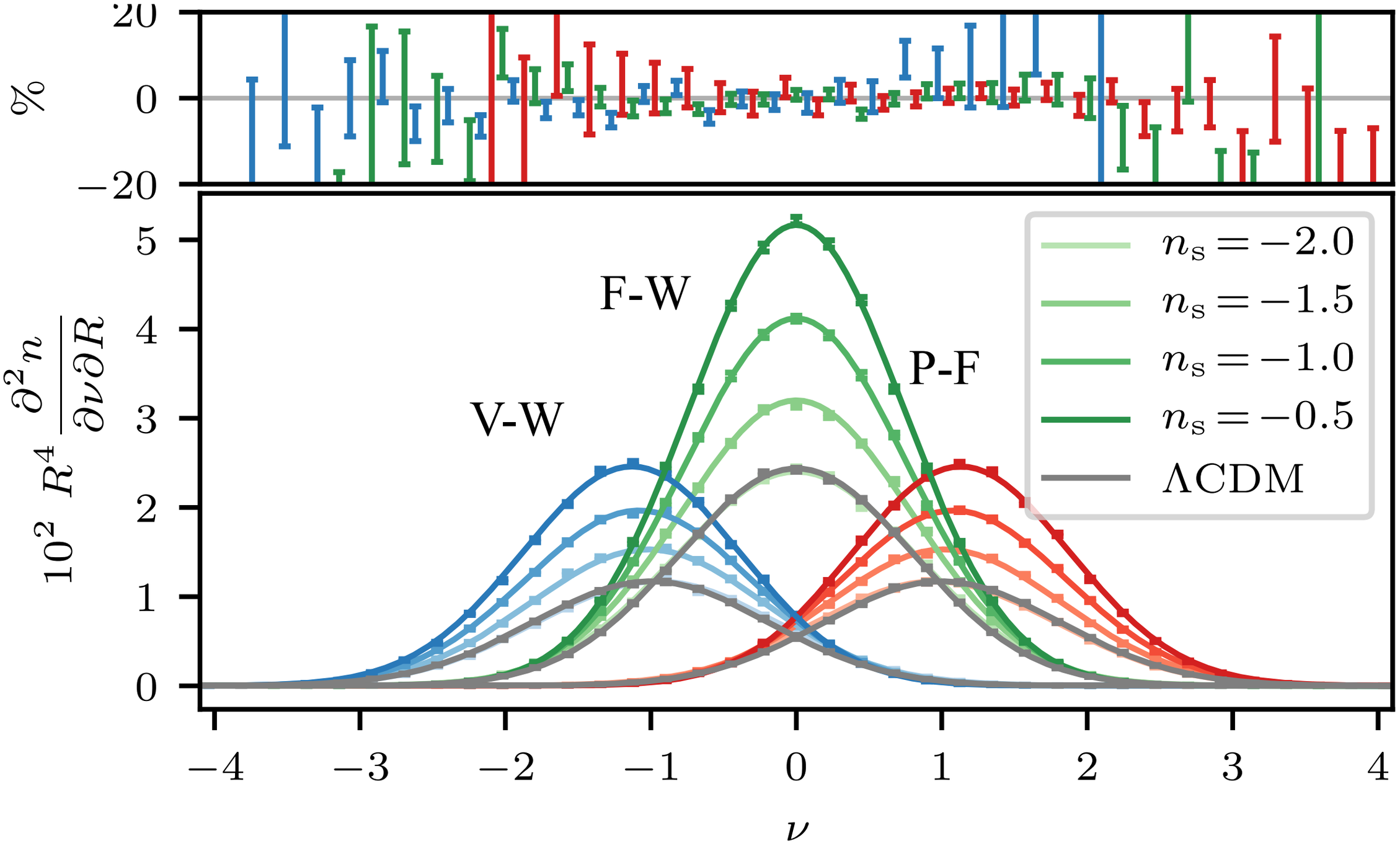
Validation: 2D event counts





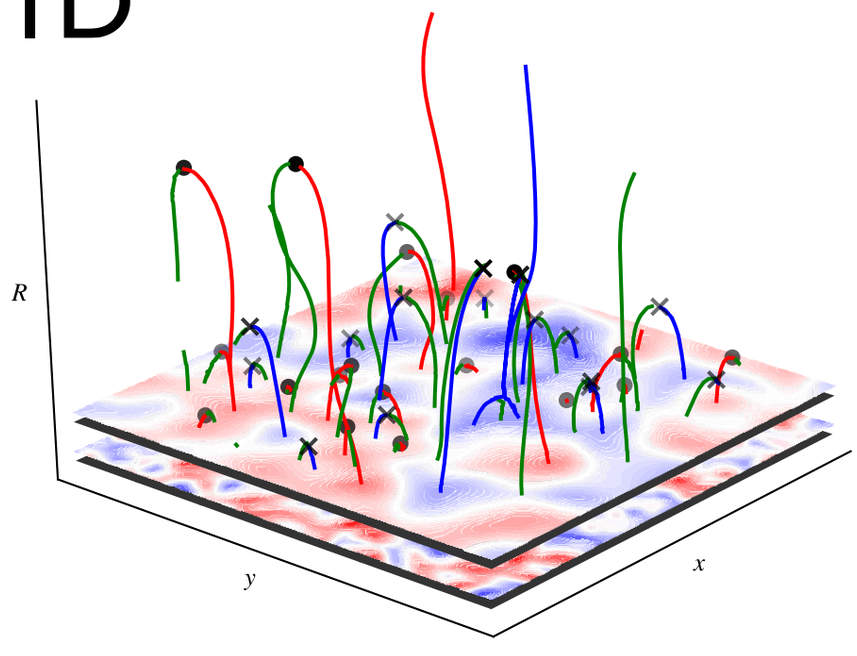
Validation: 3D event counts

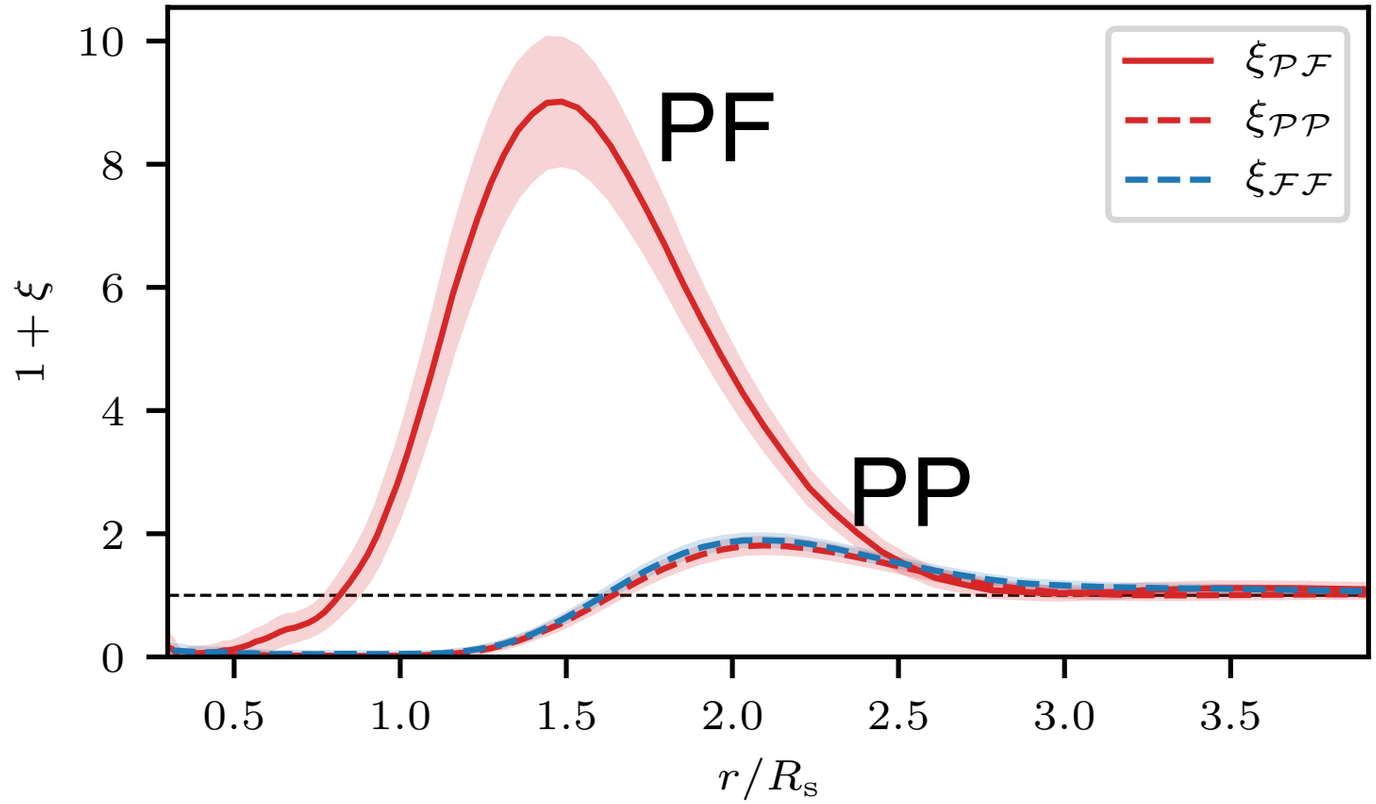




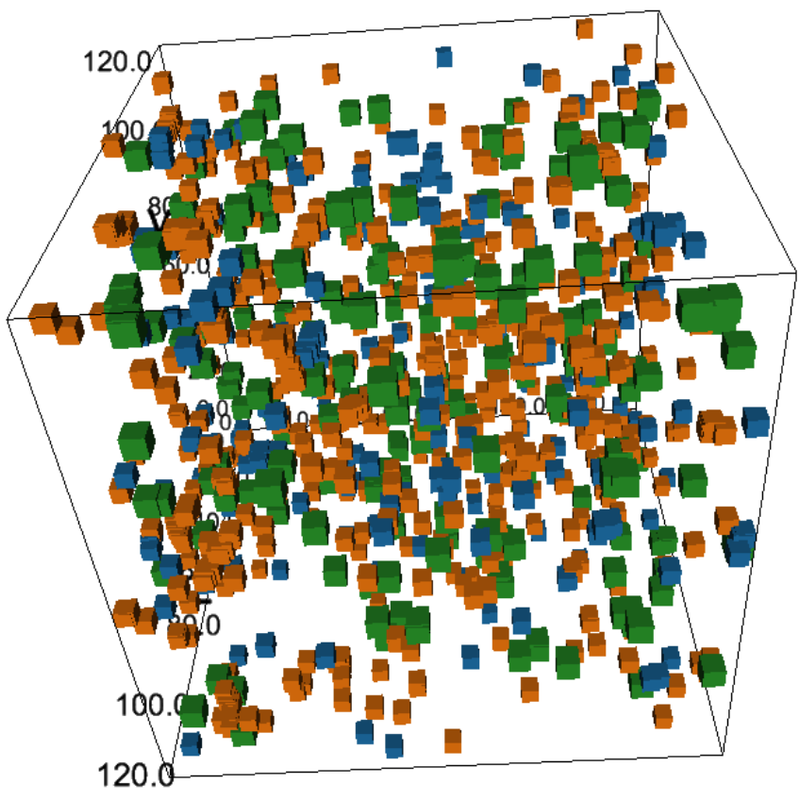
Two-point clustering of events

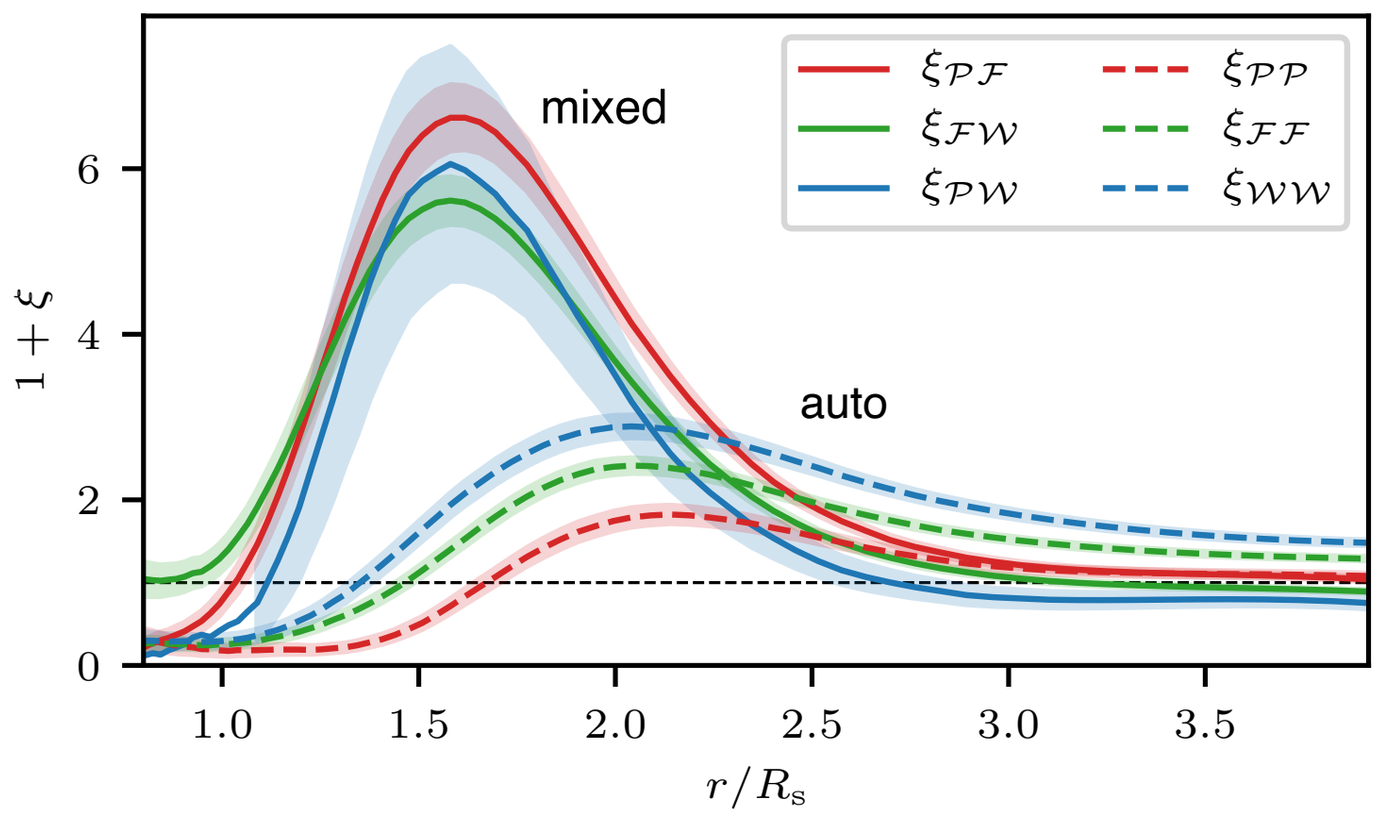
2+1D





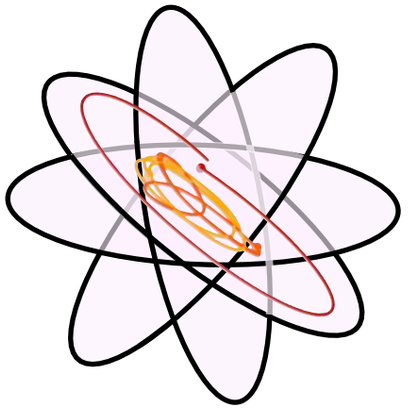
3+1D





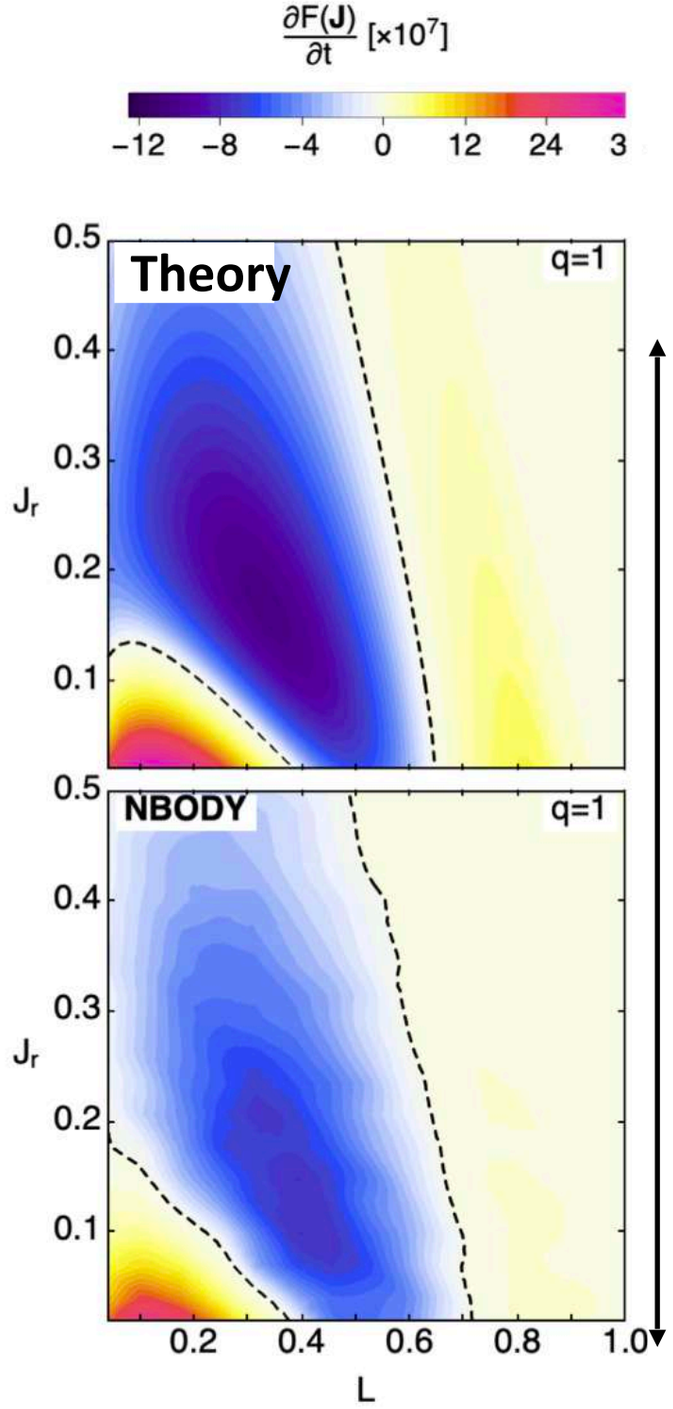
Dynamically hot systems





Theoretical prediction

N-body measurement

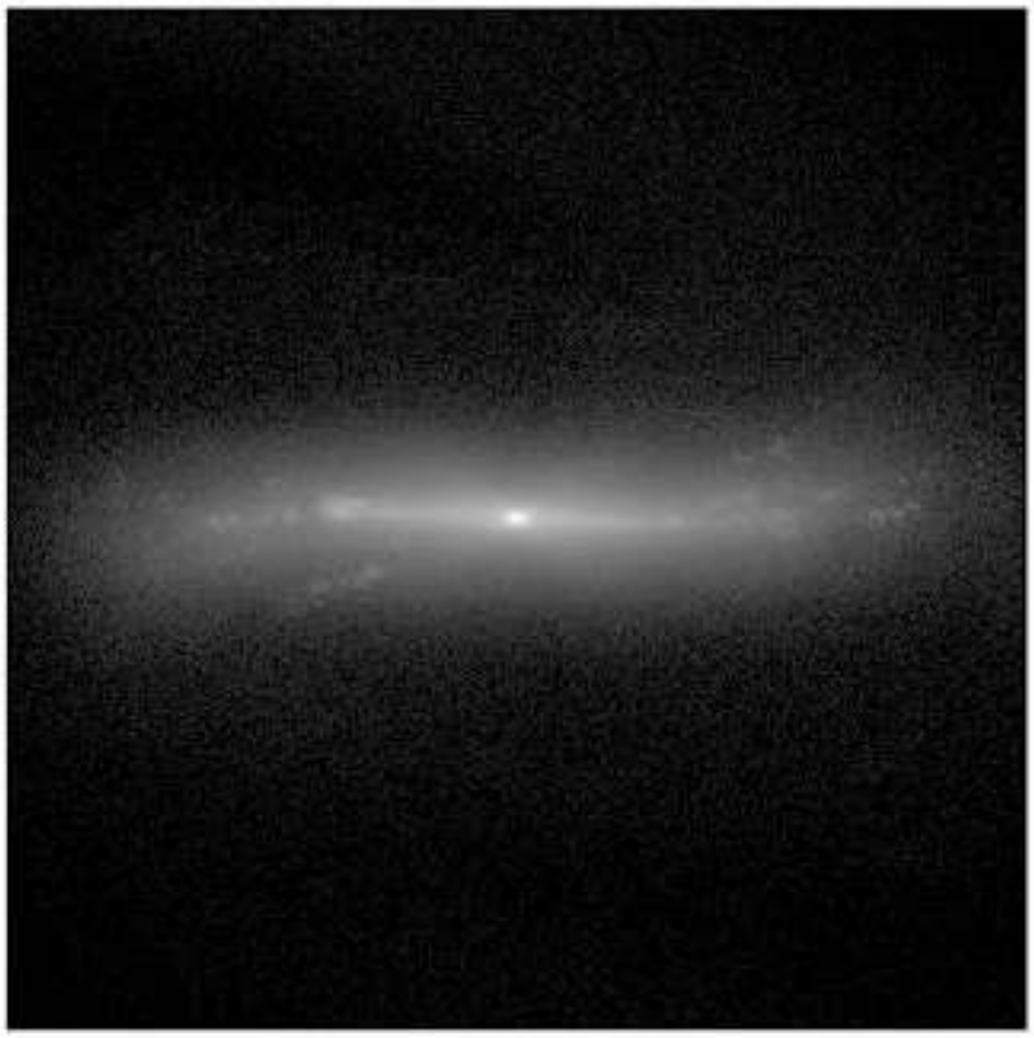


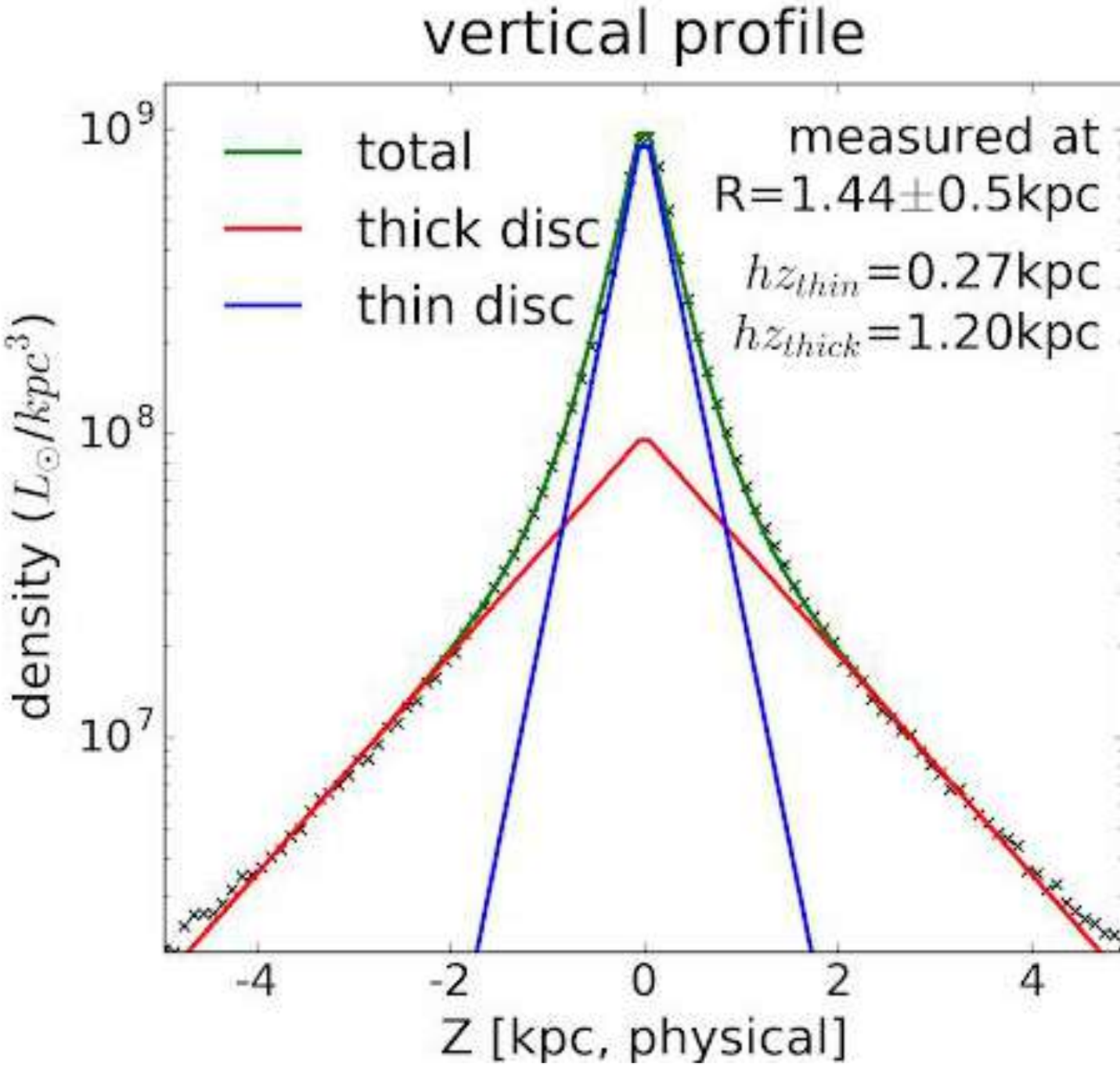
