Charting the nanohertz gravitational wave sky with pulsar timing arrays

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Gyeongju, Korea

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+ **Reggie Bernardo** (AS → APCTP)
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Outline

- Stochastic gravitational-wave background (GWB)
- Astrophysical and cosmological sources
- Observation

  CMB, laser interferometry (LIGO, Virgo, KAGRA, LISA, Taiji, TianQin,..), pulsar timing array (NANOGrav, PPTA, EPTA, ITPA, SKA,..), astrometry (VLBI, Gaia,..)
- Our recent works in PTAs
- Conclusion

Cosmological GW spectral energy density

tion h_{ij} is gauged to be transverse-traceless. The latter can be decomposed into two polarization unit tensors as

$$h_{ij}(\eta,\vec{x}) = \sum_{\lambda=+,\times} \int \frac{d^3\vec{k}}{(2\pi)^{\frac{3}{2}}} h_\lambda(\eta,\vec{k}) \epsilon_{ij}^{\lambda}(\hat{k}) e^{i\vec{k}\cdot\vec{x}}, \qquad (2)$$

where $h_\lambda(\eta,\vec{k})$ is a Gaussian random field that defines the power spectrum of tensor perturbation,

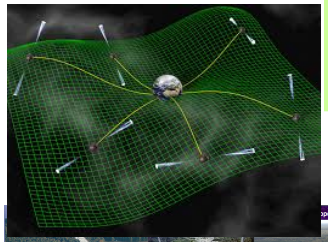
$$\langle h_\lambda(\eta,\vec{k}) h^*_{\lambda'}(\eta,\vec{k}')\rangle = \delta(\vec{k}-\vec{k}') \frac{2\pi^2}{k^3} \mathcal{P}_h^{\lambda\lambda'}(\eta,k). \qquad (3)$$

In the following, we will assume that $\mathcal{P}_h^{\lambda\lambda'}(\eta,k) = \delta_{\lambda\lambda'}\mathcal{P}_h(\eta,k)$. Then, the spectral energy density of the GWs relative to the critical density is given by

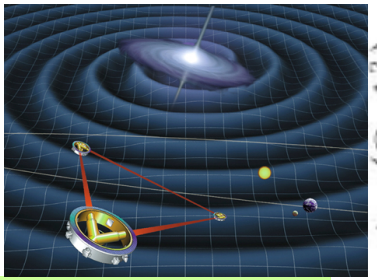
$$\Omega_{\rm GW}(\eta,k,\hat{k}) \equiv \frac{k}{\rho_c}\frac{d\rho_{\rm GW}}{dk d^2\hat{k}} = \frac{1}{96\pi}\left(\frac{k}{aH}\right)^2 \bar{\mathcal{P}}_h(\eta,k), \quad (4)$$

where $\rho_c = 3M_p^2H^2$ and the overbar denotes taking a time-average. For k-modes that re-enter the horizon

