Probing Primordial Parity-Violating Trispectrum with Intrinsic Galaxy/Halo Shapes

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Is parity violated in cosmology?

- Physics of inflation could generate parity-violating statistics: axion-like fields,…

Sorbo2011, Barnaby+2012, Shiraishi 2016,…

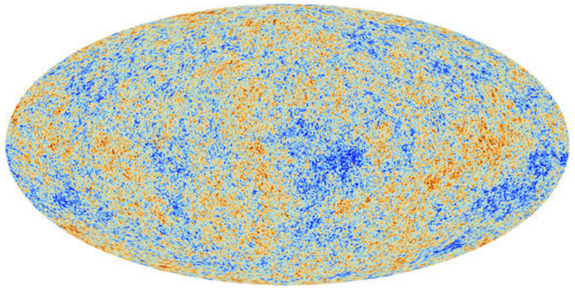
- "Scalar" parity violation = parity-odd part of trispectum of curvature perturbation $\zeta(\mathbf{k})$:

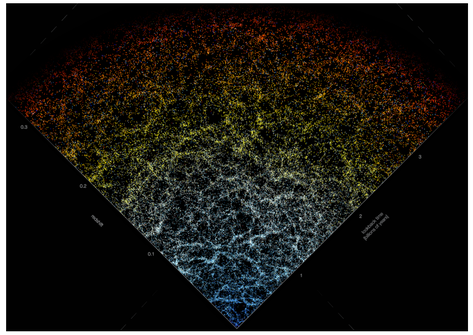
"Vector" PV, see Junsup's talk.

$$\langle\zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)\zeta(\mathbf{k}_3)\zeta(\mathbf{k}_4)\rangle = (2\pi)^3\delta^{\rm D}_{\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3+\mathbf{k}_4}T_\zeta(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3,\mathbf{k}_4)$$

$$T_\zeta(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3,\mathbf{k}_4) = T_+(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3,\mathbf{k}_4) + \underbrace{T_-(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3,\mathbf{k}_4)}_{\text{(pure imaginary)}} \qquad \mathcal{P}[T_\pm] = \pm T_\pm$$

- The CMB and LSS would carry imprints of the parity violation Shiraishi 2016, Cahn+2023





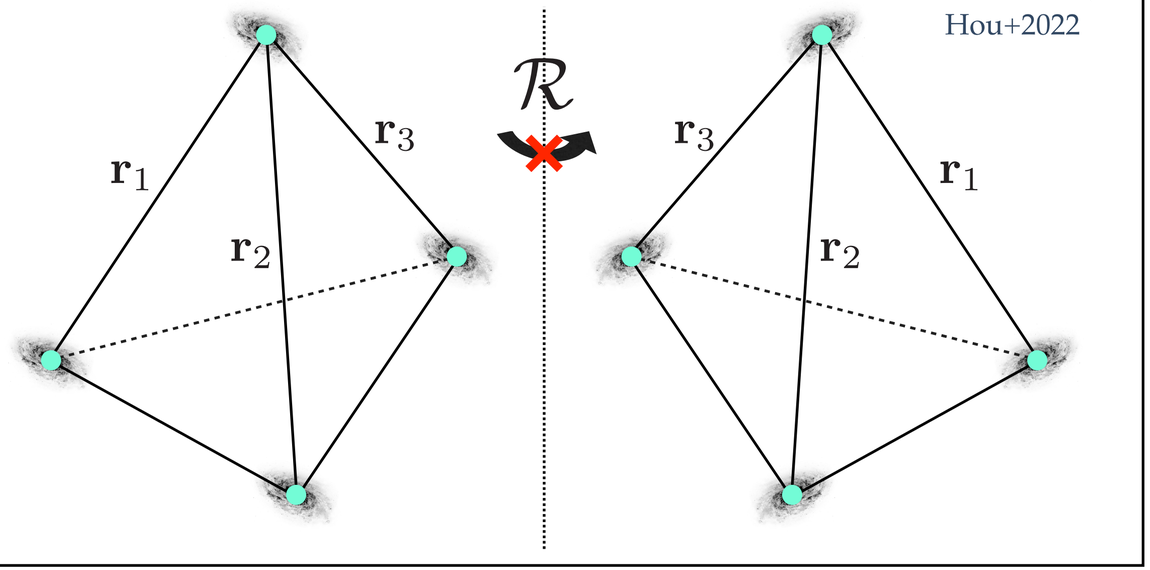
Galaxy shapes can probe PV at 2PCF/PS

What else can we use to look for parity violation in cosmology?

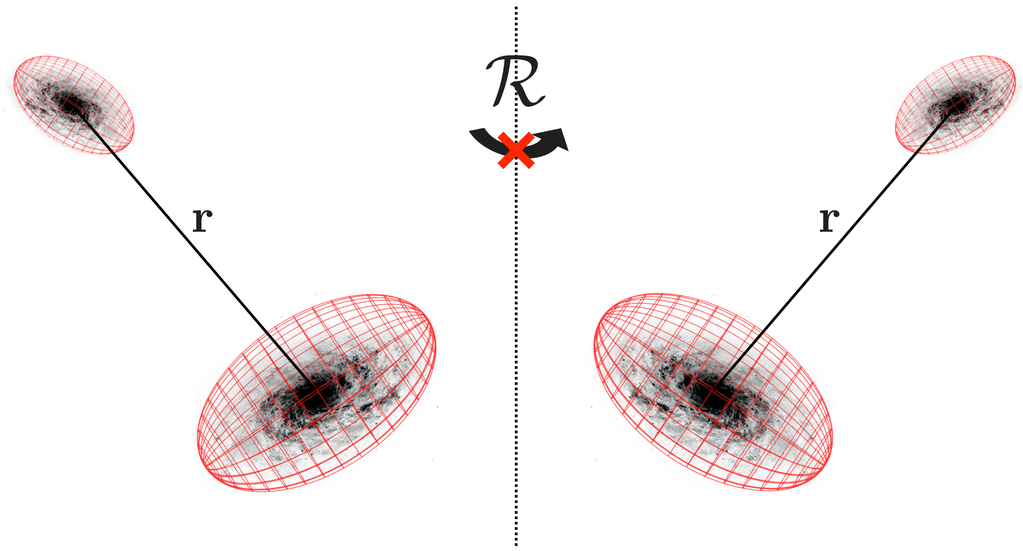
Galaxy shapes!

Scalar 4PCF / Trispecturm

Cahn+2023, Philcox 2022, Hou+2022



Tensor 2PCF / Power Spectrum



★ There are some configurations whose mirrored images can NOT be realized by rotations if we add information on galaxy shapes to the positions

→ 2PCF / PS of shapes is sensitive to PV!

Intrinsic Alignments of Galaxies/Halos

- Intrinsic Alignments (IAs)

  = Intrinsic correlations of galaxy/halo "shapes" with surrounding large-scale structure.