Qualitative agreement betweenTheory and **average** of **100** NBODY simulations

Tep et al. 2024

Q$\sim$ 1 confounding factor for joint thick+thin growth

$M_{*,\,disc} = 4.53 \times 10^{10} M_{\odot}$

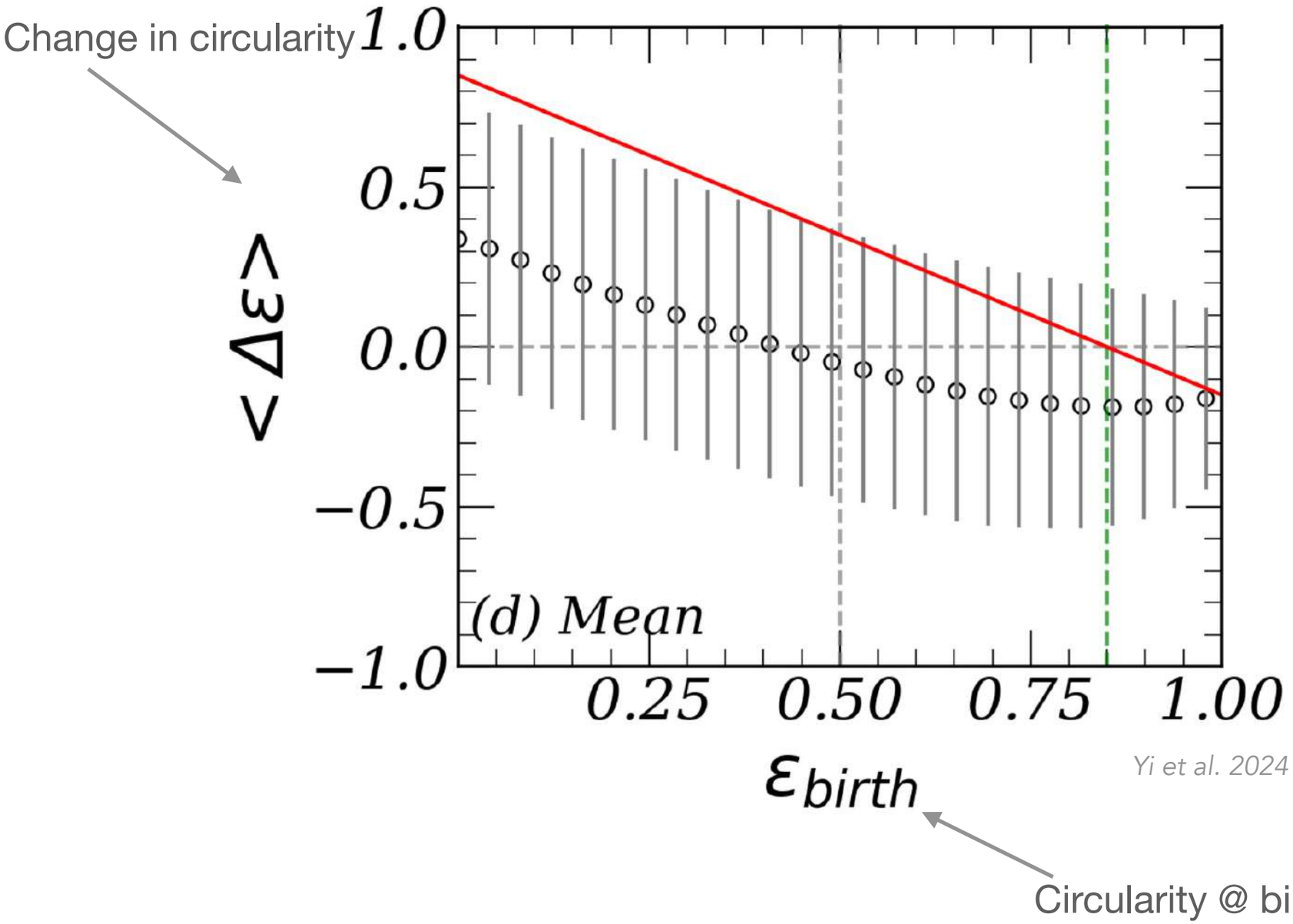




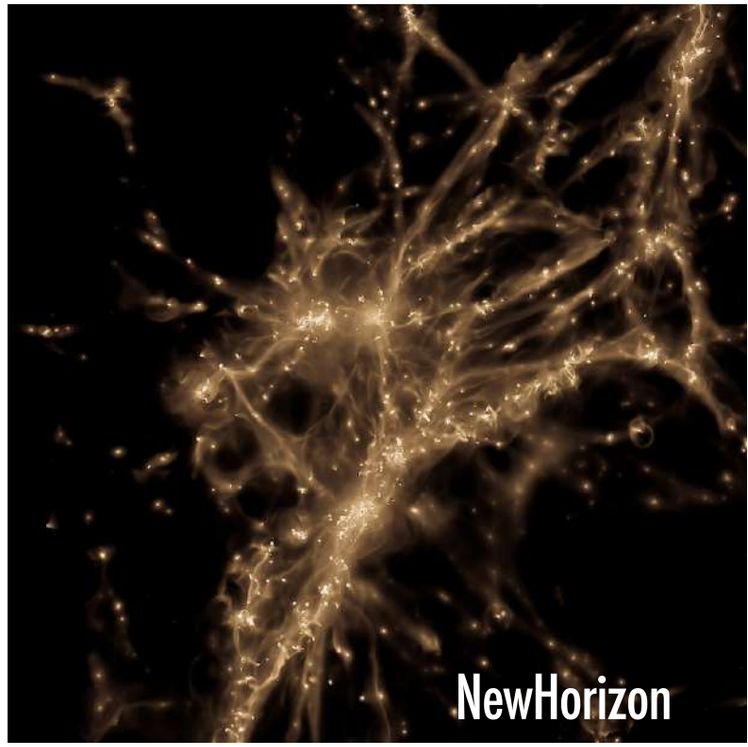
Both star formation and vertical orbital diffusion regulated by (Q $\rightarrow$ 1) **confounding** factor.

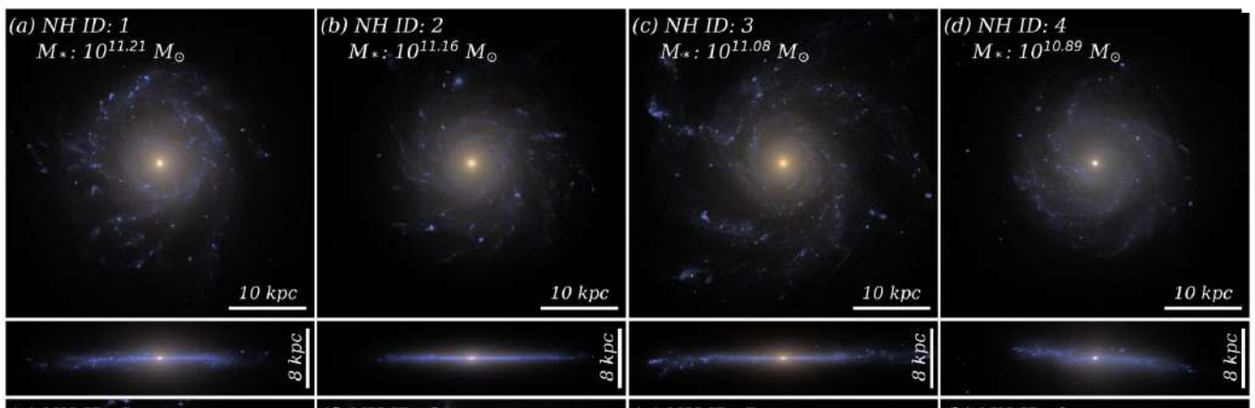
Stellar thick disc = **secular remnant** of (self regulated) disc settling process.