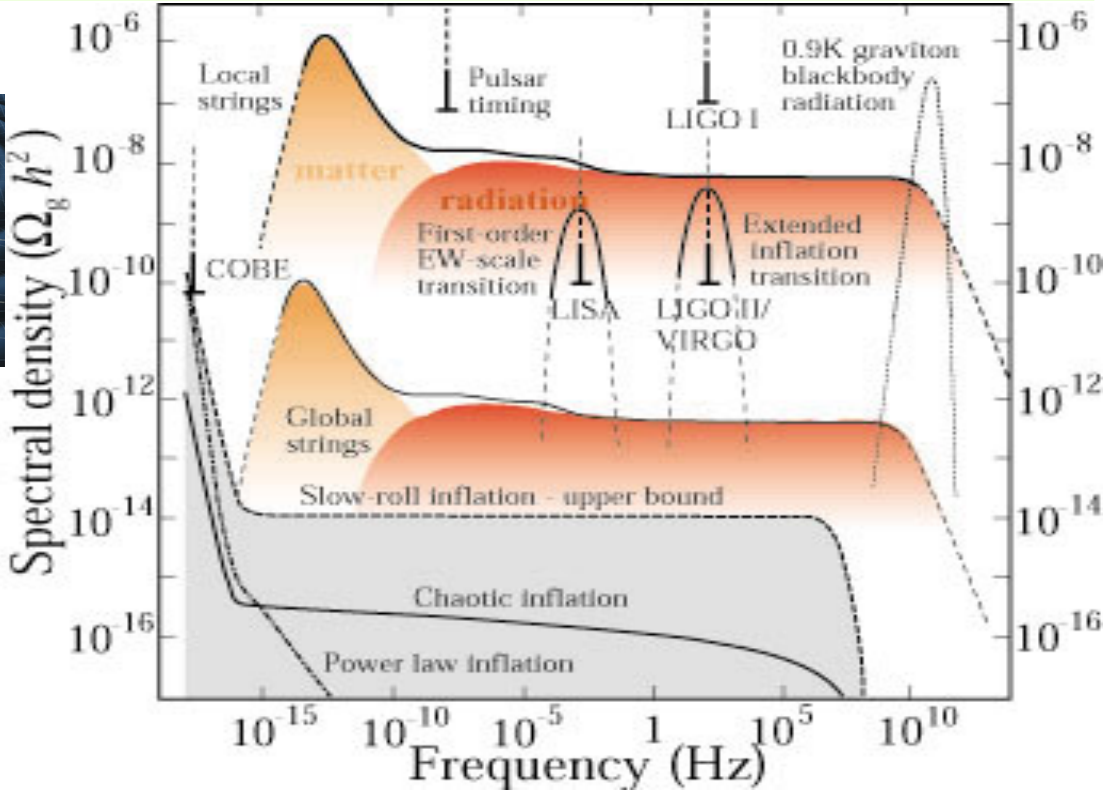
Cosmological sources for gravitational waves