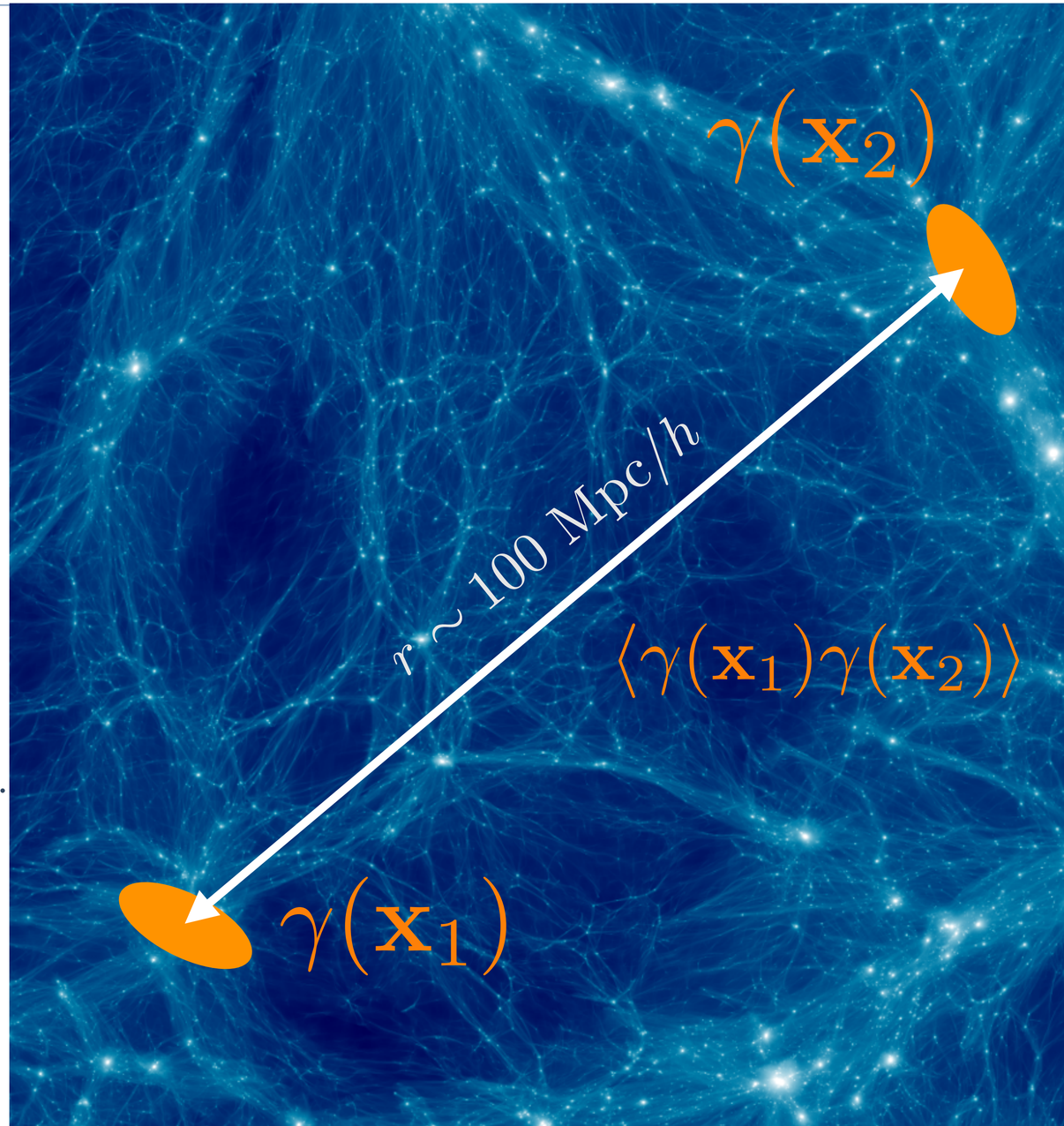
- IAs have been detected at ~ O(100) Mpc/h scales

⇒ Gravity/Primordial origin Mandelbaum+2006,…

  - Baryonic physics: ~10 Mpc/h at maximum.

- Increasing interest in IAs as a cosmological probe
  - PNG: Schmidt+2015, Akitsu+2021, TK&Takada2023,…
  - Growth rate: Taruya&Okumura2020, Okumura&Taruya2023,…
  - BAO: Chisari&Dvorkin2013, Okumura+2019, Xu+2023,…
  - GWs: Schmidt+2014, Akitsu+2023, Philcox+2024,…

★ This work: Search for Scalar PV with IA



Galaxy Shapes as Rank-Two Tensor Fields

- We approximate a galaxy/halo shape as a triaxial ellipsoid:

$$I_{ij}(\mathbf{x}_{\rm g}) \sim \int {\rm d}^3x \rho(\mathbf{x})(\mathbf{x}-\mathbf{x}_{\rm g})_i(\mathbf{x}-\mathbf{x}_{\rm g})_j$$

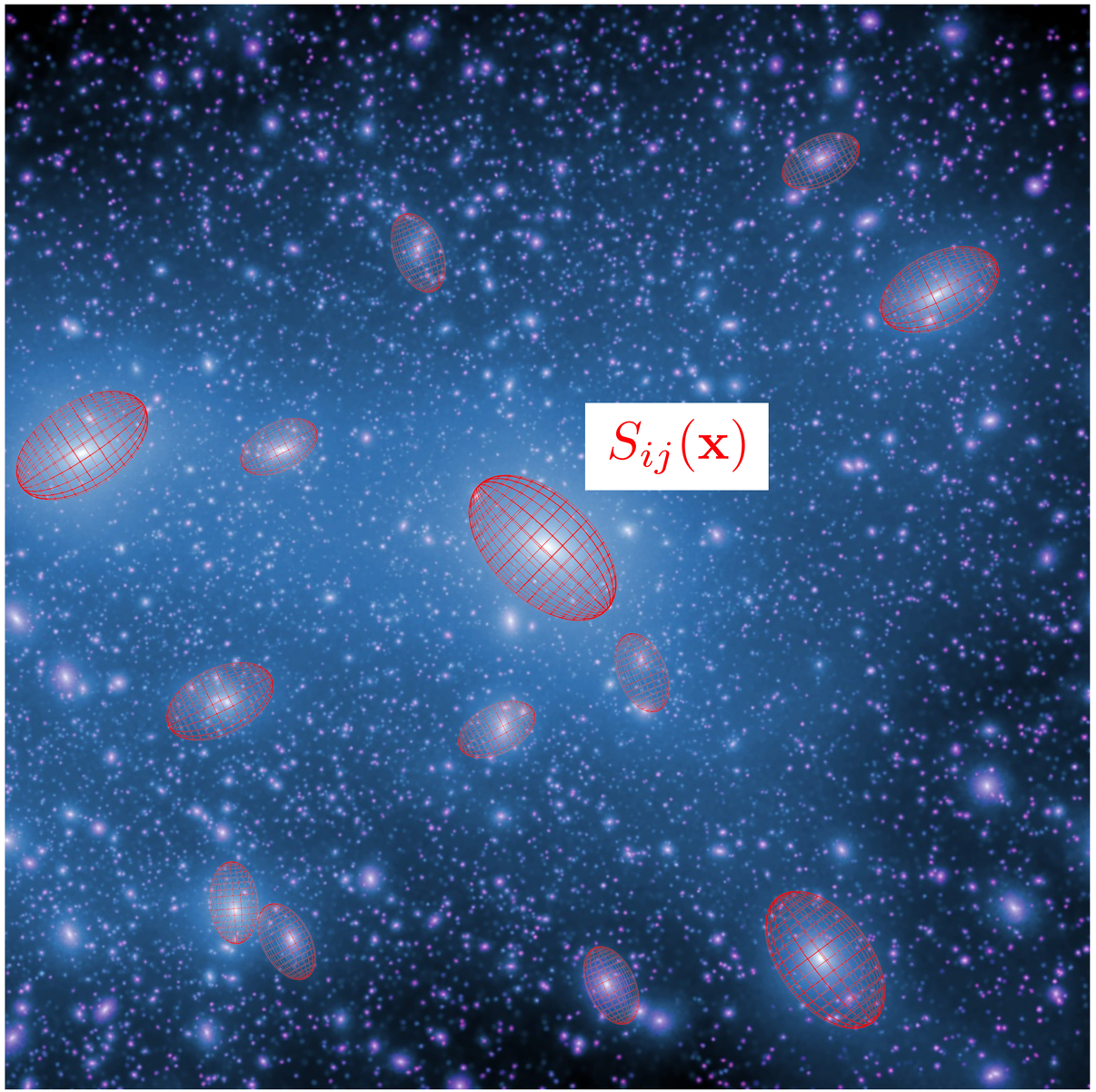
* 6 d.o.f. = 1 (trace) + 5 (trace-free)