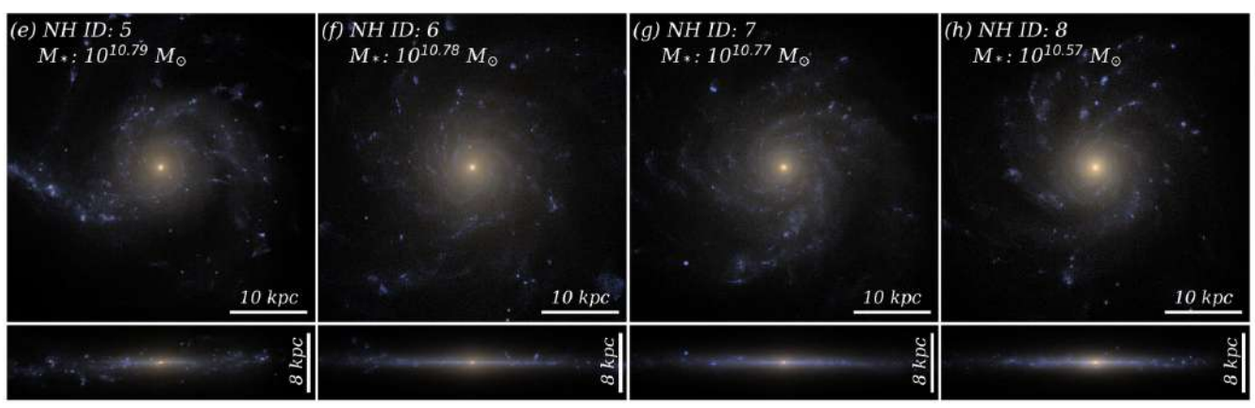
Measured orbital diffusion in cosmological simulation NewHorizon



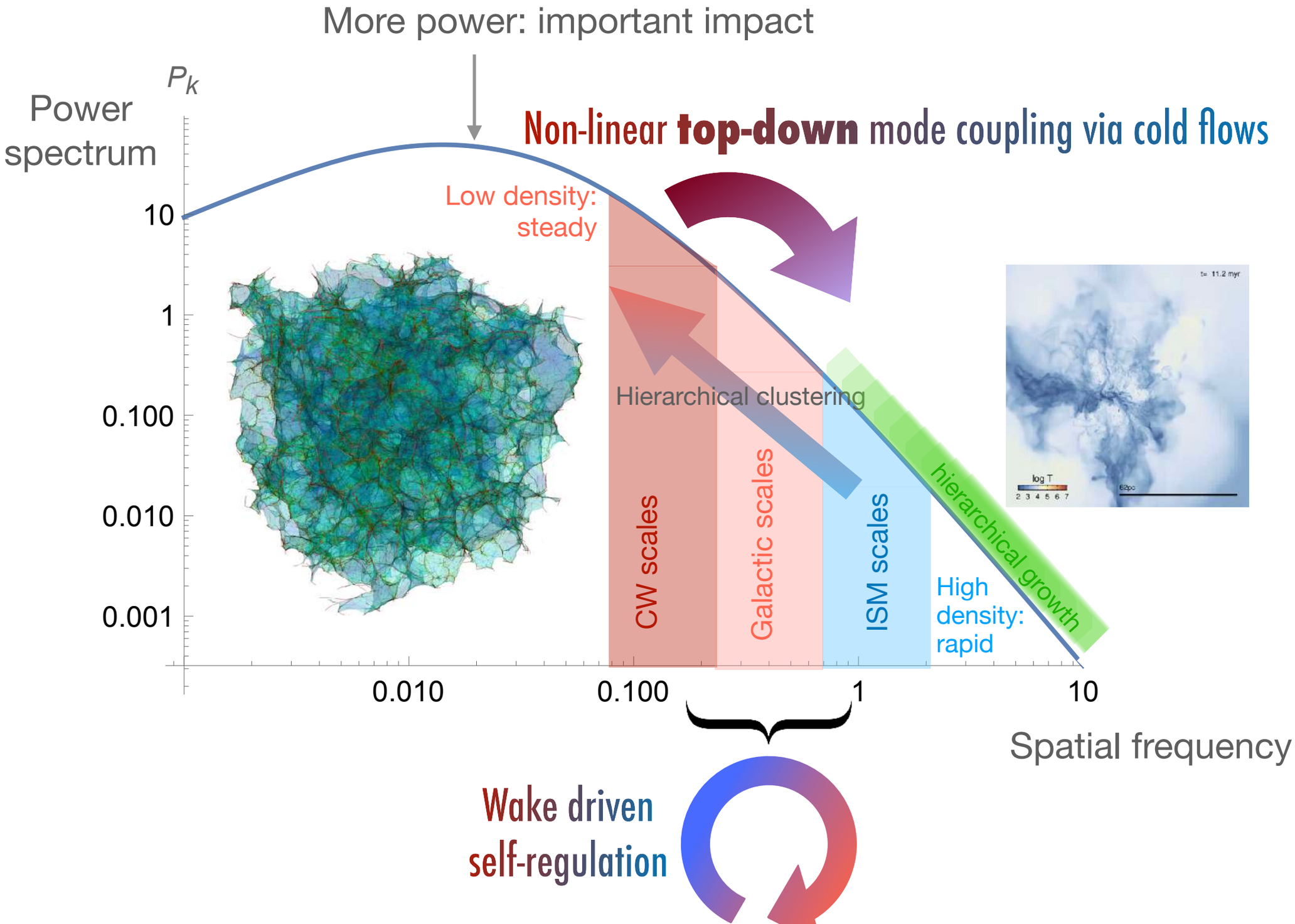
Yi et al. 2024

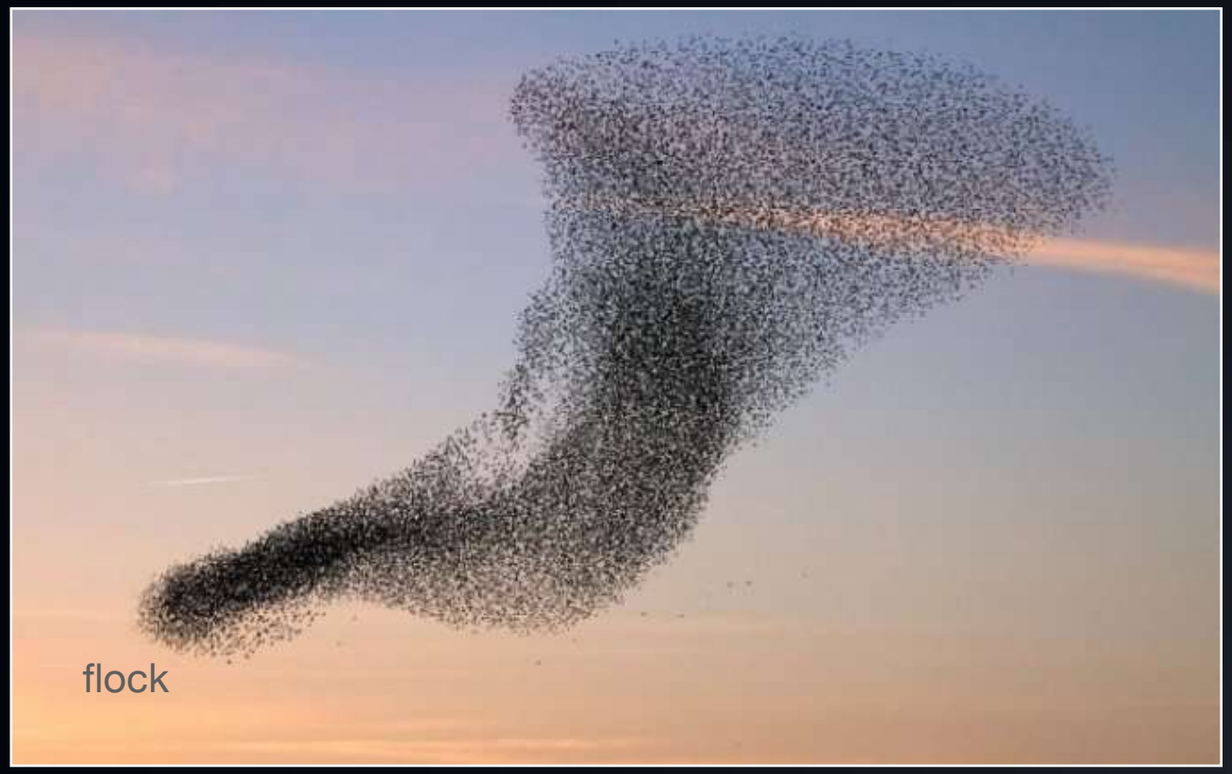






On galactic scales, the Shape of initial P_k is such that galaxies inherit stability from LSS via gas inflow, which, in turn, sets up CGM engine/reservoir required to self-regulate thin discs





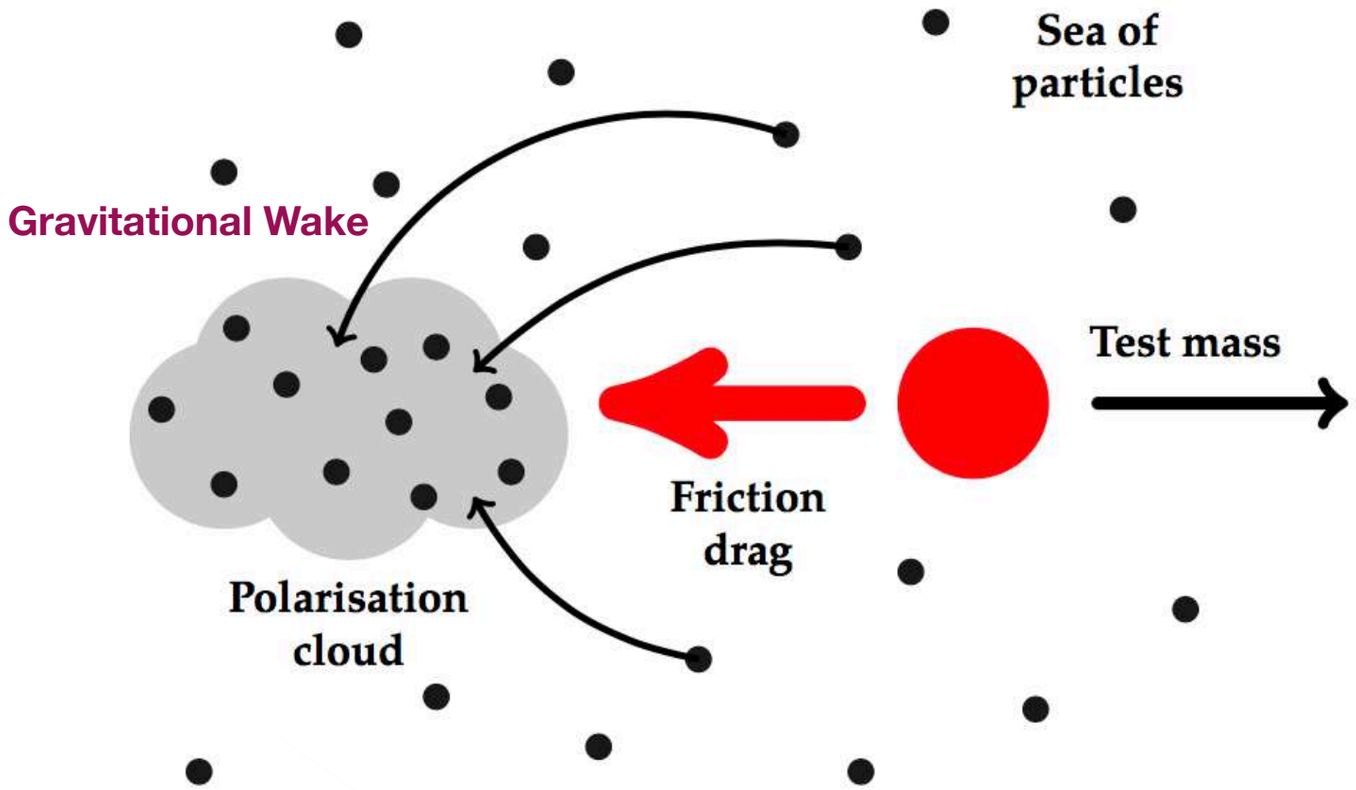


* **Emergence:** arising of novel coherent (unlikely) structures through self-organisation

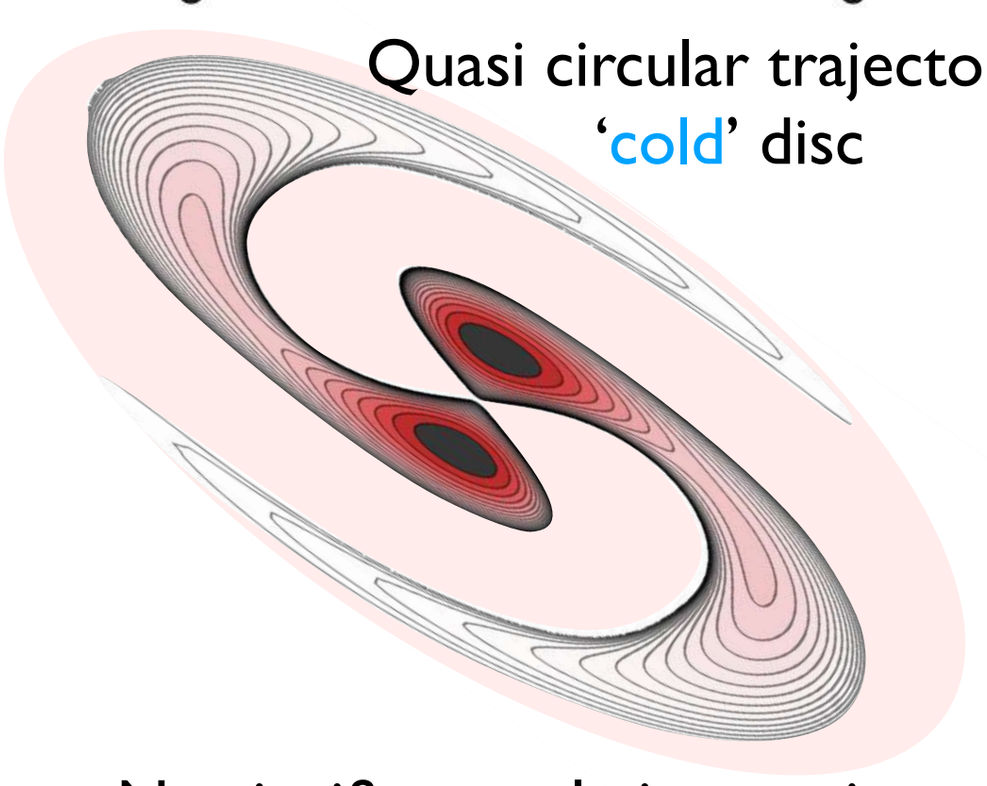
*Near **phase transition** in **open dissipative** systems.*

The **whole** does **not** simply behave like the **sum** of its parts!