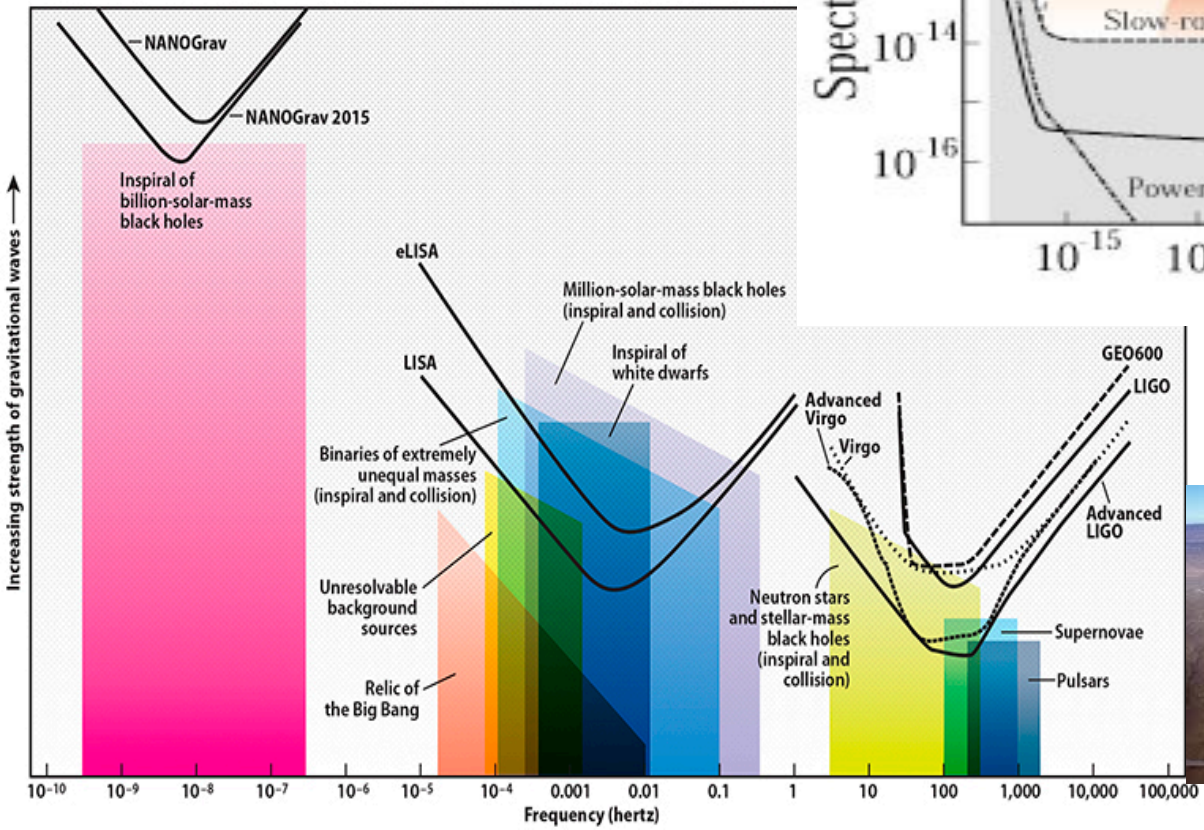






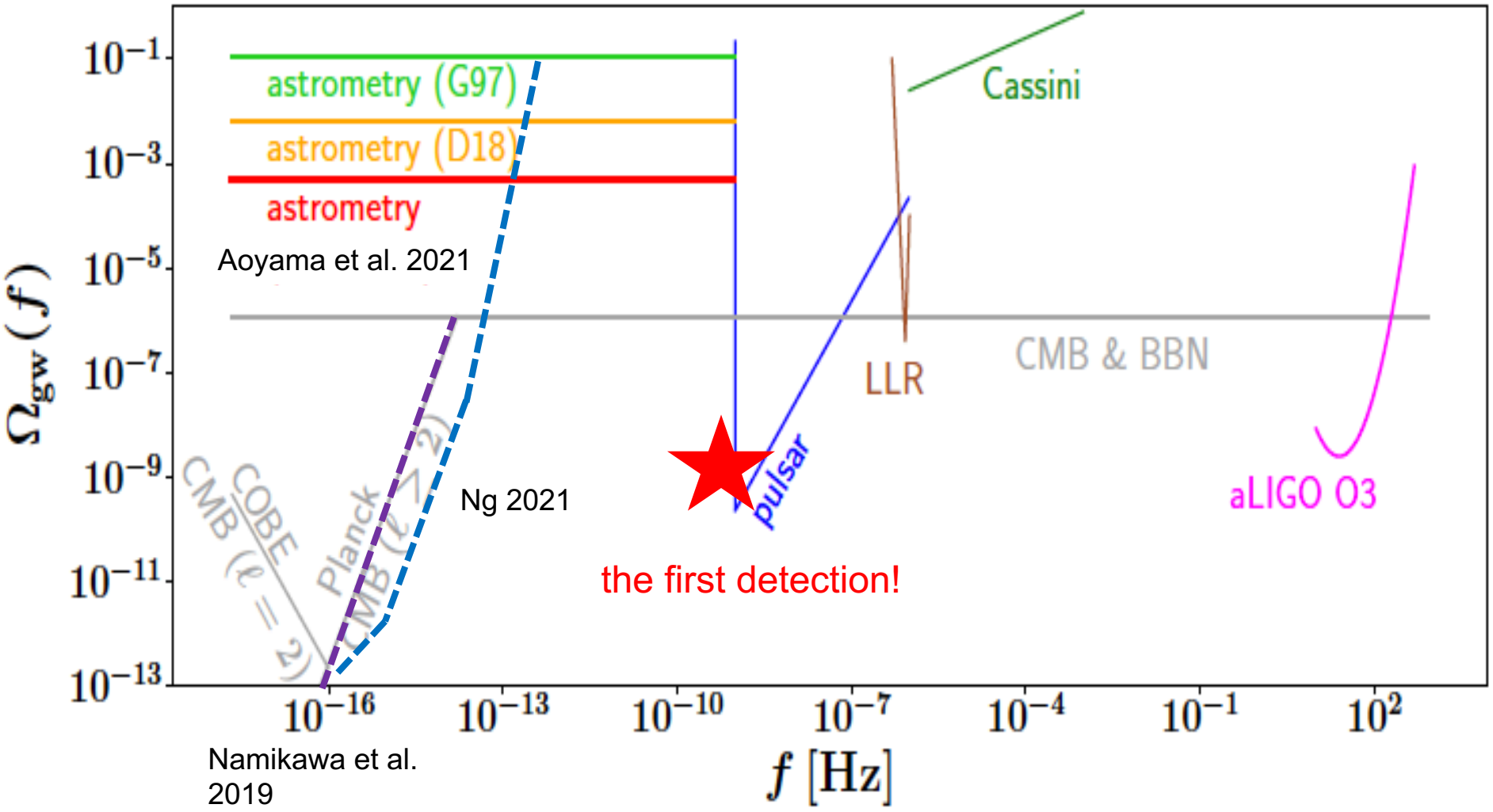
Astrophysical sources







Current Upper Bounds



CMB&BBN – extra degrees of freedom
Cassini+LLR – radar launching

CMB – anisotropy + B-mode polarization
aLIGO – direct detection of wave strain
Pulsar – Shapiro time delay
Astrometry – gravitational lensing effect

GW detection in LIGO-Virgo-KAGRA



The method adopted in current GW experiments for detecting SGWB is to correlate responses of different detectors to the GW strain amplitude. This allows us to filter out detector noises and obtain a large signal-to-noise ratio [4]. Let $h(\vec{x})$ be the response of a detector located at $\vec{x}$ with a pair of arms d_{ij}; then, we have

GW strain

$$h(\vec{x}) \equiv d^{ij} h_{ij}(\vec{x}) = \sum_{\lambda=+,\times} \int \frac{d^3\vec{k}}{(2\pi)^{\frac{3}{2}}} h_\lambda(\vec{k}) F^\lambda(\hat{k}) e^{i\vec{k}\cdot\vec{x}}, \tag{6}$$

where $F^\lambda = d^{ij}\epsilon^\lambda_{ij}(\hat{k})$ is the beam-pattern function. Hence, using Eq. (3) the response correlation between two detectors a and b, located at $\vec{x}_a = \vec{x} + \vec{r}/2$ and $\vec{x}_b = \vec{x} - \vec{r}/2$ respectively, is given by