Size — 2 distortions + 3 angles

- Shape tensor fields:

$$S_{ij}(\mathbf{x}) = {\rm TF}\left[I_{ij}(\mathbf{x})\right]$$

Rank-two tensor under 3D rotation



Modeling of IA: Linear Alignment

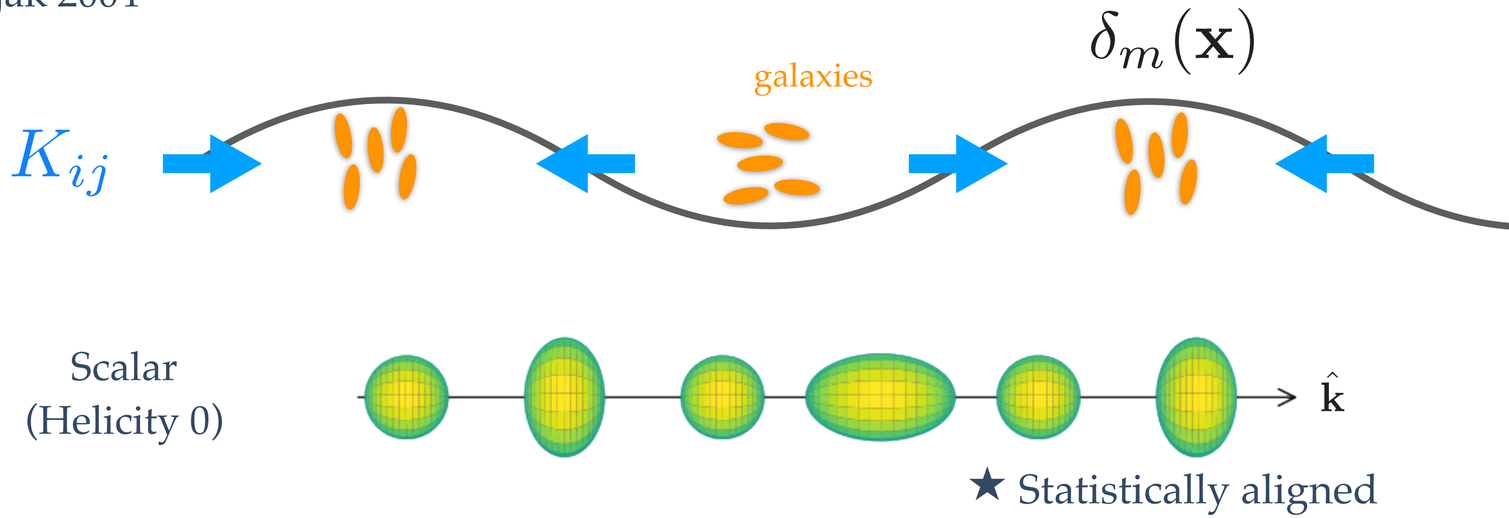
- Linear Alignment Model: Hirata & Seljak 2004

$$S_{ij}(\mathbf{x}) = c_1 K^{(1)}_{ij}(\mathbf{x})$$

shapes — (large-scale) tidal field

$$K_{ij}(\mathbf{x}) = \left(\frac{\partial_i \partial_j}{\partial^2} - \frac{\delta_{ij}}{3}\right) \delta_m(\mathbf{x})$$

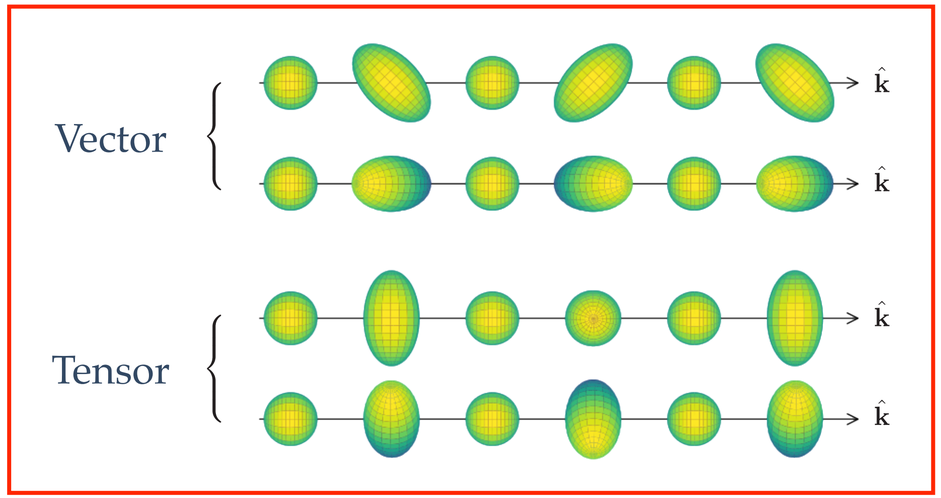
matter



- c.f. $\delta_{\rm g}(\mathbf{x}) = b_1 \delta_{\rm m}(\mathbf{x})$

  - LA
    = longitudinal scalar mode (helicity 0)
    = parity-even

  - Linear theory is insufficient!



- Perturbation Theory
- N-body simulation

New: Parity-Odd Components

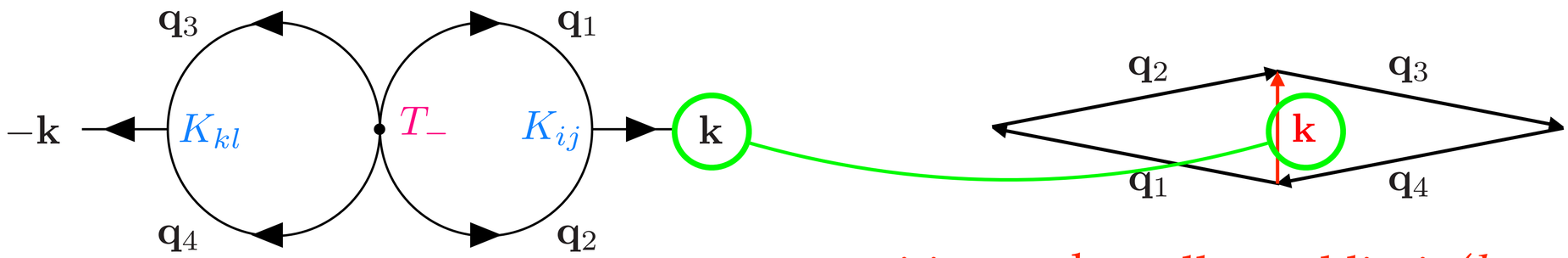
- Perturbative, bias expansion: Blazek+2019, Vlah+2020,…

$$S_{ij}(\mathbf{x}) = \underbrace{S_{ij}^{(1)}(\mathbf{x})}_{\text{linear alignment}} + \underbrace{S_{ij}^{(2)}(\mathbf{x}) + S_{ij}^{(3)}(\mathbf{x})}_{\text{up to } \mathcal{O}(\delta_m^4)} + \cdots$$

$$P_{ij,kl}(\mathbf{k}) = P_{ij,kl}^{(11)}(\mathbf{k}) + P_{ij,kl}^{(22)}(\mathbf{k}) + P_{ij,kl}^{(13)}(\mathbf{k}) + \mathcal{O}(\delta_m^4)$$

★ Projection on the helicity basis:

$$P_-^{(\lambda)}(k) = \int_{\mathbf{q}_1,\cdots,\mathbf{q}_4} (2\pi)^3\delta^{\rm D}_{\mathbf{k}-\mathbf{q}_{12}}(2\pi)^3\delta^{\rm D}_{\mathbf{k}+\mathbf{q}_{34}} \underbrace{\left[\left[\mathbb{P}_{ij,kl}^{(\lambda,-)}(\hat{\mathbf{k}})\right]^* K_{ij}^{(2)}(\mathbf{q}_1,\mathbf{q}_2)K_{kl}^{(2)}(\mathbf{q}_3,\mathbf{q}_4)\right]}_{\text{(bias-dependent) second-order kernels}} \underbrace{iT_-^{\delta}(\mathbf{q}_1,\mathbf{q}_2,\mathbf{q}_3,\mathbf{q}_4)}_{\text{parity-odd trispectrum}}$$



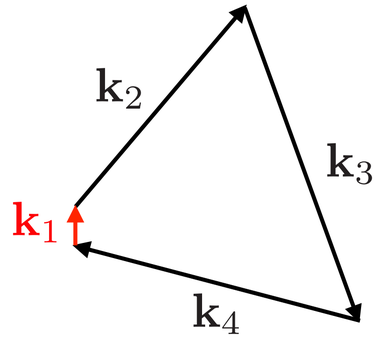
sensitive to the collapsed limit ($k \to 0$) of T_-!