Chandrasekhar polarisation





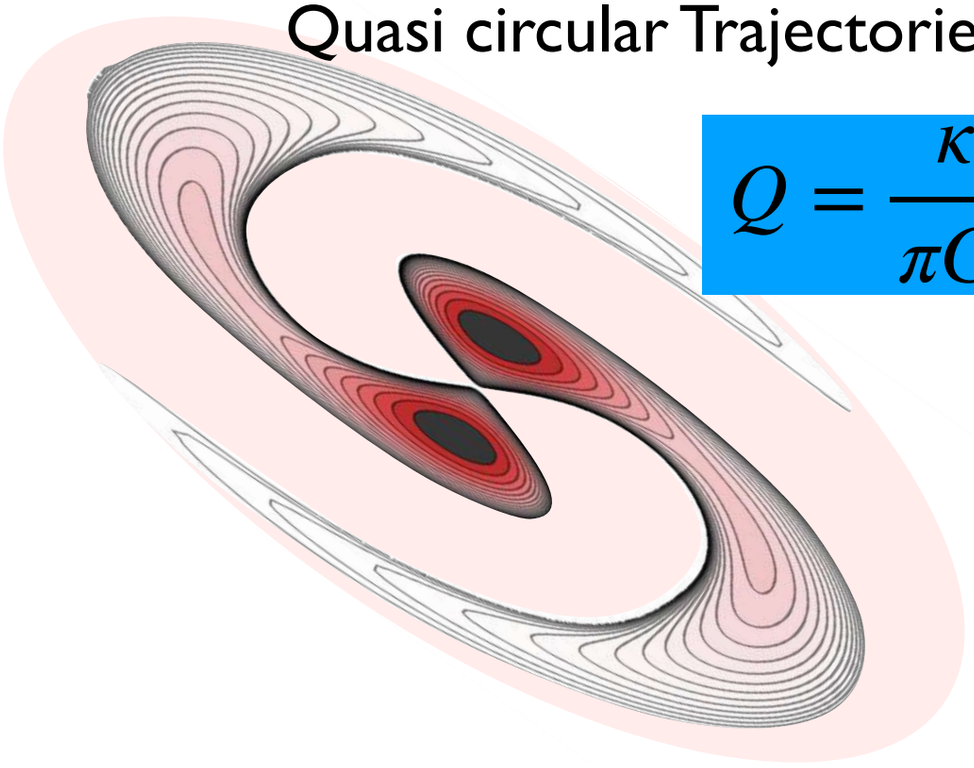


Quasi circular trajectories:
'cold' disc

→ No significant relative motion to oppose gravitation



Quasi circular Trajectories: 'cold' disc

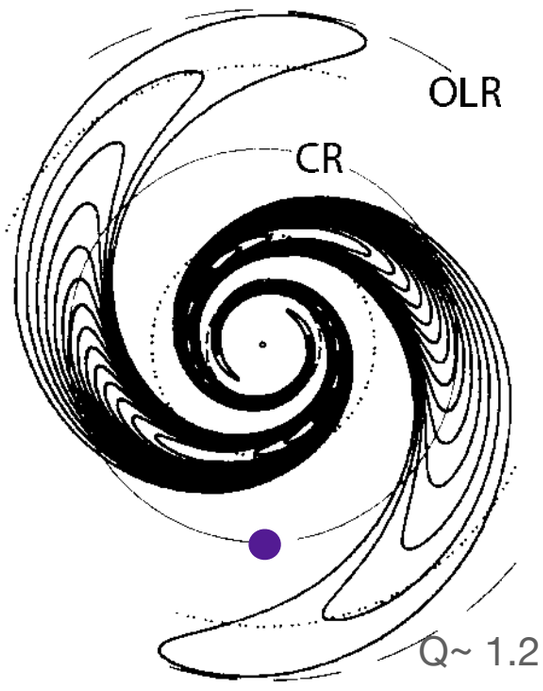


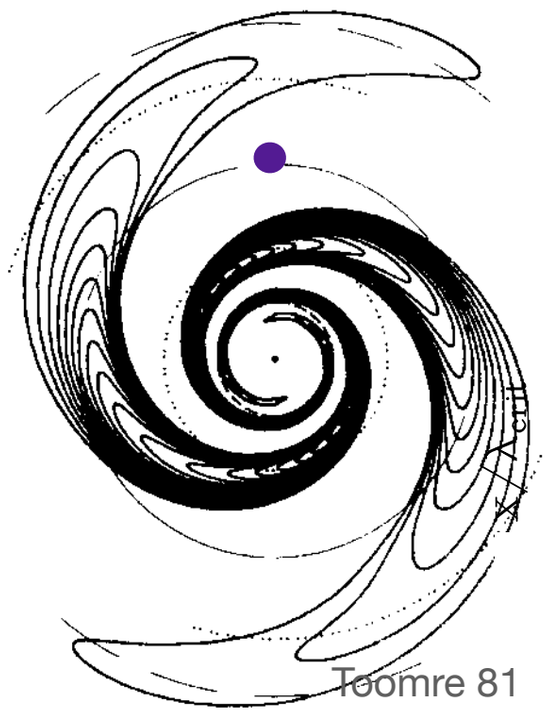
$$Q = \frac{\kappa\sigma}{\pi G\Sigma} \to 1$$

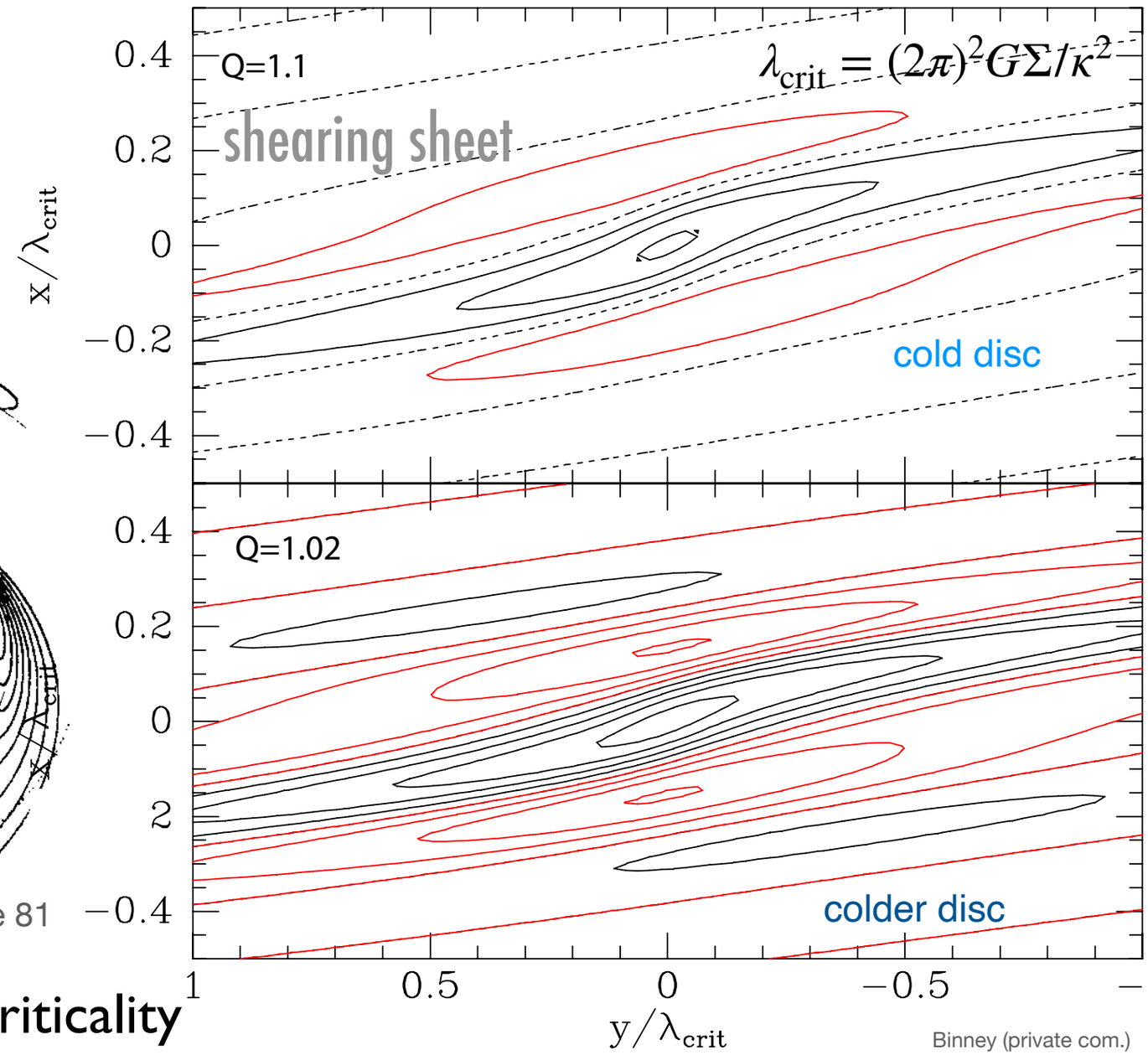
- colder disc means **larger** wake
- colder disc means **stronger** wake
- colder disc means **shorter** secular time

Mass in **wake** = mass in perturbation **X 30** !!

Kalnajs



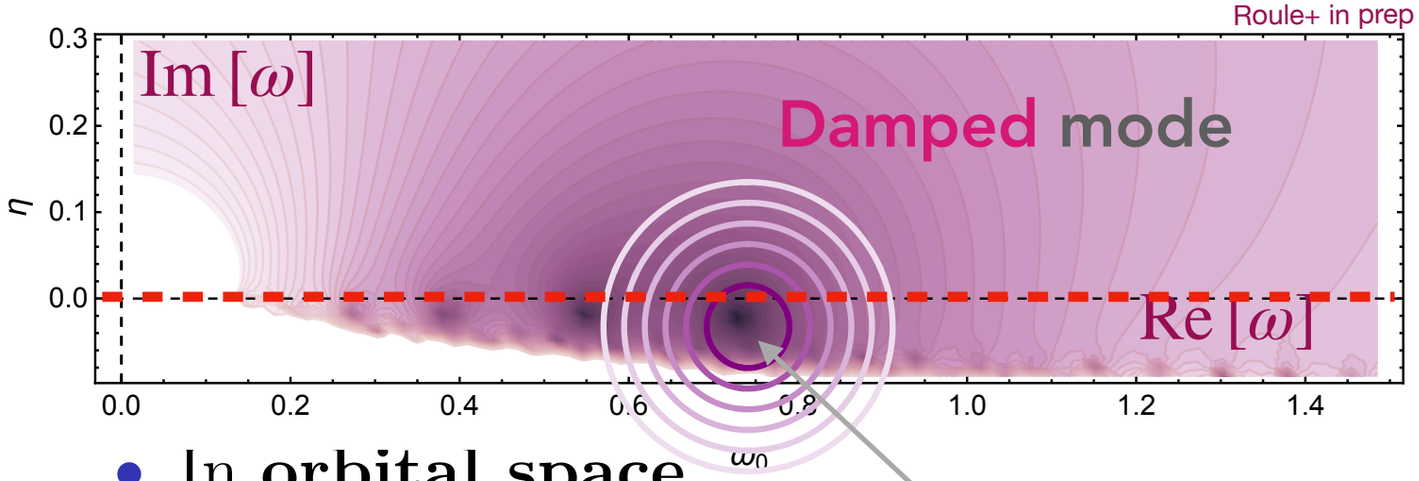




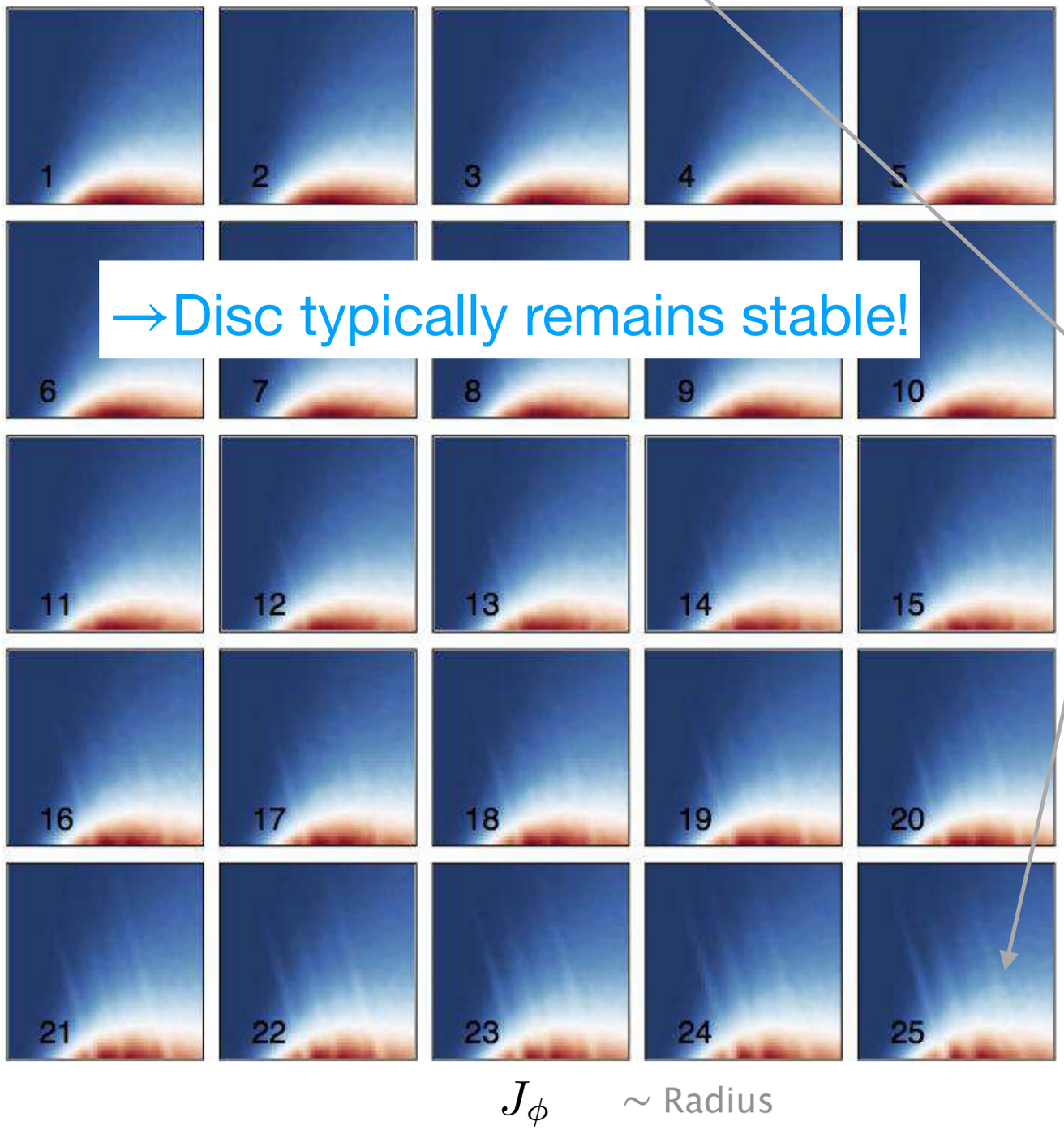
Binney (private com.)