$$\langle h_a(\vec{x}_a) h_b(\vec{x}_b)\rangle = \sum_{\lambda=+,\times} \int \frac{d^3\vec{k}}{(2\pi)^3} P_h(k) F^\lambda_a(\hat{k}) F^\lambda_b(\hat{k}) e^{i\vec{k}\cdot\vec{r}}, \tag{7}$$

GW power spectrum

$$P_h(k) \equiv (2\pi^2/k^3)\mathcal{P}_h(k)$$

Astrometric observation of SGWB

Astrometry is another indirect search for a SGWB using precise measurements of the positions, distances, and motions of celestial objects [17–25]. The subject has recently attracted a lot of attention due to high-precision astrometric data made by the *Gaia* space optical observatory [26]. A SGWB will induce fluctuations of the apparent angular positions of stars, which are independent of the star distances and correlated over the sky [17, 19, 20]. The rms fluctuation can be estimated as given by the GW characteristic strain amplitude,

$$\delta_{\rm rms} \sim h_c(f), \quad \text{with} \quad \Omega_{\rm GW} = \frac{2\pi^2}{3H_0^2} h_c^2 f^2 \,. \tag{1}$$

This leads to a rms angular velocity,

$$\omega_{\rm rms} \sim f\delta_{\rm rms} \sim H_0\sqrt{\Omega_{\rm GW}}\,, \tag{2}$$

which is constant for a flat spectral energy density.