Case study

- We consider the following two trispectrum templates as examples:

1. Squeezed-type model: Coulton+2024

$$T_-(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3,\mathbf{k}_4) = g_- i\left[\hat{\mathbf{k}}_1\cdot\left(\hat{\mathbf{k}}_2\times\hat{\mathbf{k}}_3\right)\right] k_1^{-1}k_2^0k_3^1 P_\phi(k_1)P_\phi(k_2)P_\phi(k_3) \mp 23 \text{ perms}$$

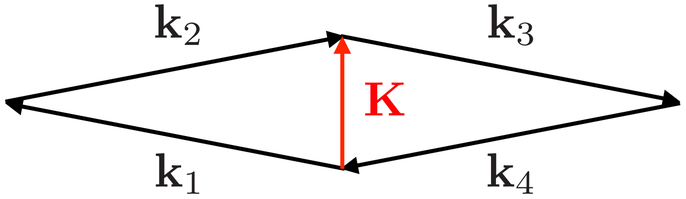


- Large amplitude in the squeezed limit ($k_i \to 0$)

2. Collapsed-type model: Shiraishi+2016

$$T_-(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3,\mathbf{k}_4) = 3\left(\frac{25}{9}\right)^3 Ai[\hat{\mathbf{K}}\cdot(\hat{\mathbf{k}}_1\times\hat{\mathbf{k}}_3)]\mathcal{F}(\hat{\mathbf{k}}_1,\hat{\mathbf{k}}_3,\hat{\mathbf{K}})P_\phi(k_1)P_\phi(k_3)P_\phi(K) \mp 23 \text{ perms}$$

$$\mathcal{F}(\hat{\mathbf{k}}_1,\hat{\mathbf{k}}_3,\hat{\mathbf{K}}) = \left[1-(\hat{\mathbf{q}}_1\cdot\hat{\mathbf{q}}_3)+\left(\hat{\mathbf{q}}_1\cdot\hat{\mathbf{K}}\right)-\left(\hat{\mathbf{q}}_3\cdot\hat{\mathbf{K}}\right)\right]$$

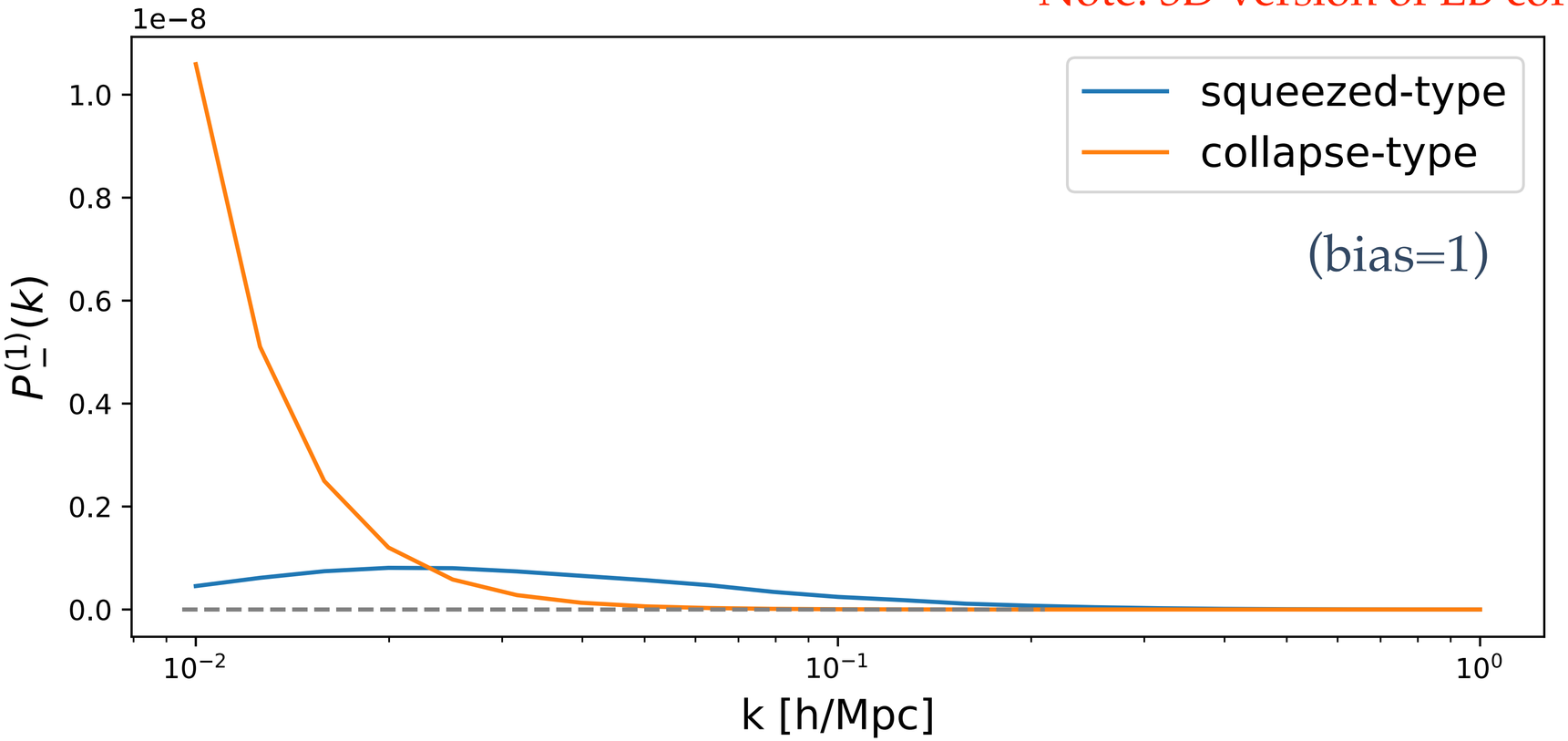


- "U(1)-gauge model": $\mathscr{L} \supset I(\phi)F\tilde{F}$
- Large amplitude in the collapsed limit ($K \to 0$)

Main Result (1)

- **<u>IA PS is more sensitive to the collapsed-type model</u>**
- Scale-dependent enhancement of the collapsed-type model in $k \to 0$ limit can be explained by "stochastic" contribution.

Note: 3D version of EB correlation/spectrum



- Perturbation Theory
- N-body simulation

N-body simulation: Setup

- Initial Conditions:
  - Modify MUSIC2-monofonIC (Hahn+2020) to add the non-Gaussian correction to the Gaussian part:

$$\Phi(\mathbf{x}) = \phi_{\rm G}(\mathbf{x}) + A\phi_{\rm NG}(\mathbf{x})$$

    - Modification so that the trispectrum T_Φ realizes the two model templates
    - Cubic correction for "squeezed"-type model (T^{1113})
    - Quadratic correction for "collapsed"-type model (T^{1122})

- N-body simulations:
  - Gadget-4 (Springel+2024), $L = 1{\rm Gpc}/h$, $N = (1024)^3$
  - Identify dark matter halos with FOF algorith, $M_{halo} \sim 10^{13} M_\odot/h$
  - Measure the shape tensor for each halo:

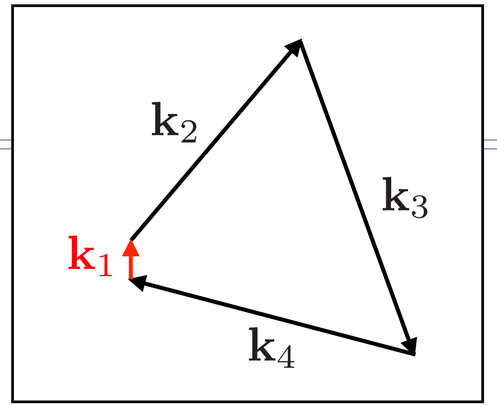
$$S_{ij}(\mathbf{x}) \sim \sum_{p \in halo} \Delta x_i^p \Delta x_j^p$$

  - Measure helicity power spectra:

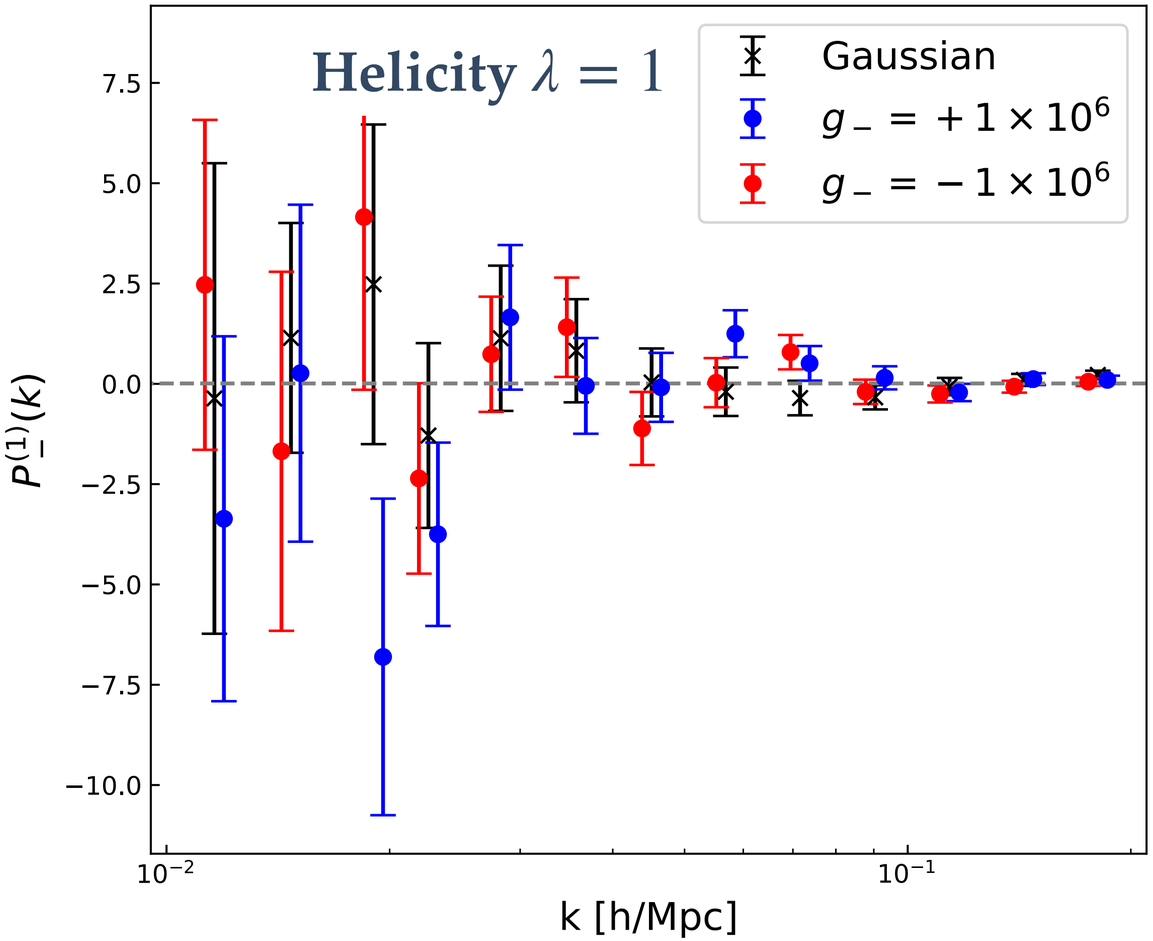
$$\hat{P}_-^{(\lambda)}(k) = \frac{1}{N_k} \sum_{\mathbf{k}} \left[\mathbb{P}_{ij,kl}^{(\lambda,-)}(\hat{\mathbf{k}}) \right]^* S_{ij}(\mathbf{k}) S_{kl}(-\mathbf{k})$$

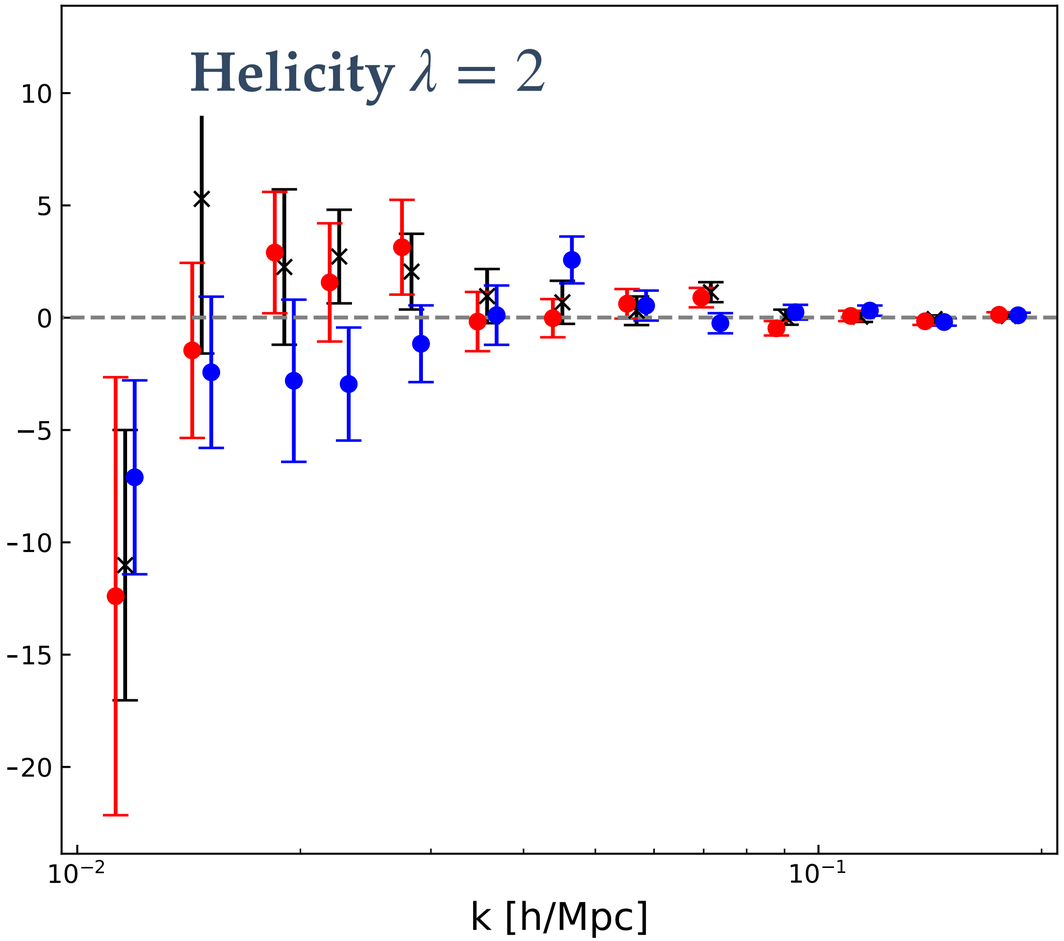


Main Result (2)

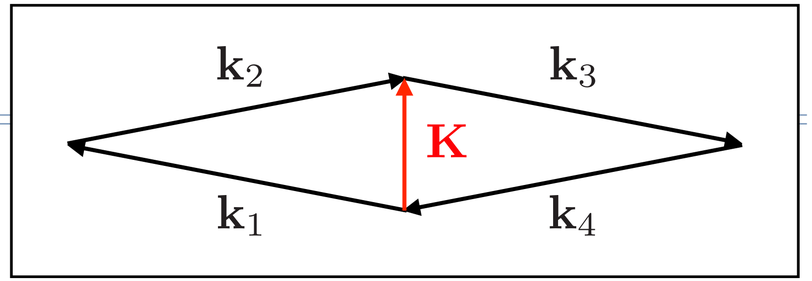


- Model 1: Squeezed-type
  - No detection
  - **IA PS is not sensitive to this model.**