→ long range correlation: self organised criticality

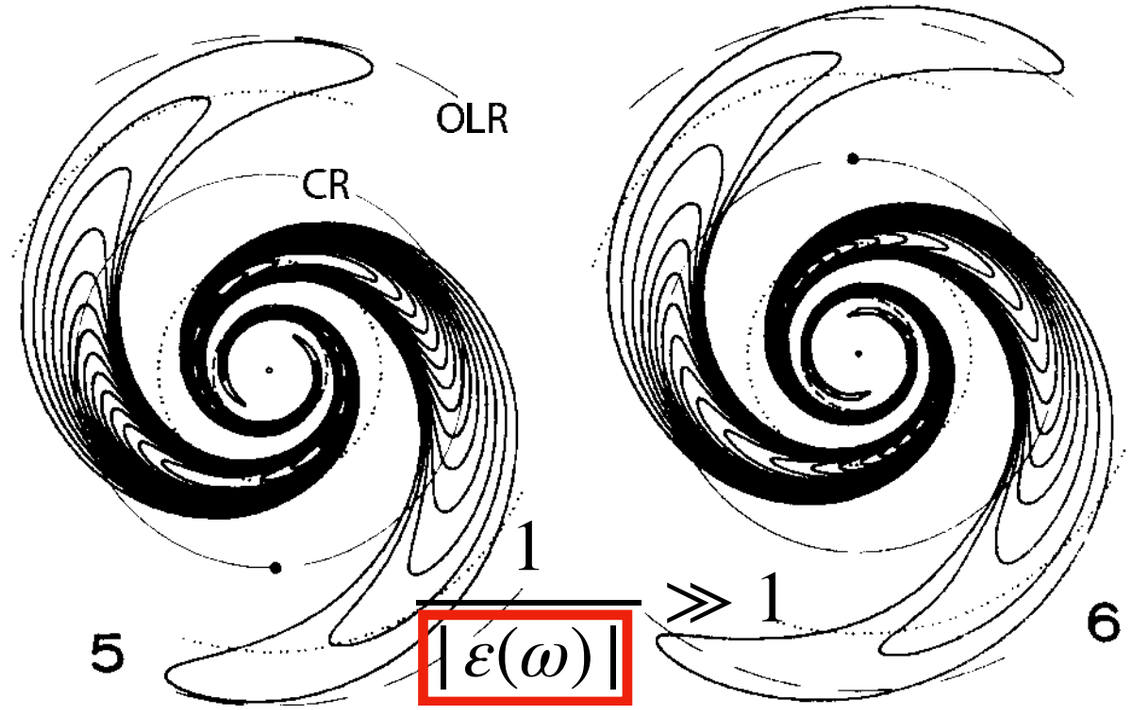
Roule+ in prep

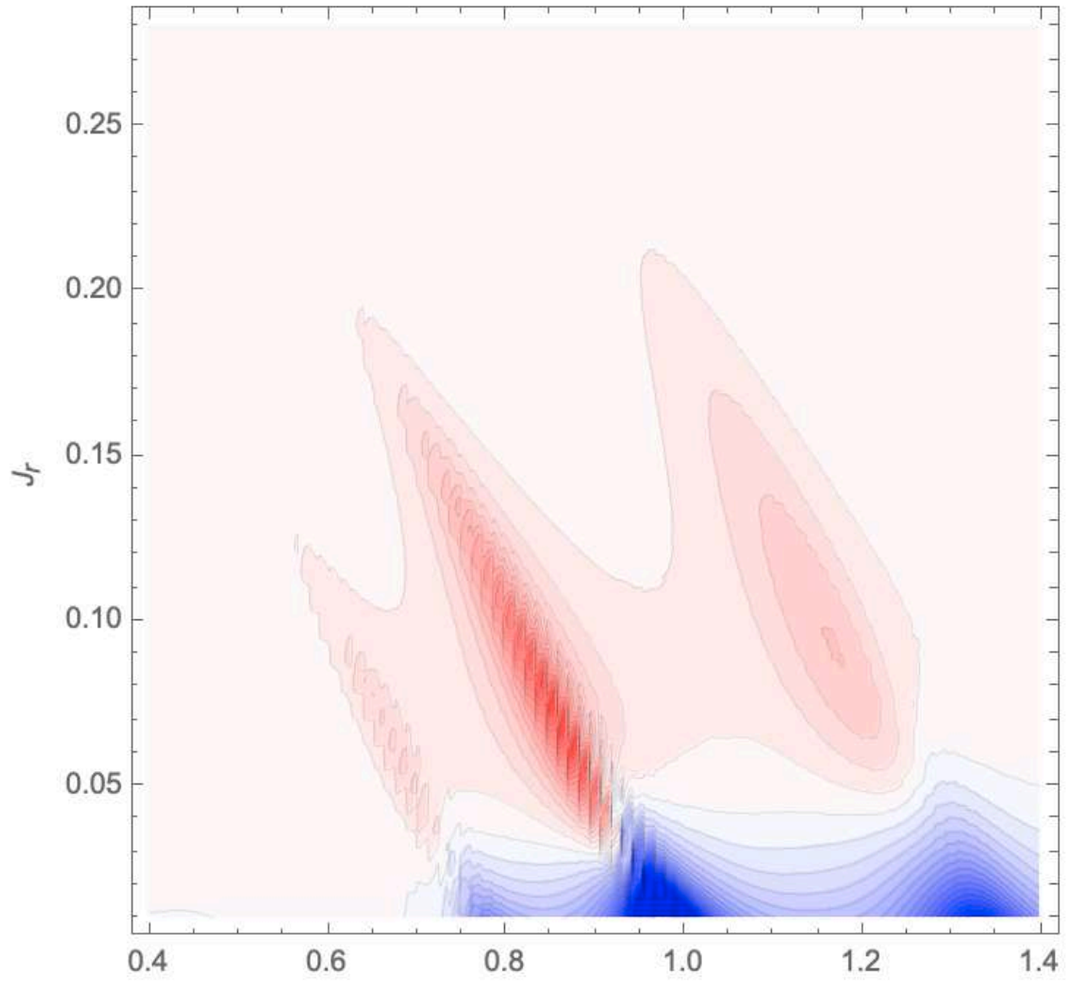


- In **orbital space**



Mass in **wake** = mass in perturbation **X 140**



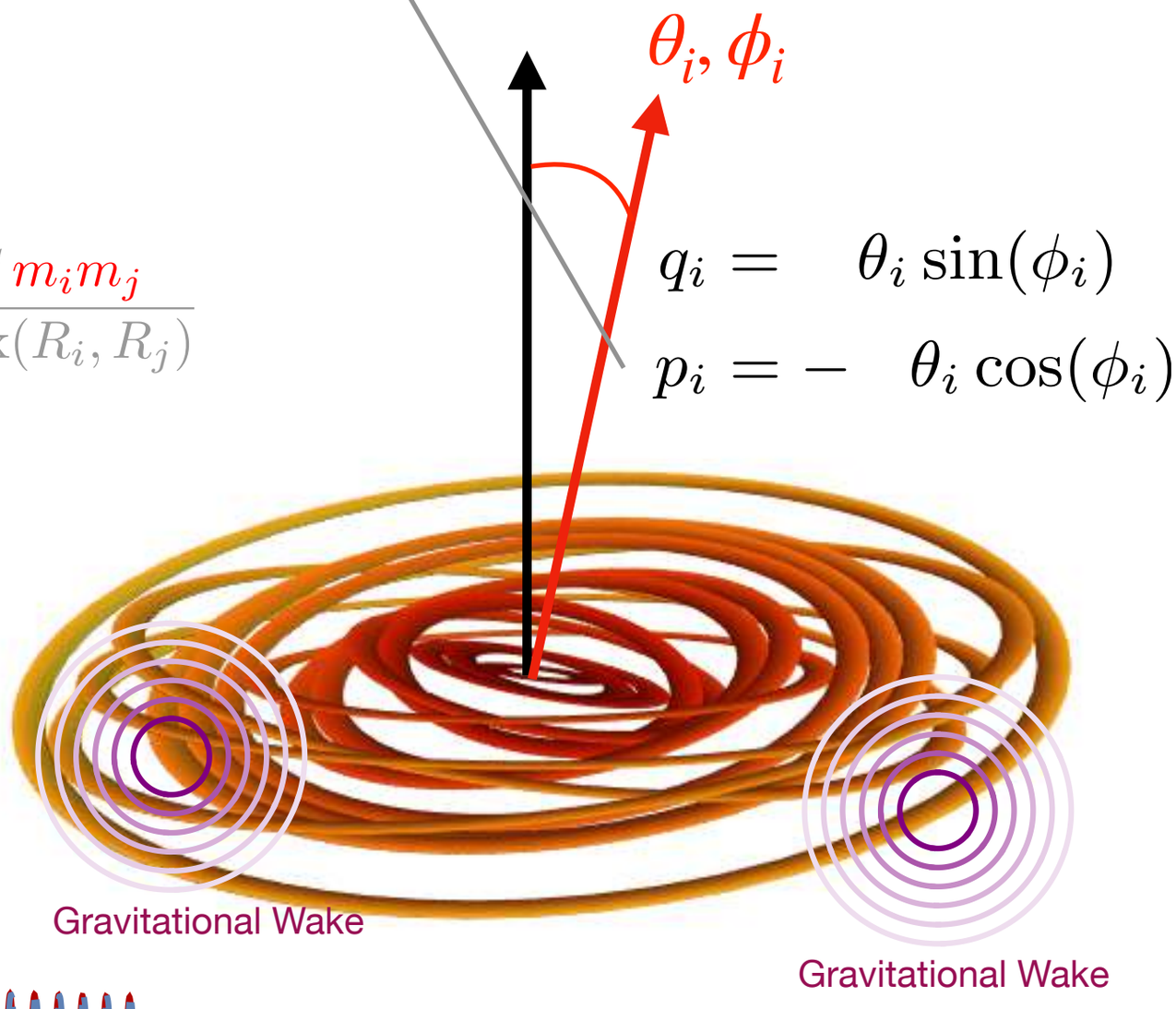


Lagrange Laplace theory of rings *(small eccentricity small inclinaison)*

x and y components of **angular momentum**

$$H(\mathbf{p},\mathbf{q}) = \frac{1}{2}\mathbf{p}^{\mathrm{T}}\cdot\mathbf{A}\cdot\mathbf{p} + \frac{1}{2}\mathbf{q}^{\mathrm{T}}\cdot\mathbf{A}\cdot\mathbf{q},$$

$$A_{ij} \propto -\frac{G\,m_i m_j}{\max(R_i,R_j)}$$

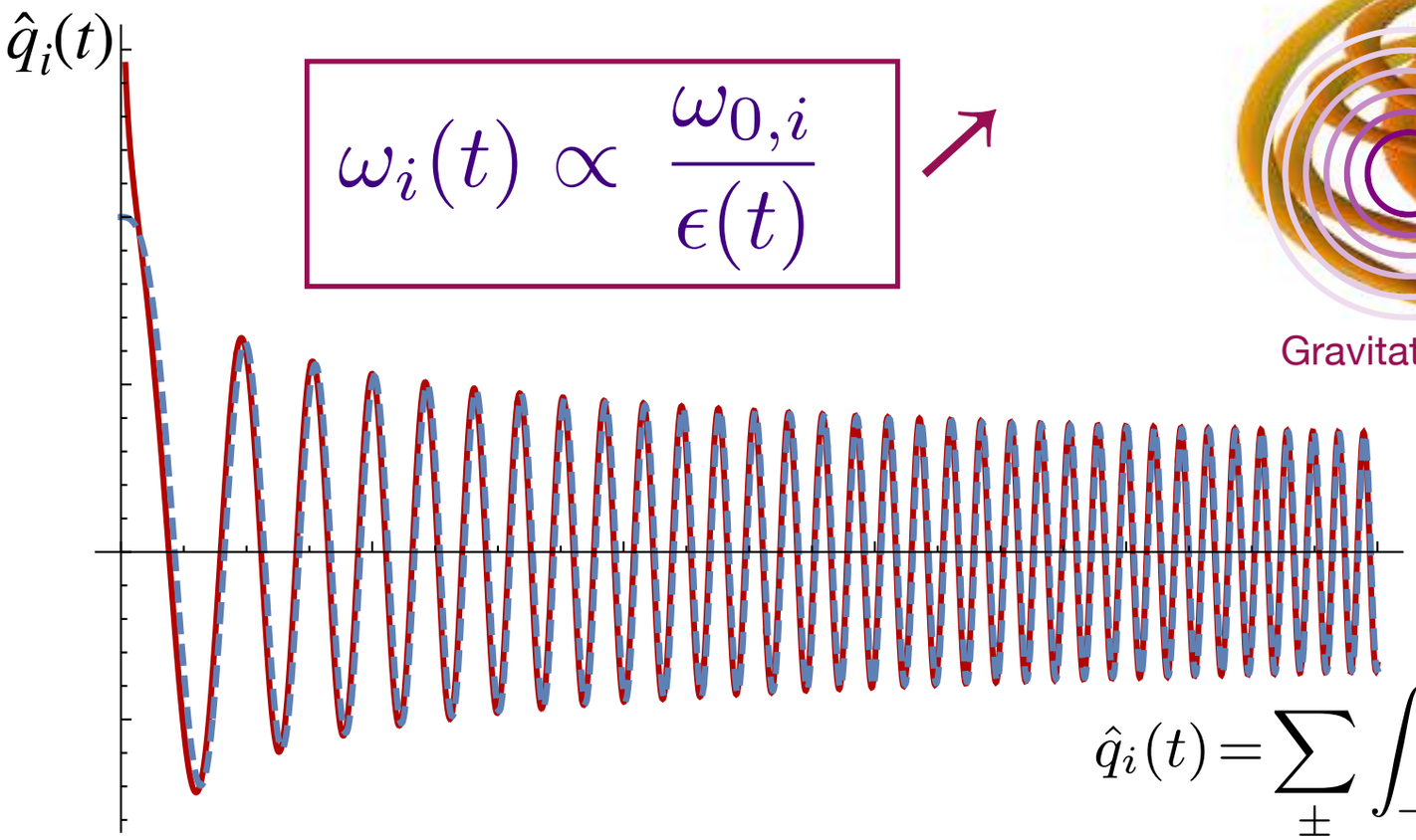


In eigenframe of A

Eigen frequency

$$\ddot{\hat{q}}_i + \omega_i^2(t)\hat{q}_i = \xi_i$$

forcing



$$\omega_i(t) \propto \frac{\omega_{0,i}}{\epsilon(t)}$$

Secular WKB solution

$$\hat{q}_i(t) = \sum_{\pm}\int_{-\infty}^{\infty} \frac{\hat{\xi}_i(t')}{\sqrt{\omega_i(t)\omega_i(t')}} \exp\left(\pm\imath\int_{t'}^{t}\omega_i(\tau)d\tau\right)dt'$$