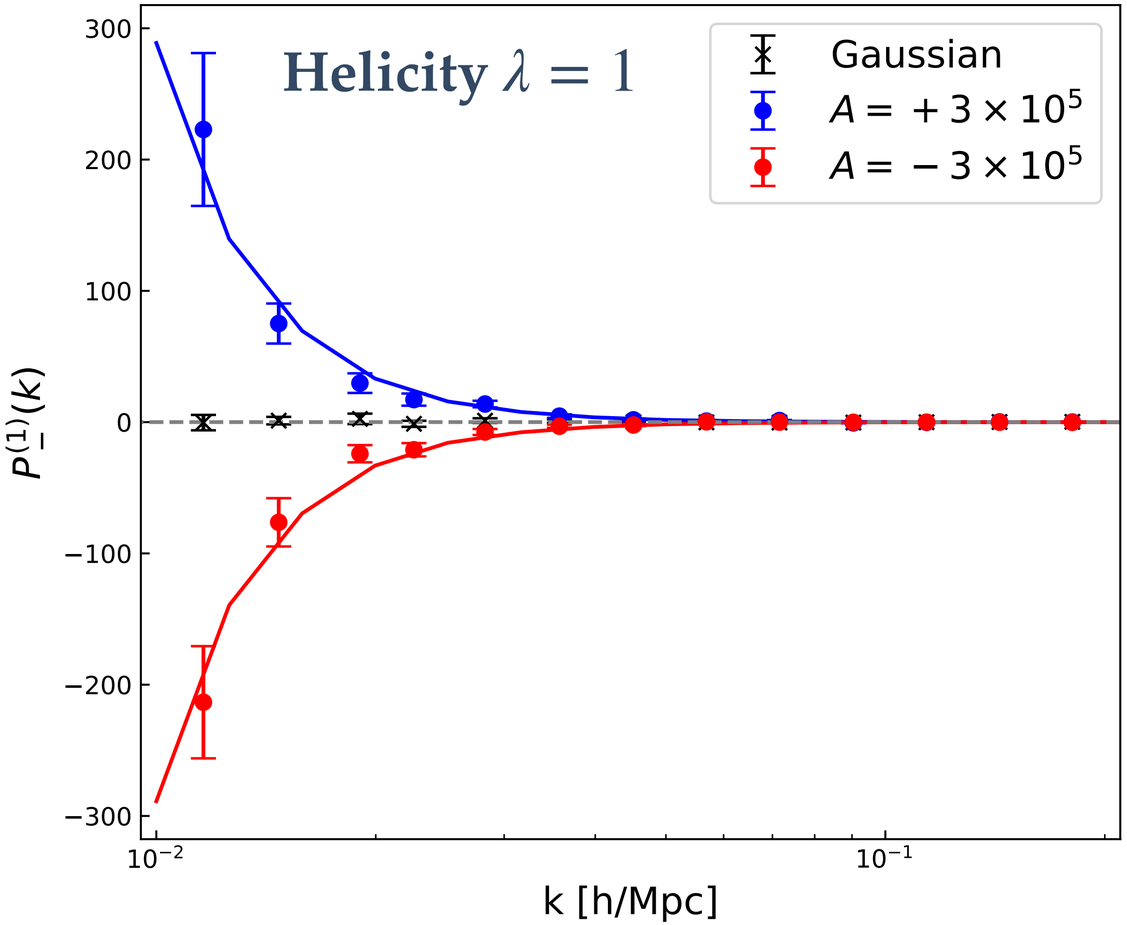


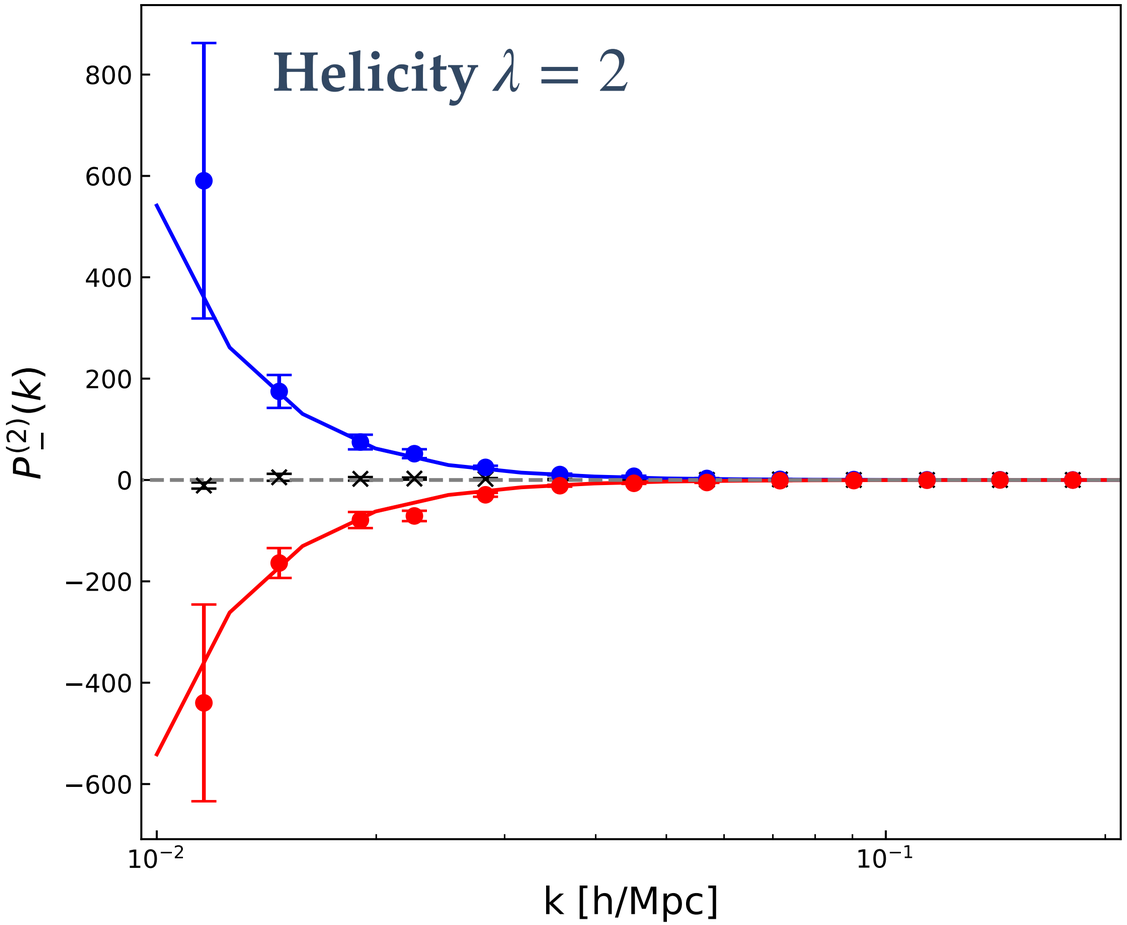


Main Result (3)



- Model 2: Collapsed-type **Strong Signal!!**
  - Scale-dependent enhancement matches the PT preditions
  - Current observational constraints: $\sigma(A) \sim 1 \times 10^4$ from CMB (Philcox&Shiraishi 2024), $\sigma(A) \sim 3 \times 10^4$ from LSS (Philcox 2022)





Conclusion

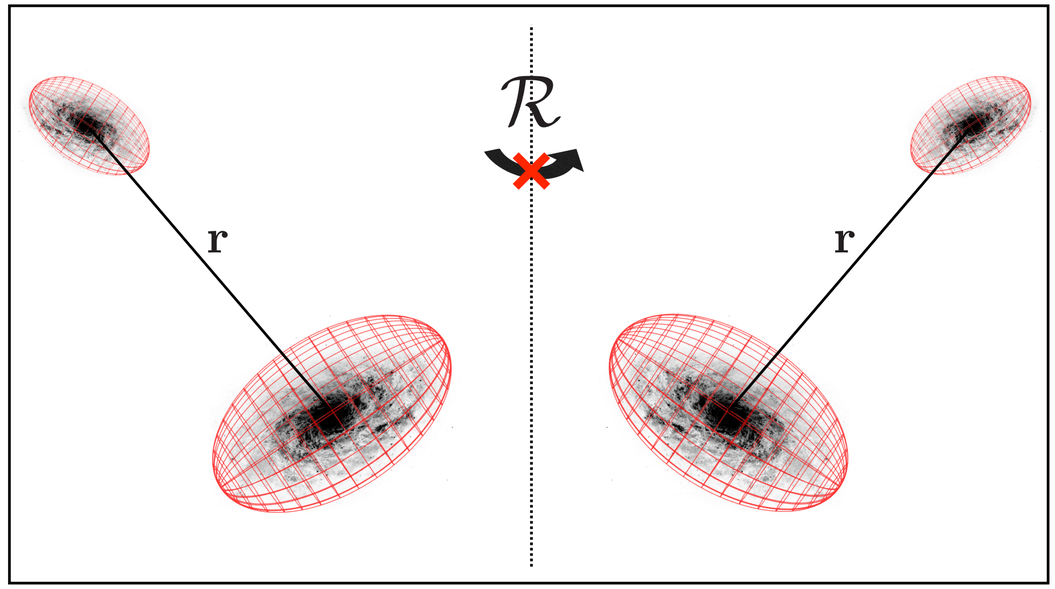
Intrinsic Alignment (IA) offers a new probe of parity violation in cosmology!

- IA power spectrum probes the *collapsed-type* scalar parity violation.

- Complementary to 4PCF analysis of galaxy density fields ~ *equilateral*

- Less systematics in covariance at large scales (dominated by Gaussian + shape noise)

- Scale-dependene can be predicted with PT (EFT) given a trispectrum in terms of stochastic bias



Appendix

Four Point Correlation Function of LSS

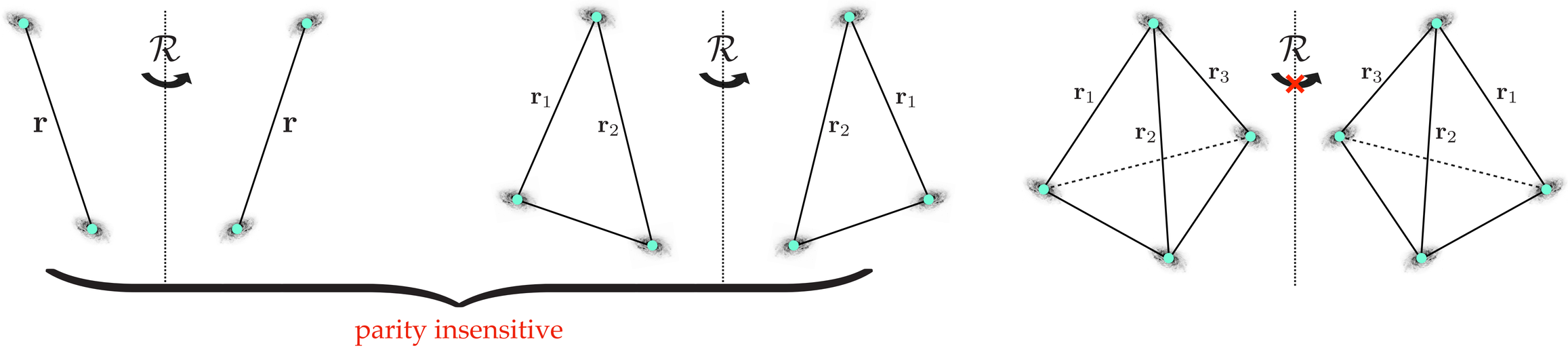
Search for parity violation with LSS: Cahn+2023

- We need at least 4PCF / Trispectrum of galaxy density field

2PCF / Power Spectrum

3PCF / Bispectrum

4PCF / Trispecturm



- Some recent 4PCF analyses reported evidence / detection of parity-violating signal.

Philcox 2022, Hou+2022

- We still do not have a conclusive understanding of whether the signal is physical origin or systematics.

Philcox&Ereza 2024 (follow-up analysis)

Helicity (SVT) Decomposition

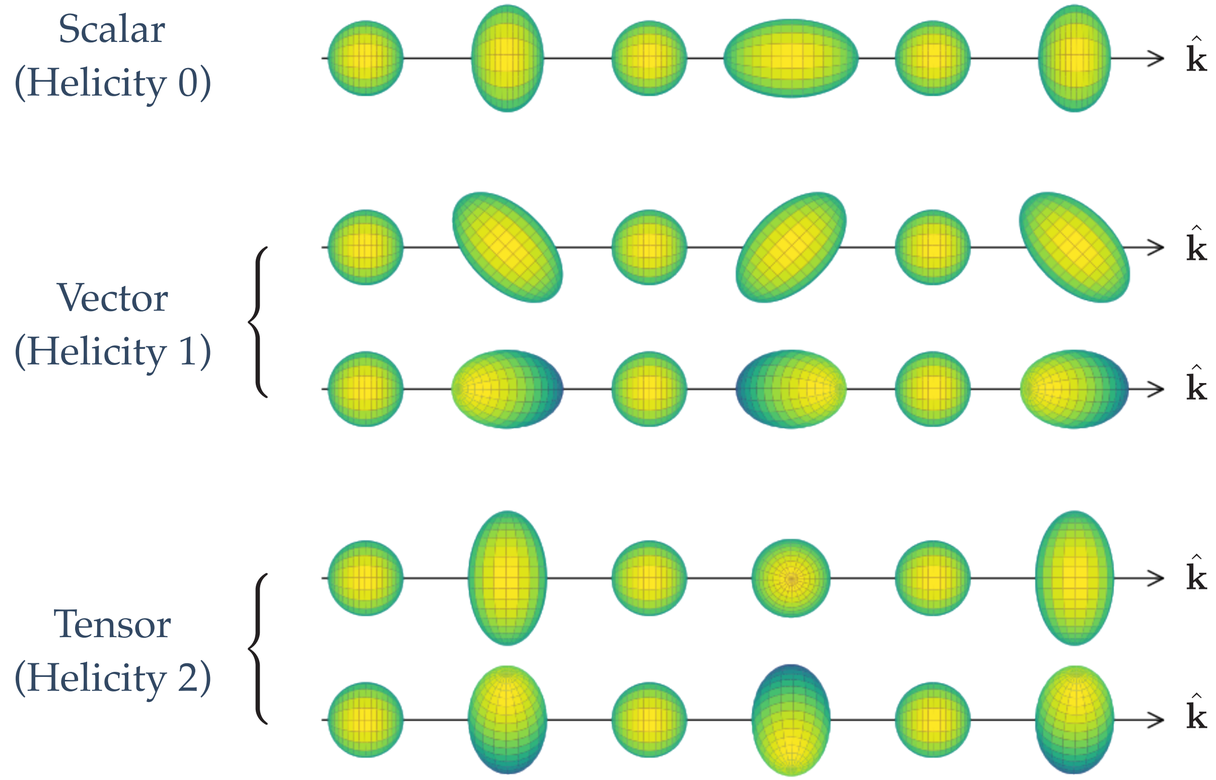
- Power spectrum of shapes:

$$\langle S_{ij}(\mathbf{k}) S_{kl}(\mathbf{k}') \rangle \equiv (2\pi)^3 \delta^{\rm D}_{\mathbf{k}+\mathbf{k}'} P_{ij,kl}(\mathbf{k})$$

- Helicity components of PS: Vlah+2020

$$P_{\pm}^{(\lambda)}(k) = \left[\mathbb{P}_{ij,kl}^{(\lambda,\pm)}(\hat{\mathbf{k}}) \right]^* P_{ij,kl}(\mathbf{k})$$

Projection onto the helicity basis



- $P_{ij,kl}(\mathbf{k})$ has 5 components $P_{\pm}^{(\lambda)}(k)$: 3 (parity-even) + 2 (parity-odd)

$\lambda = 0, 1, 2$ $\quad S, V, T$ $\qquad$ $\lambda = 1, 2$ $\quad V, T$

$$P_{\pm}^{(\lambda)} = \frac{1}{2}\left(P_{\rm RR}^{(\lambda)} - P_{\rm LL}^{(\lambda)} \right)$$

Modeling of IA: Beyond Linear Alignment

- Perturbative, bias expansion: Blazek+2019, Vlah+2020,…

$$S_{ij}(\mathbf{x}) = S_{ij}^{(1)}(\mathbf{x}) + S_{ij}^{(2)}(\mathbf{x}) + S_{ij}^{(3)}(\mathbf{x}) + \cdots$$

$$S_{ij}^{(1)} = c_1 \underbrace{\mathrm{TF}\left[\Pi_{ij}^{[1]}\right]^{(1)}}_{K_{ij}^{(1)}}, \qquad \Pi_{ij}^{[1]}(\mathbf{x}) = \frac{\partial_i \partial_j}{\partial^2}\delta_m(\mathbf{x})$$

$K_{ij}^{(1)}$ (linear tidal field = linear alignment)

$$S_{ij}^{(2)} = c_1 \underbrace{\mathrm{TF}\left[\Pi_{ij}^{[1]}\right]^{(2)}}_{\text{Non-linear evolution of matter}} + c_{2,1} \underbrace{\mathrm{TF}\left[\Pi_{ij}^{[2]}\right]^{(2)}}_{\text{Advection}} + c_{2,2} \underbrace{\mathrm{TF}\left[\Pi_{ik}^{[1]}\Pi_{kj}^{[1]}\right]^{(2)}}_{\text{Tidal torquing}} + c_{2,3} \underbrace{\mathrm{TF}\left[\Pi_{kk}^{[1]}\Pi_{ij}^{[1]}\right]^{(2)}}_{\text{Density weight}}$$

- Power spectrum up to $\mathcal{O}(\delta_m^4)$:

$$P_{ij,kl}(\mathbf{k}) = P_{ij,kl}^{(11)}(\mathbf{k}) + P_{ij,kl}^{(22)}(\mathbf{k}) + P_{ij,kl}^{(13)}(\mathbf{k}) + \mathcal{O}(\delta_m^4)$$