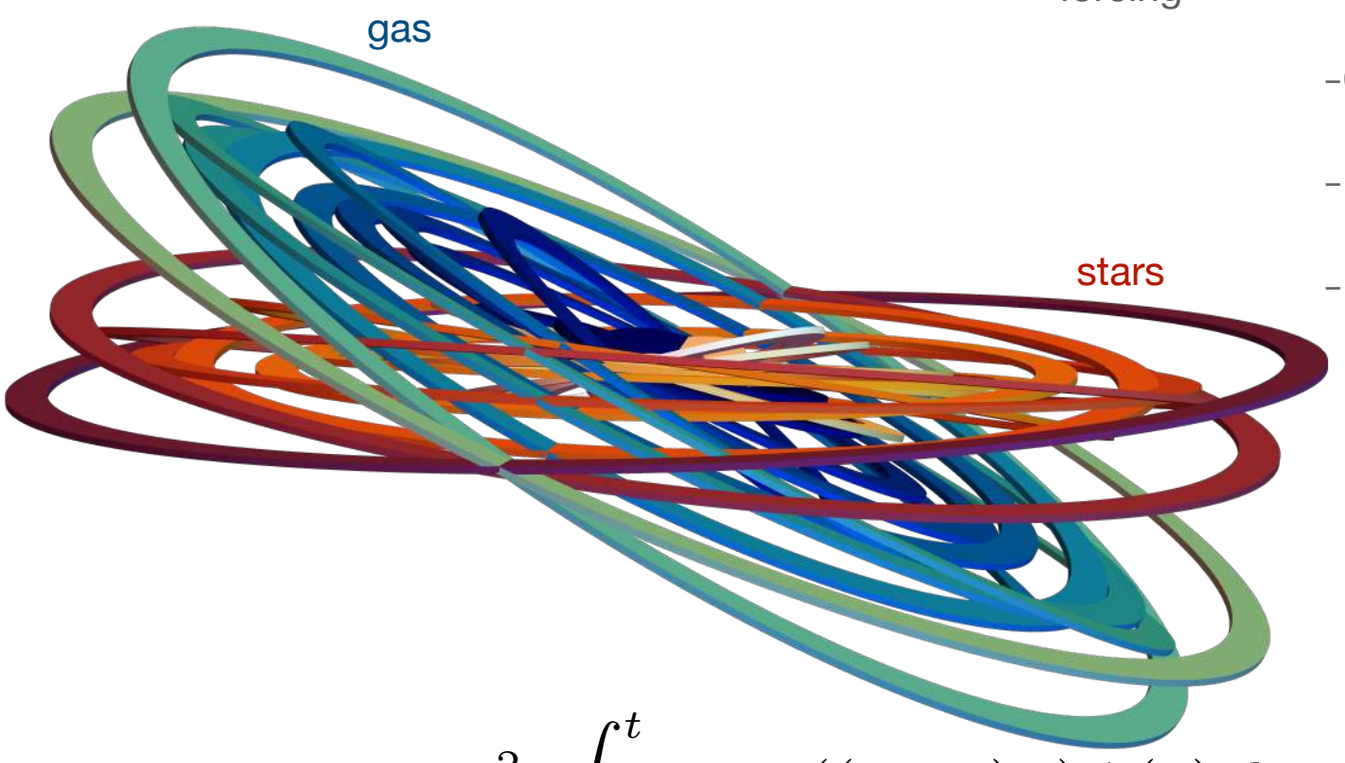
Ring Toy model: gas + star coupling

gravitational coupling

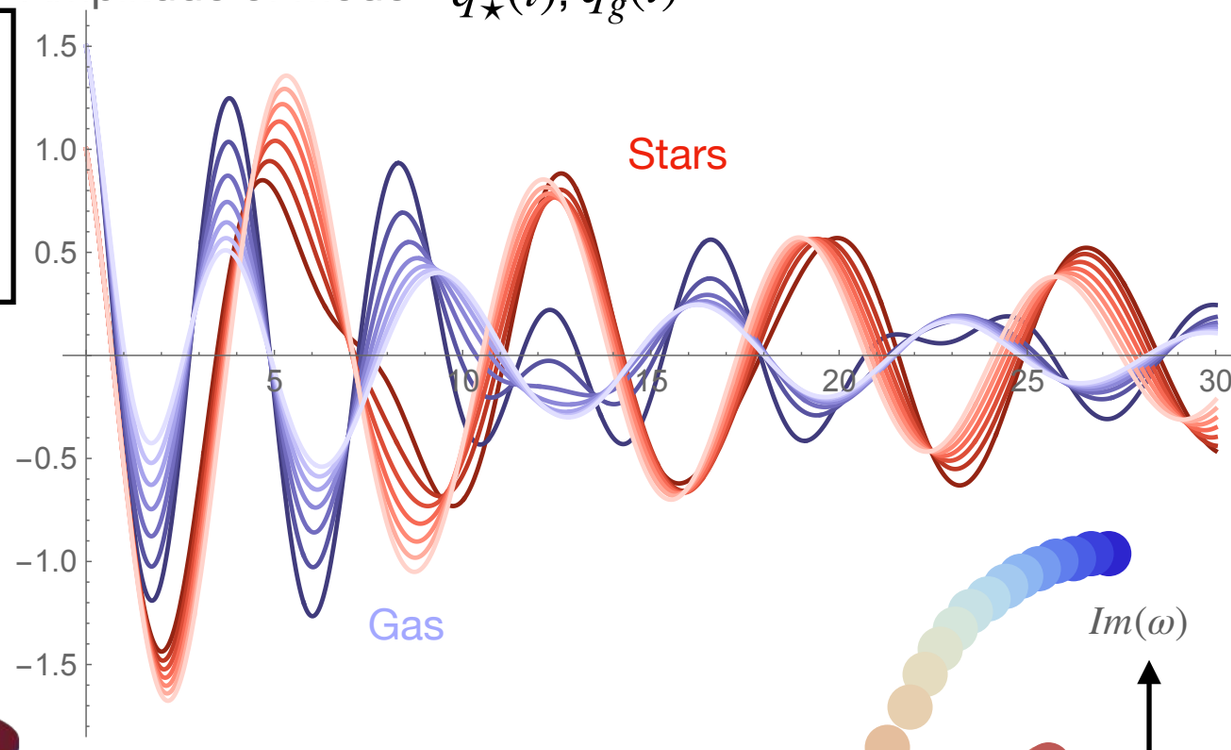
damping

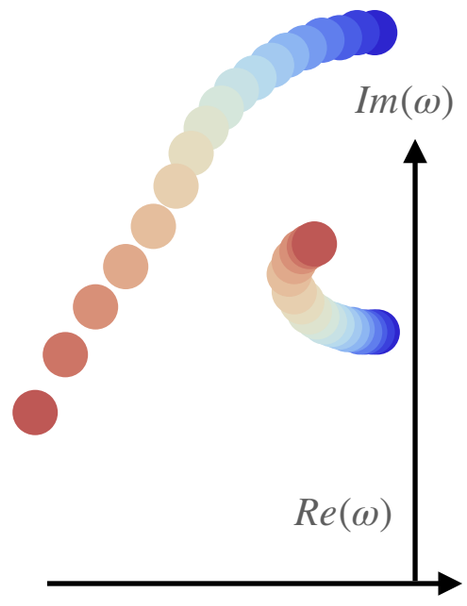
$$\ddot{q}_\star + \omega_\star^2 q_\star + \omega_{\star g}^2 q_g = 0\,,$$
$$\ddot{q}_g + \omega_g^2 \hat{q}_g + \omega_{\star g}^2 q_\star + \eta \dot{q}_g = \xi\,,$$

forcing



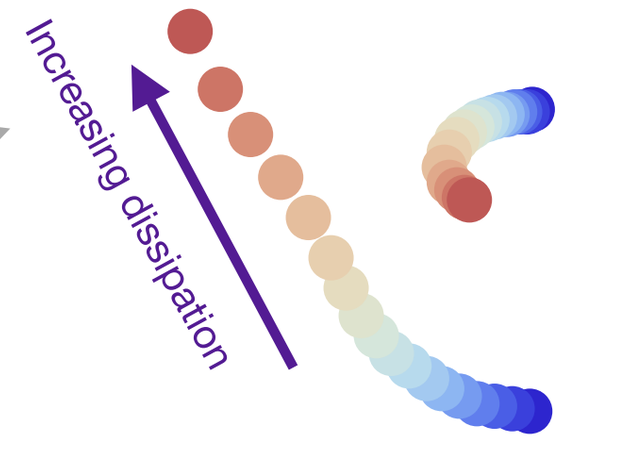
Amplitude of mode $\hat{q}_\star(t), \hat{q}_g(t)$





$$q_\star(t) = -\sum_{\omega \in S_4} \frac{\omega_{g\star}^2 \int_{-\infty}^{t} \exp\left((t-\tau)\omega\right) \xi(\tau)\, d\tau}{\eta\left(3\omega^2 + \omega_\star^2\right) + 2\omega\left(2\omega^2 + \omega_g^2 + \omega_\star^2\right)}\,,$$

$$S_4 = \{\omega \,|\, \left(\omega^2 + \omega_\star^2\right)\left(\omega\left(\eta + \omega\right) + \omega_g^2\right) = \omega_{g\star}^4\},$$



*Dissipation in gas **also** brings down the ★ modes*

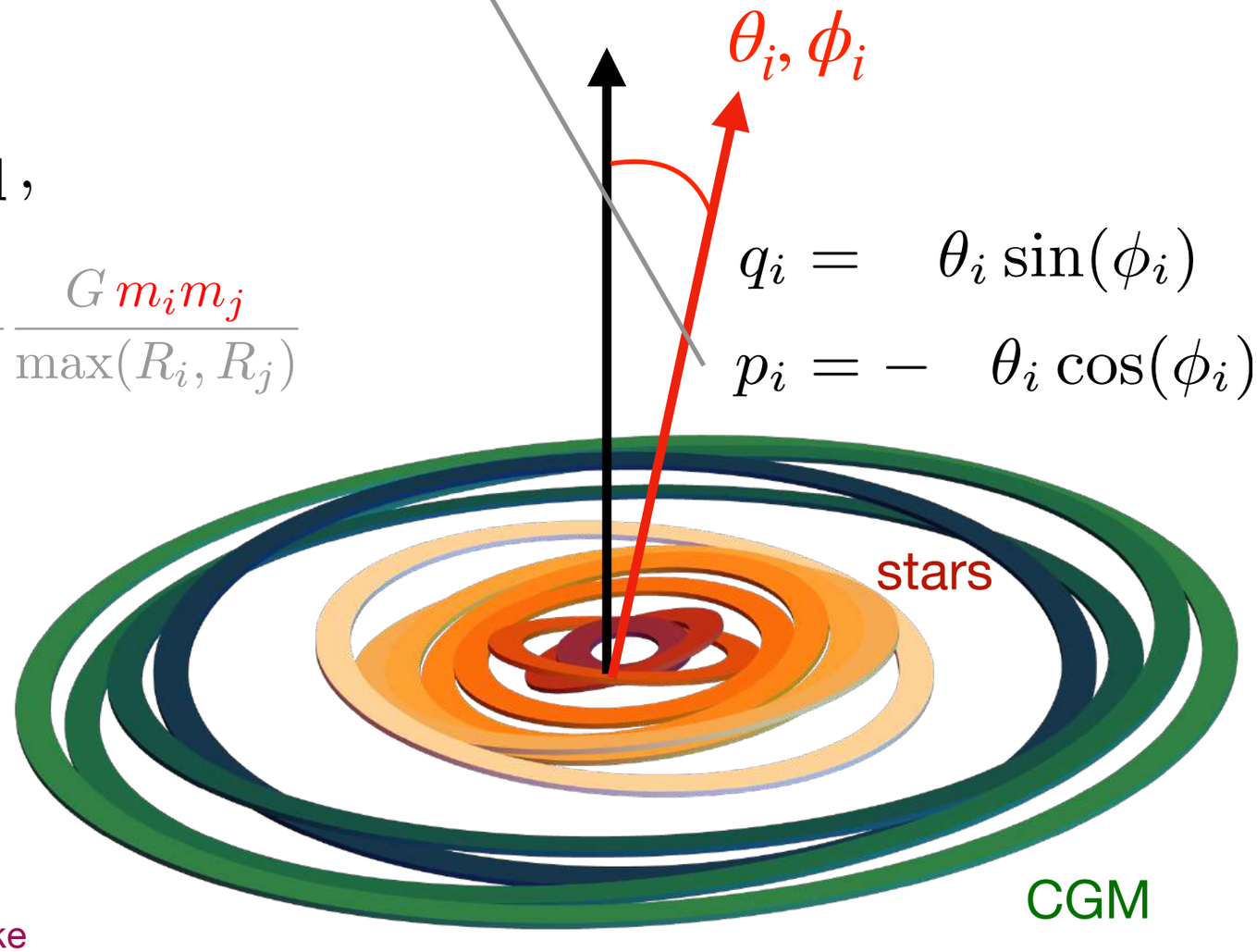
See also Bertin Romeo (1988) 195, 105-113

Lagrange Laplace theory of rings *(small eccentricity small inclinaison)*

x and y components of **angular momentum**

$$H(\mathbf{p},\mathbf{q}) = \frac{1}{2}\mathbf{p}^{\mathrm{T}}\cdot\mathbf{A}\cdot\mathbf{p} + \frac{1}{2}\mathbf{q}^{\mathrm{T}}\cdot\mathbf{A}\cdot\mathbf{q},$$

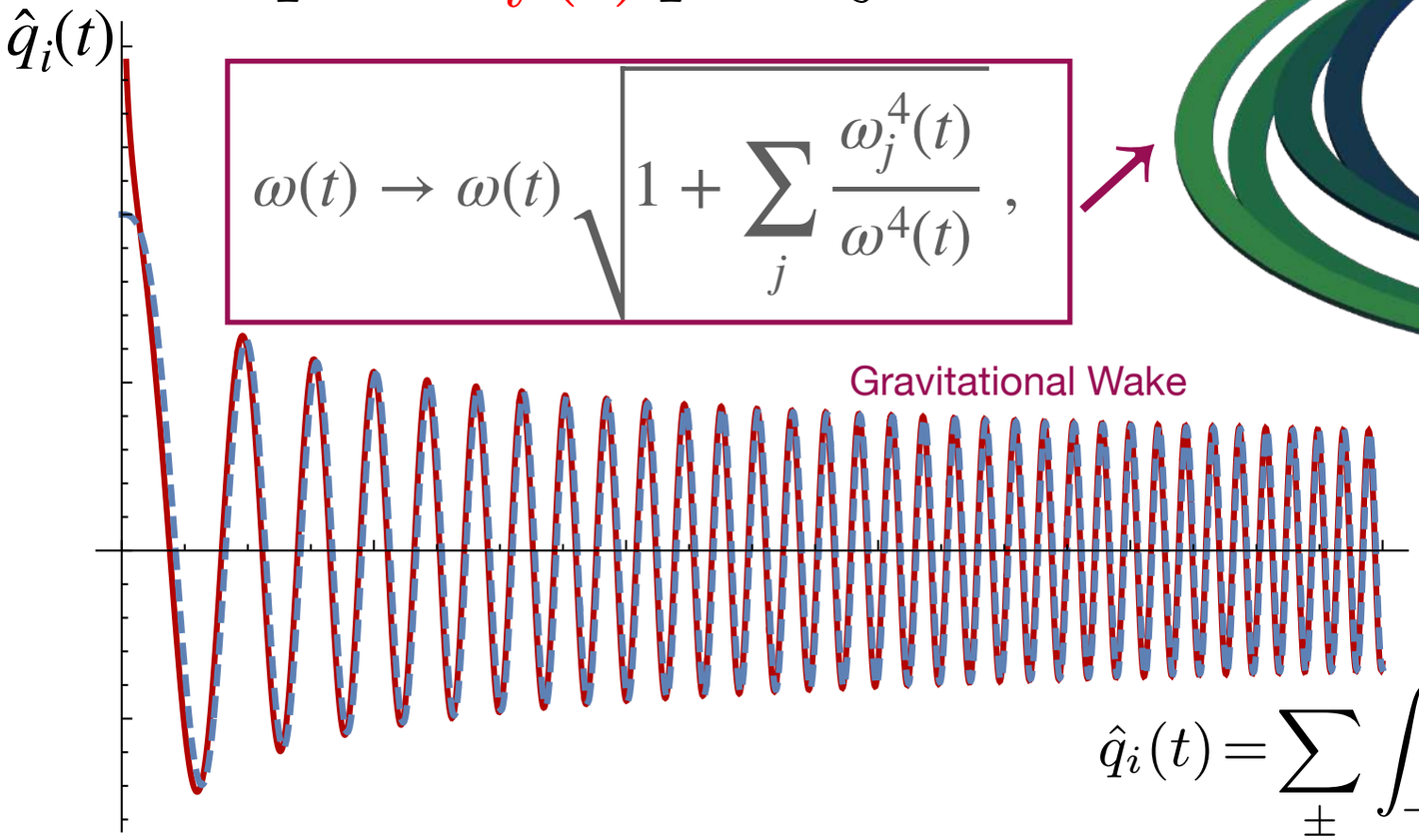
$$A_{ij} \propto -\frac{G\,m_i m_j}{\max(R_i,R_j)}$$



In eigenframe of A

Eigen frequency — forcing

$$\ddot{\hat{q}}_i + \omega_i^2(t)\hat{q}_i = \xi_i$$



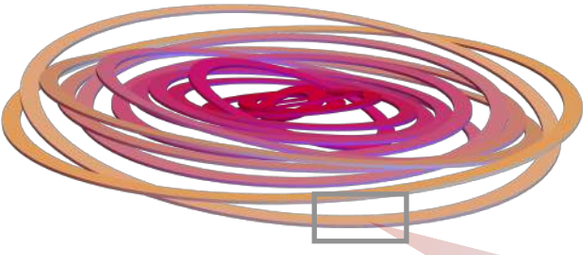
$$\omega(t) \to \omega(t)\sqrt{1+\sum_j \frac{\omega_j^4(t)}{\omega^4(t)}},$$

Secular WKB solution

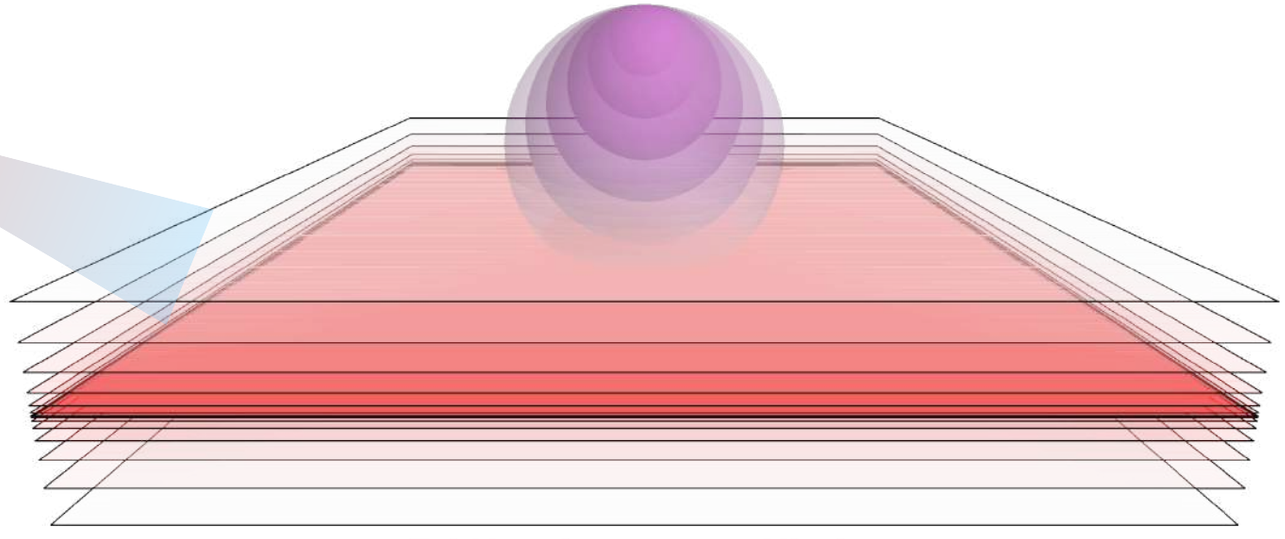
$$\hat{q}_i(t) = \sum_{\pm}\int_{-\infty}^{\infty} \frac{\hat{\xi}_i(t')}{\sqrt{\omega_i(t)\omega_i(t')}} \exp\left(\pm\imath\int_{t'}^{t}\omega_i(\tau)d\tau\right)dt'$$