New: Parity-Odd Components

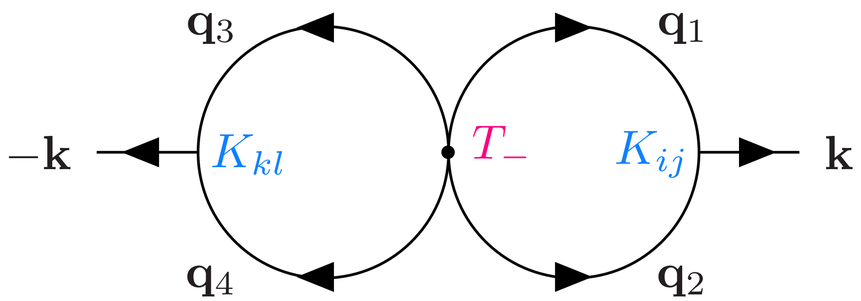
- Parity-odd components are extracted by the projection:

$$P_-^{(\lambda)}(k) = \left[\mathbb{P}_{ij,kl}^{(\lambda,-)}(\hat{\mathbf{k}})\right]^* P_{ij,kl}(\mathbf{k})$$

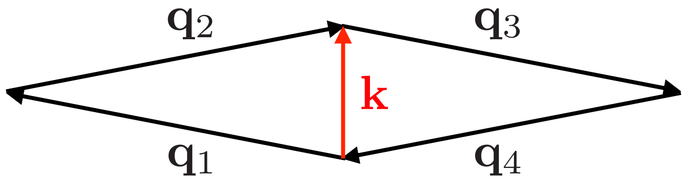
$$= \cancelto{0}{\left[\mathbb{P}_{ij,kl}^{(\lambda,-)}(\hat{\mathbf{k}})\right]^* P_{ij,kl}^{(11)}(\mathbf{k})} + \left[\mathbb{P}_{ij,kl}^{(\lambda,-)}(\hat{\mathbf{k}})\right]^* P_{ij,kl}^{(22)}(\mathbf{k}) + \cancelto{0}{\left[\mathbb{P}_{ij,kl}^{(\lambda,-)}(\hat{\mathbf{k}})\right]^* P_{ij,kl}^{(13)}(\mathbf{k})} + \mathcal{O}(\delta_m^4)$$

★ Only $P^{(22)}(k)$ survives because $P^{(11)}(k)$ and $P^{(13)}(k)$ are helicity 0 (parity-even) up to $\mathcal{O}(\delta_m^4)$.

$$P_-^{(\lambda)}(k) = \int_{\mathbf{q}_1,\cdots,\mathbf{q}_4} (2\pi)^3\delta^{\rm D}_{\mathbf{k}-\mathbf{q}_{12}}(2\pi)^3\delta^{\rm D}_{\mathbf{k}+\mathbf{q}_{34}} \underbrace{\left[\left[\mathbb{P}_{ij,kl}^{(\lambda,-)}(\hat{\mathbf{k}})\right]^* K_{ij}^{(2)}(\mathbf{q}_1,\mathbf{q}_2)K_{kl}^{(2)}(\mathbf{q}_3,\mathbf{q}_4)\right]}_{\text{(bias-dependent) second-order kernels}} \underbrace{iT_-^{\delta}(\mathbf{q}_1,\mathbf{q}_2,\mathbf{q}_3,\mathbf{q}_4)}_{\text{parity-odd trispectrum}}$$



$\sim$



sensitive to the collapsed limit ($k \to 0$) of T_-!

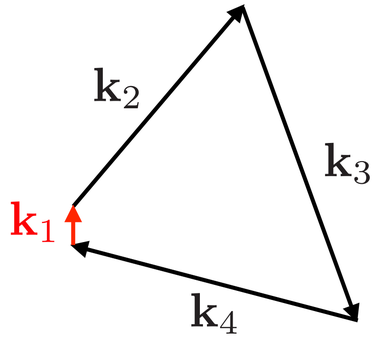
Case study

- We consider the following two trispectrum templates as examples:

1. Squeezed-type model: Coulton+2024

$$T_-(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3,\mathbf{k}_4) = g_- i\left[\hat{\mathbf{k}}_1\cdot\left(\hat{\mathbf{k}}_2\times\hat{\mathbf{k}}_3\right)\right] k_1^{-1}k_2^0k_3^1 P_\phi(k_1)P_\phi(k_2)P_\phi(k_3) \mp 23 \text{ perms}$$

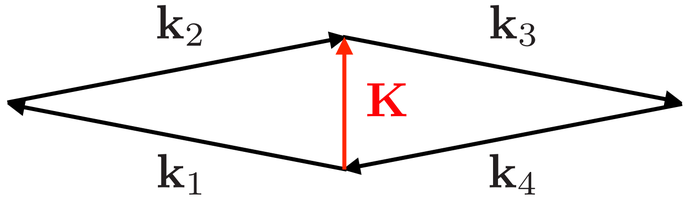
- c.f. $T_{g_{\rm NL}}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3,\mathbf{k}_4) = g_{\rm NL}P_\phi(k_1)P_\phi(k_2)P_\phi(k_3) + 23 \text{ perms}$
- Large amplitude in the squeezed limit ($k_i \to 0$)



2. Collapsed-type model: Shiraishi+2016

$$T_-(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3,\mathbf{k}_4) = 3Ai[\hat{\mathbf{K}}\cdot(\hat{\mathbf{q}}_1\times\hat{\mathbf{q}}_3)]\left[1-(\hat{\mathbf{q}}_1\cdot\hat{\mathbf{q}}_3)+\left(\hat{\mathbf{q}}_1\cdot\hat{\mathbf{K}}\right)-\left(\hat{\mathbf{q}}_3\cdot\hat{\mathbf{K}}\right)\right] P_\zeta(k_1)P_\zeta(k_3)P_\zeta(K) \mp 23 \text{ perms}$$

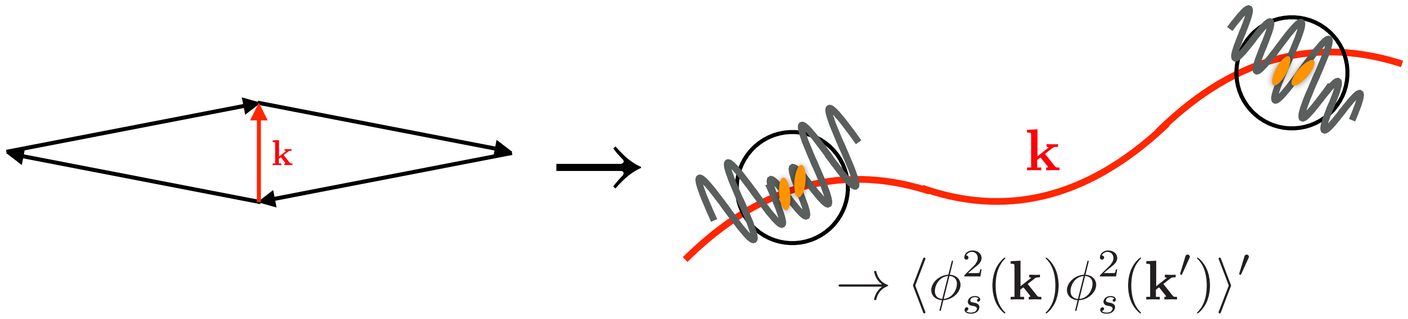
- "U(1)-gauge model": $\mathscr{L} \supset I(\phi)F\tilde{F}$
- c.f. $T_{\tau_{\rm NL}}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3,\mathbf{k}_4) = \frac{25}{9}\tau_{\rm NL}P_\phi(k_1)P_\phi(k_3)P_\phi(K) + 11 \text{ perms}$
- Large amplitude in the collapsed limit ($K \to 0$)



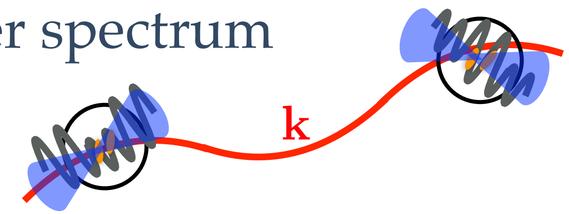
Physical Interpretation

- Collapsed-limit of T = large-scale correlation of small-scale variances = "stochasticity"
  - $\tau_{\rm NL}$ cosmology

    Smith&LoVerde2010
    Baumann+2013
    Assassi+2015,…



- IA: *quadrupole* of small-scale power spectrum



- Cutoff-dependent term has to be renormalized by adding a new operator: $P_-^{(22)}(k) \overset{\rm UV}{\to} c_\Lambda^2 P_\phi(k)$

$$S_{ij}(\mathbf{x}) \supset b_\Psi \Psi_{ij}(\mathbf{x}) \qquad \langle \delta_m \Psi \rangle = 0 \qquad P_\Psi(k) = P_\phi(k)$$

stochastic bias