*Growth of CGM component **also** brings down the ★ modes*

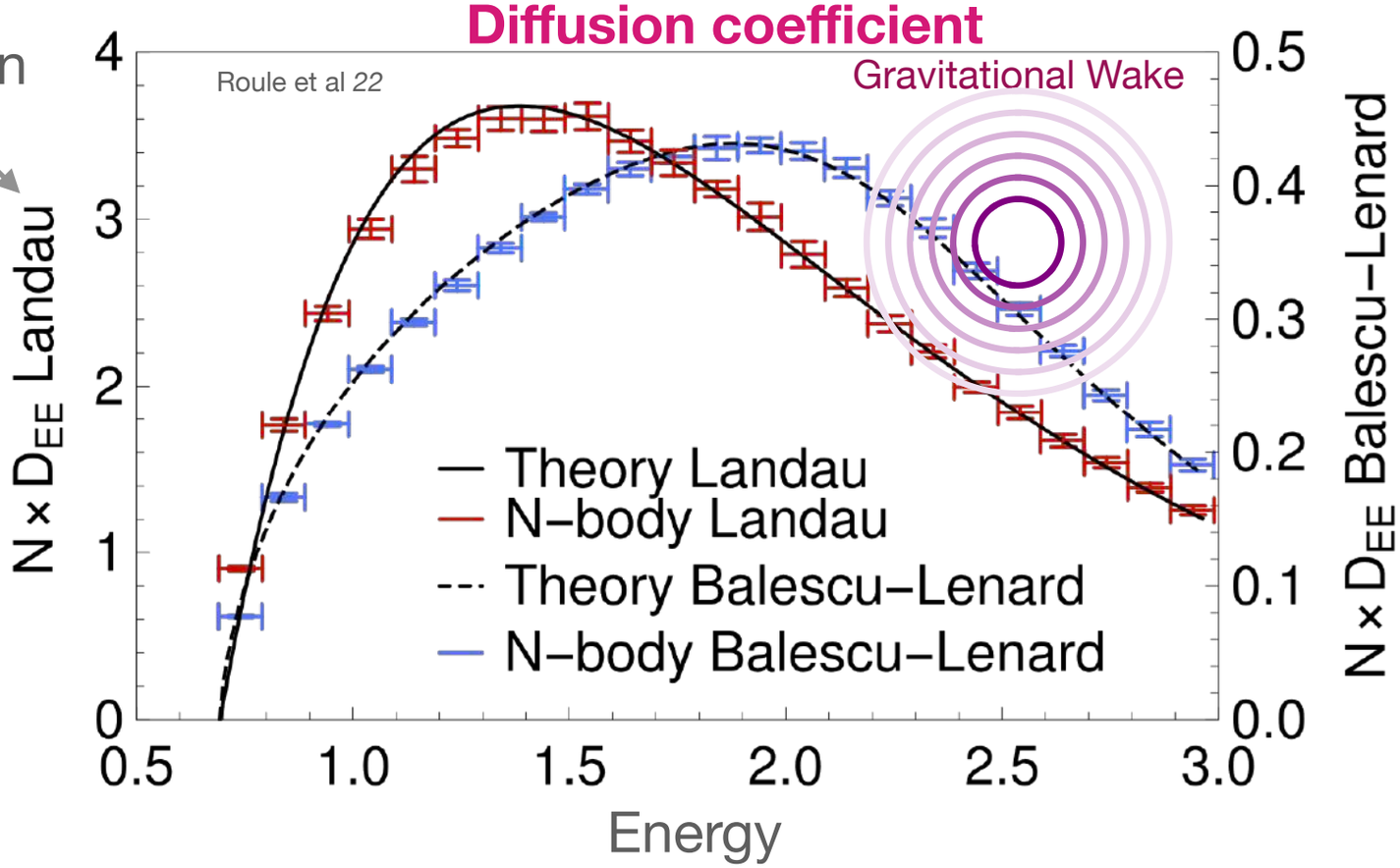
Plane Toy model: dressing damps vertical diffusion



Kinetic theory of (toy model) parallel planes with and w/o dressing



Without polarisation



With polarisation:

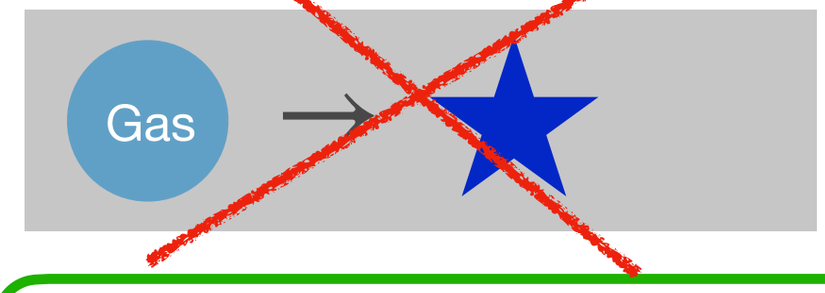
rate of diffusion
x 1/ 10

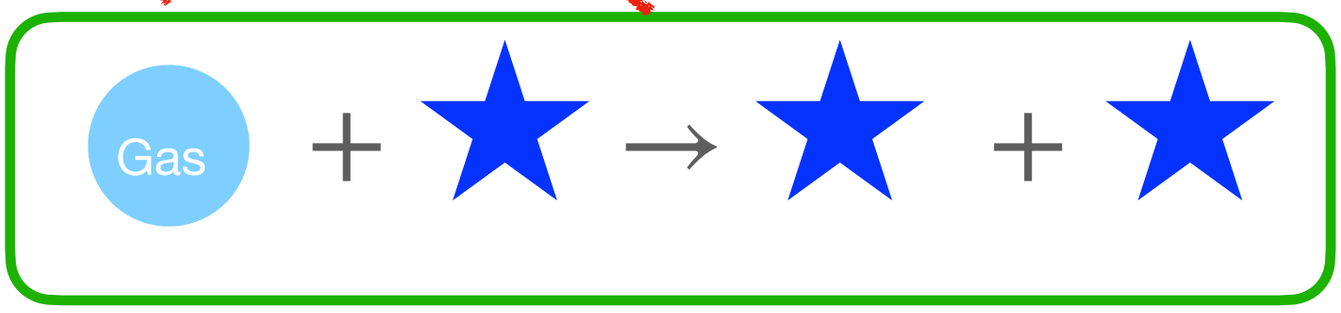
Polarisation stiffen coupling between planes → wakes stiffen disc

Why finite thickness? Chemistry of emergence

Let us write down effective (closed loop) production rate for cold stellar component

Auto-catalysis of the cold component (via wakes) converts kinetic evolution into a logistic differential equation.





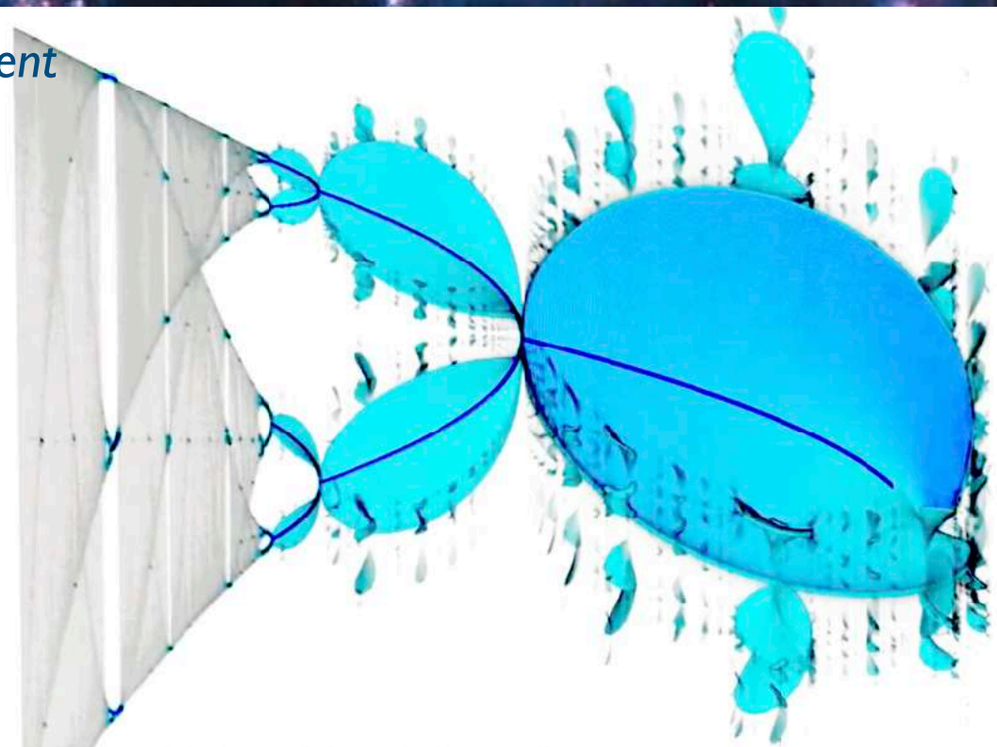
$$\frac{d}{dt}\bigstar = r\bigstar(1-\bigstar)$$

control parameter

Logistic *ODE (cf Ecology, Chaos, Covid, Innovation etc..)*

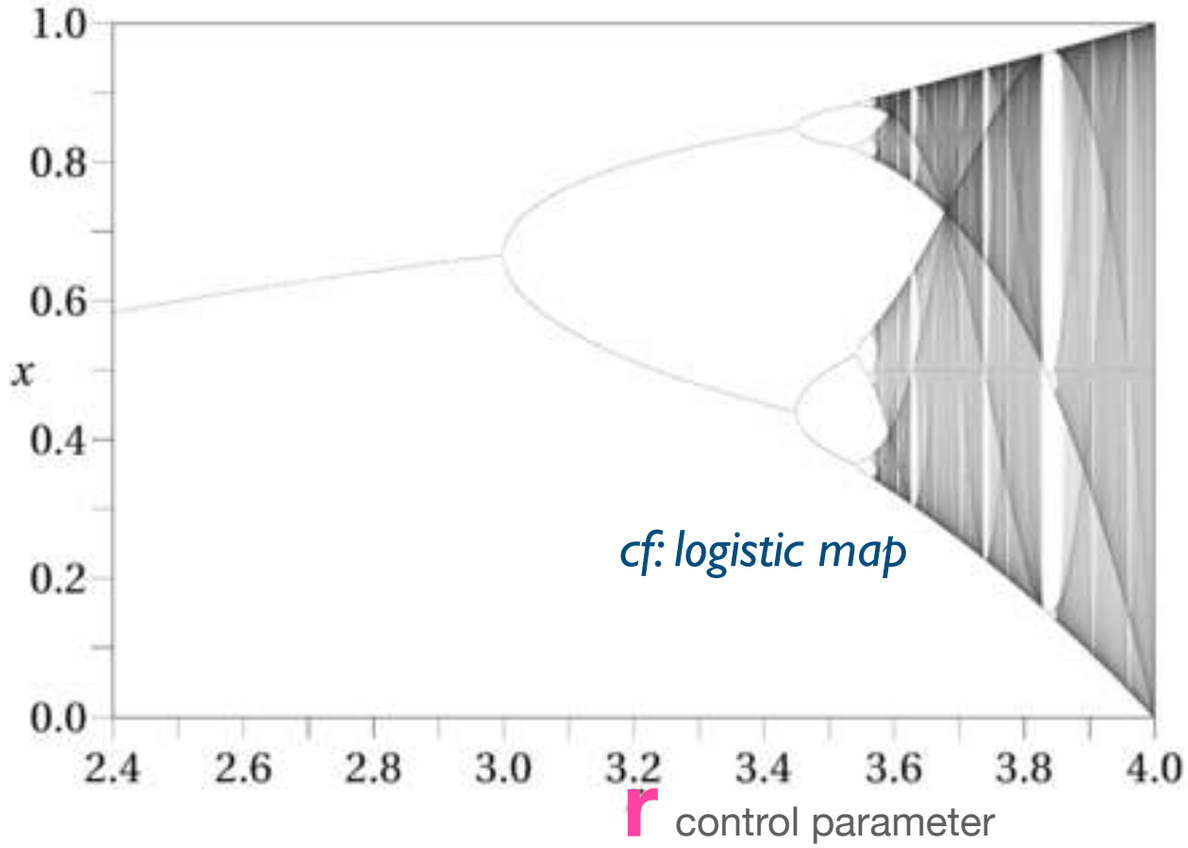
= Simplest **quadratic** model for self -regulation

= Taylor expansion of effective production rate

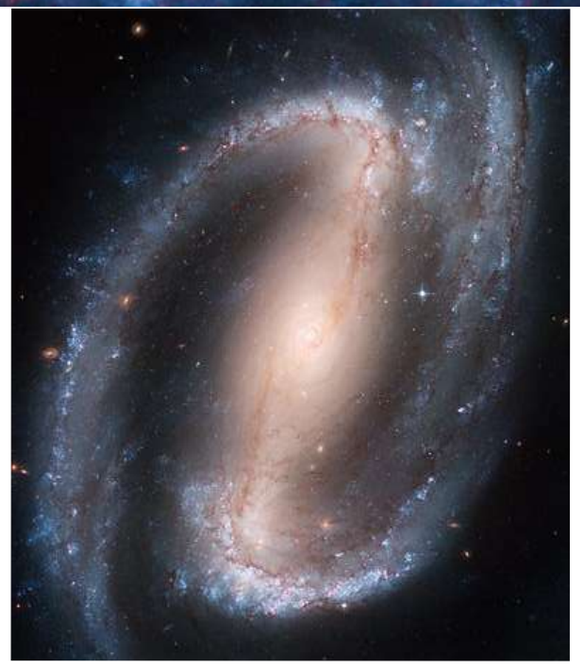


Link to Mandelbrot Set (Veritassium 2021)

★ = cold stellar component



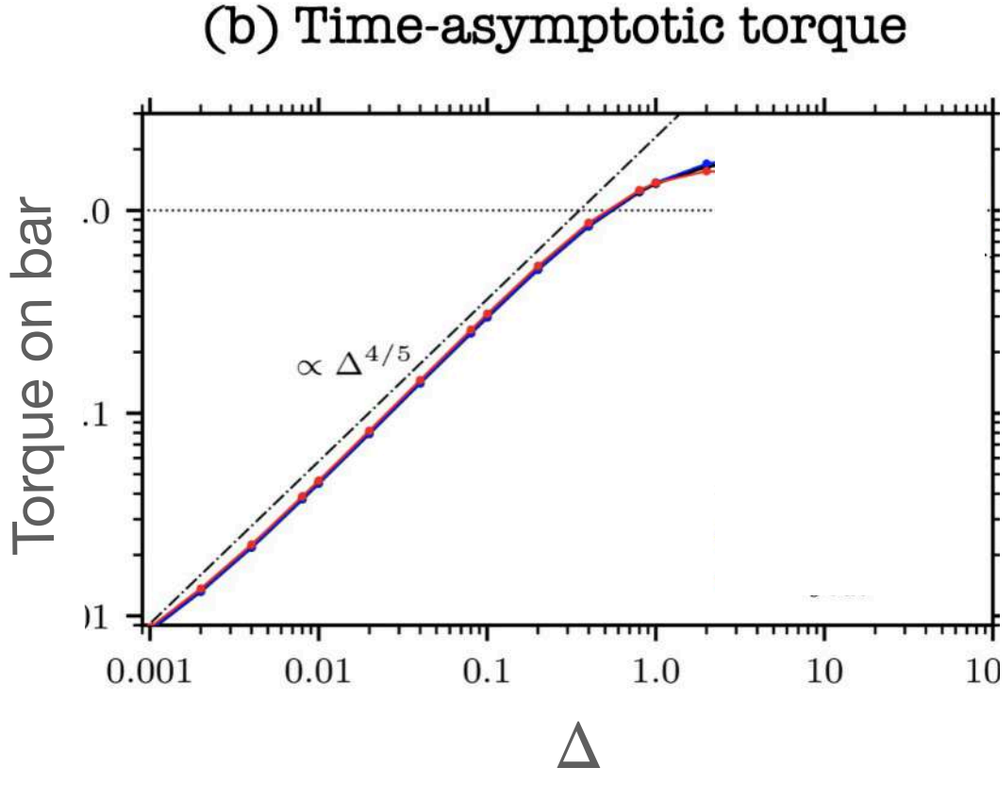
Impact of subgrid physics on bar survival?

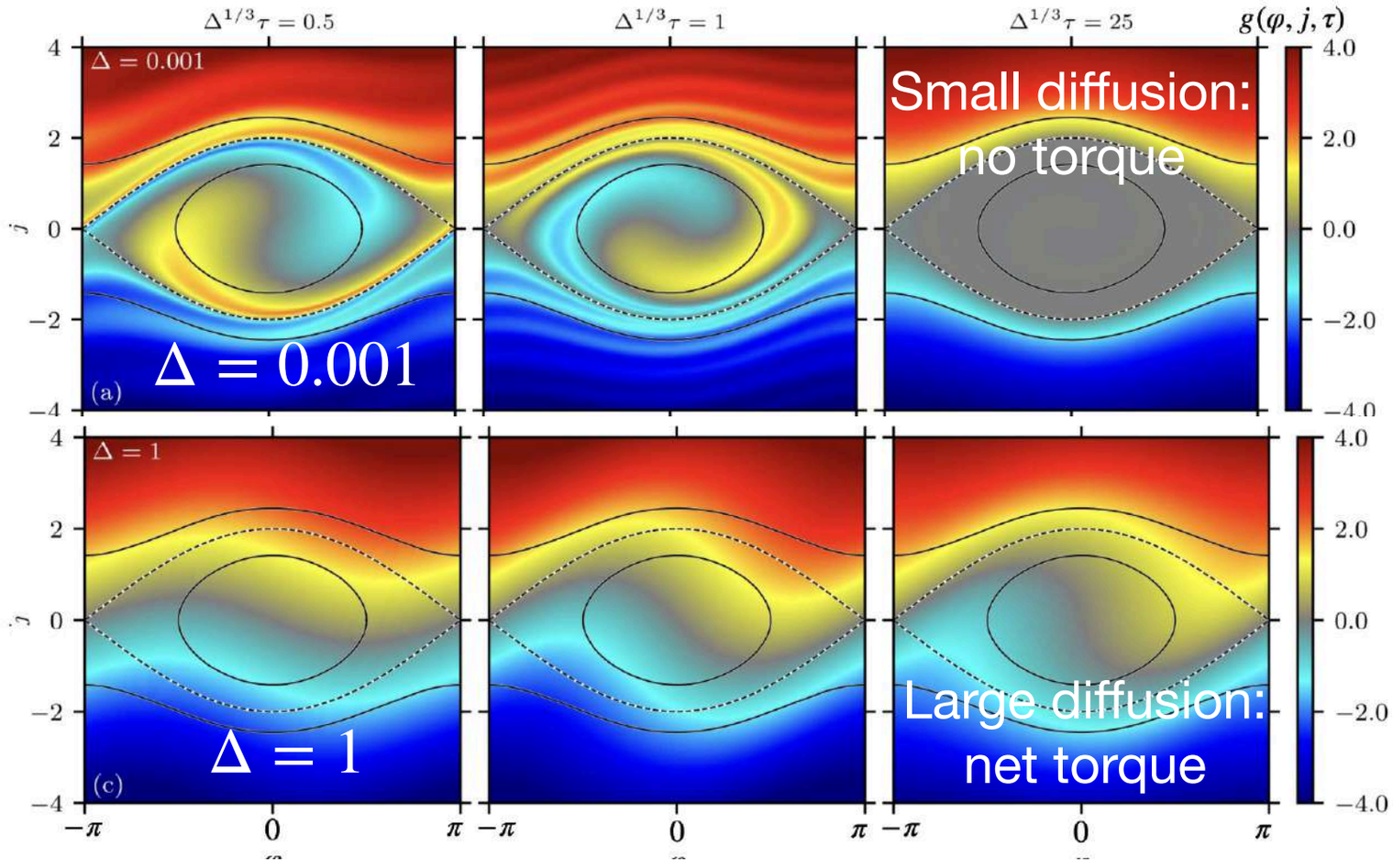


$$\Delta \propto \int P_k(k\Omega)\,|\,\hat{W}(kR)\,|^2 dk = \int^{2\pi/R} P_k(k\Omega)dk \implies \Delta_{\rm subgrid} < \Delta_{\rm real}$$

Truncation @ high frequency

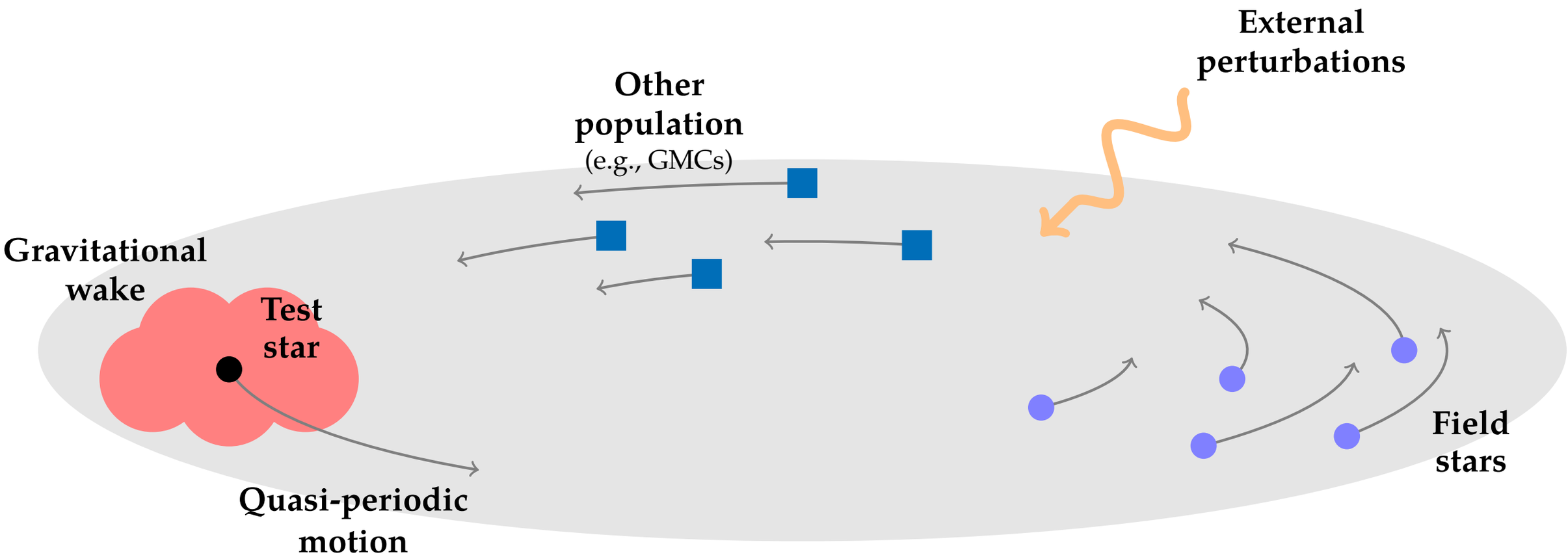
$$\frac{\partial f}{\partial \tau} + j\frac{\partial f}{\partial \varphi} - \sin\varphi \frac{\partial f}{\partial j} = \Delta \frac{\partial^2 f}{\partial j^2}.$$





increased resolution means larger torque $\implies$ smaller bar growth (if donkey)

Evolution on cosmic timescales



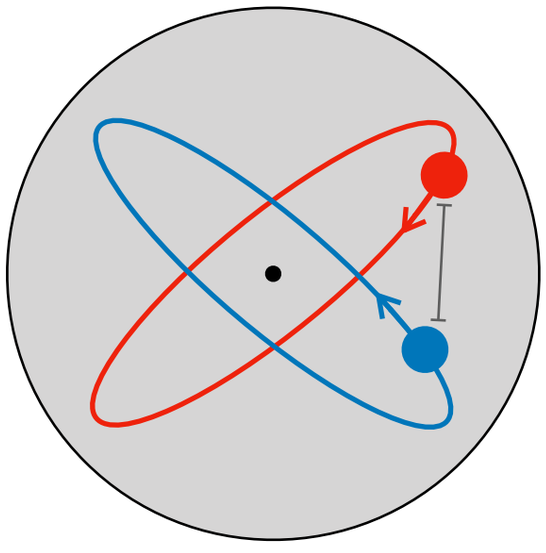
Galaxies are:

+ **Inhomogeneous** (complex trajectories)	*Angle-action coordinates*
+ **Relaxed** (equilibrium states)	*Quasi-stationary states*
+ **Resonant** (orbital frequencies)	*Fast timescale vs. cosmic timescale*
+ **Degenerate** (in some regions)	*Frequency commensurability*
+ **Self-gravitating** (amplification of perturbations)	*Linear response theory*
+ **Discrete** (finite-N effects)	*Nature vs. Nurture*
+ **Perturbed** (effects of the environment)	

Perspectives - Balescu–Lenard

Degenerate

$$\mathbf{k}\cdot\boldsymbol{\Omega}(\mathbf{J}) = \mathrm{cst}\,.$$



Quasi-Keplerian

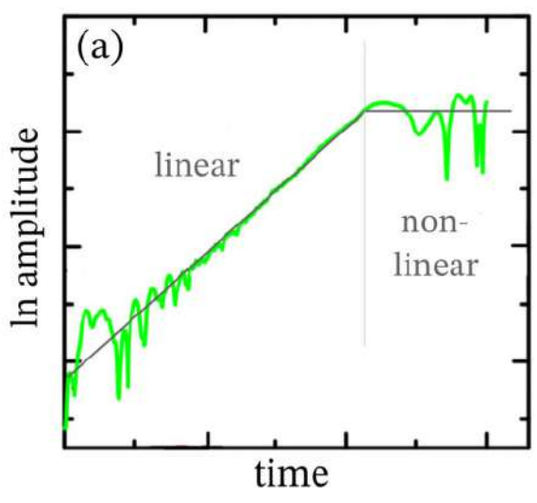
Harmonic potential

Read+(2006), Kocsis+(2015)

Beyond quasi-linear

$$\delta F \sim F$$

Hamilton (2024)



Violent relaxation

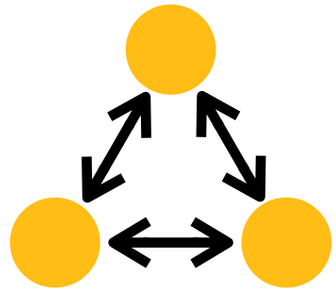
Non-linear saturation

Lynden-Bell (1967), Krommes (2002)

Kinetic blockings



$$\left.\frac{\partial F}{\partial t}\right|_{\mathrm{BL}} = \frac{1}{N}\times 0$$



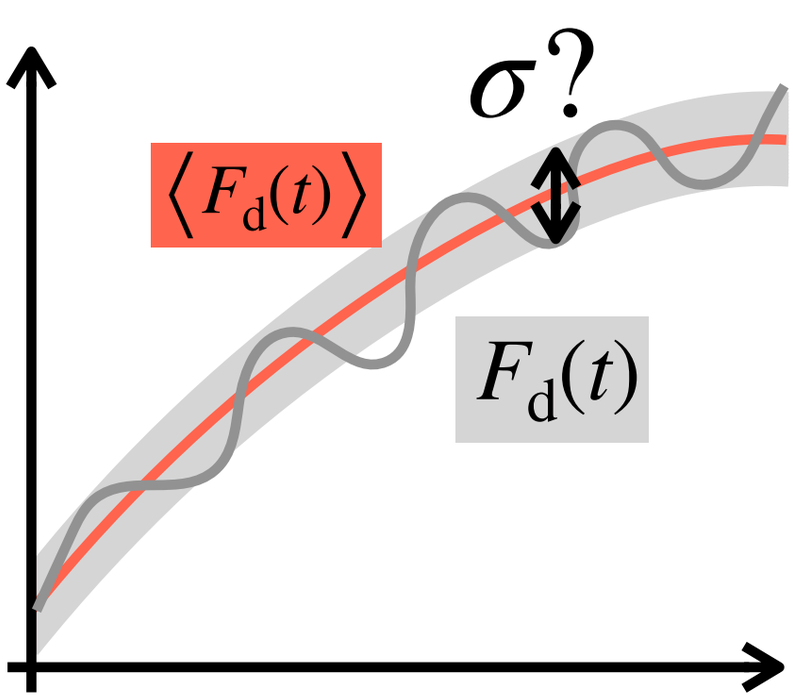
$$\frac{\partial F}{\partial t} = \frac{1}{N^2}\times -$$

Fouvry & Roule (2024) - Phys. Rev. E

Perspectives - Theory

Large deviations

$$\mathbb{P}\left(\{F_{\rm d}\}_t = \{F\}_t\right) = \,?$$



Feliachi+(2022)

Marginal stability

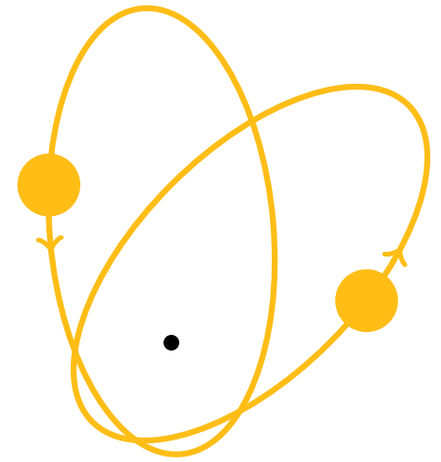
Stable → Unstable

$$\frac{1}{|\,\boldsymbol{\varepsilon}(\mathbf{k}\cdot\boldsymbol{\Omega})\,|} \to \infty$$

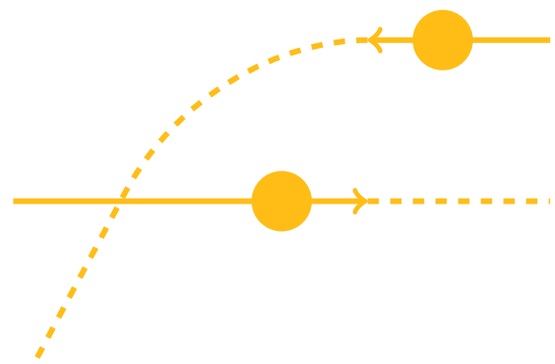
Balescu–Lenard is
``**deeply flawed**''

Weinberg (1993), Hamilton+(2023)

Local *vs* resonant



Cold spiral galaxies



Hot globular clusters

Marcos+(2017), Tep+(2022)

Perspectives - Astrophysics

External noise

Dressed **Fokker-Planck**

$$\frac{\partial F}{\partial t} \simeq \frac{|\delta\psi_{\rm ext}|^2}{|\varepsilon(\omega)|^2}$$

Cosmic **environment**

SNe, turbulence

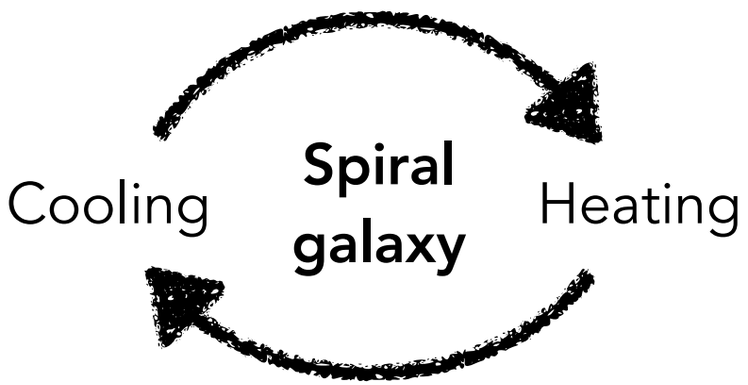
Binney+(1988), Weinberg (2001), Johnston+(2017)

Cooling processes

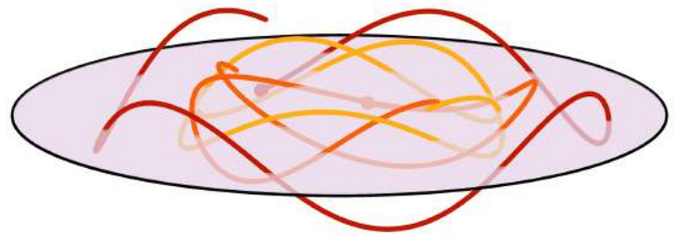
Star formation



Self-regulation



Thickened geometry



Morphological transformation

Dejonghe (1998), Fouvry+(2017)