Suppose we monitor the positions of N stars in an observation with an angular resolution of $\Delta\theta$ over a time period of T. Then, the sensitivity of detecting an angular motion of a star will be $\Delta\theta/(T\sqrt{N})$ for a frequency $f < 1/T$, which means that we can detect a SGWB with [20]

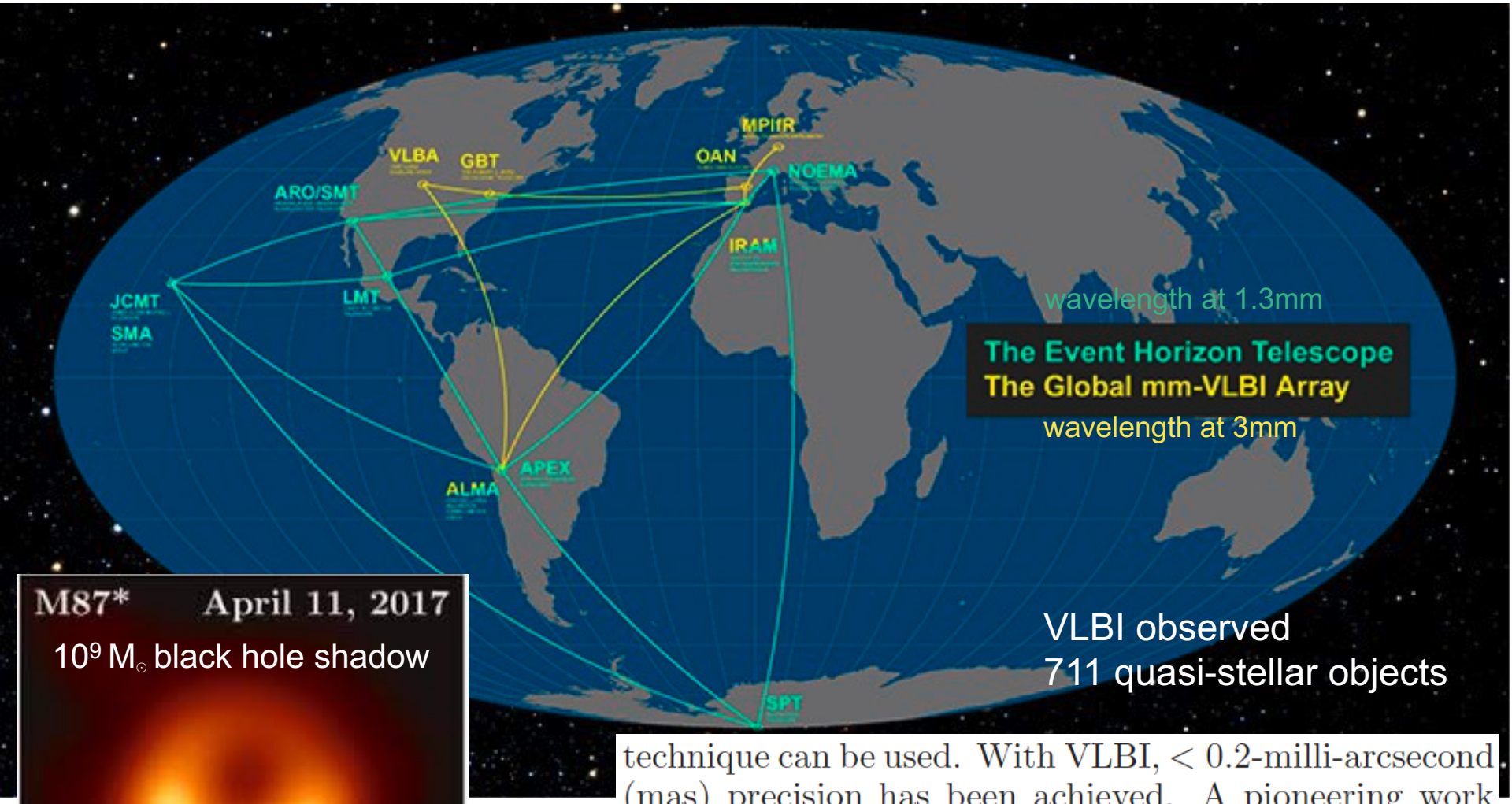
$$\Omega_{\rm GW}(f < 1/T) \sim \left(\frac{\Delta\theta}{T}\right)^2 \frac{1}{NH_0^2}\,. \tag{3}$$

$\Delta\theta \sim \lambda/D$

λ observing wavelength

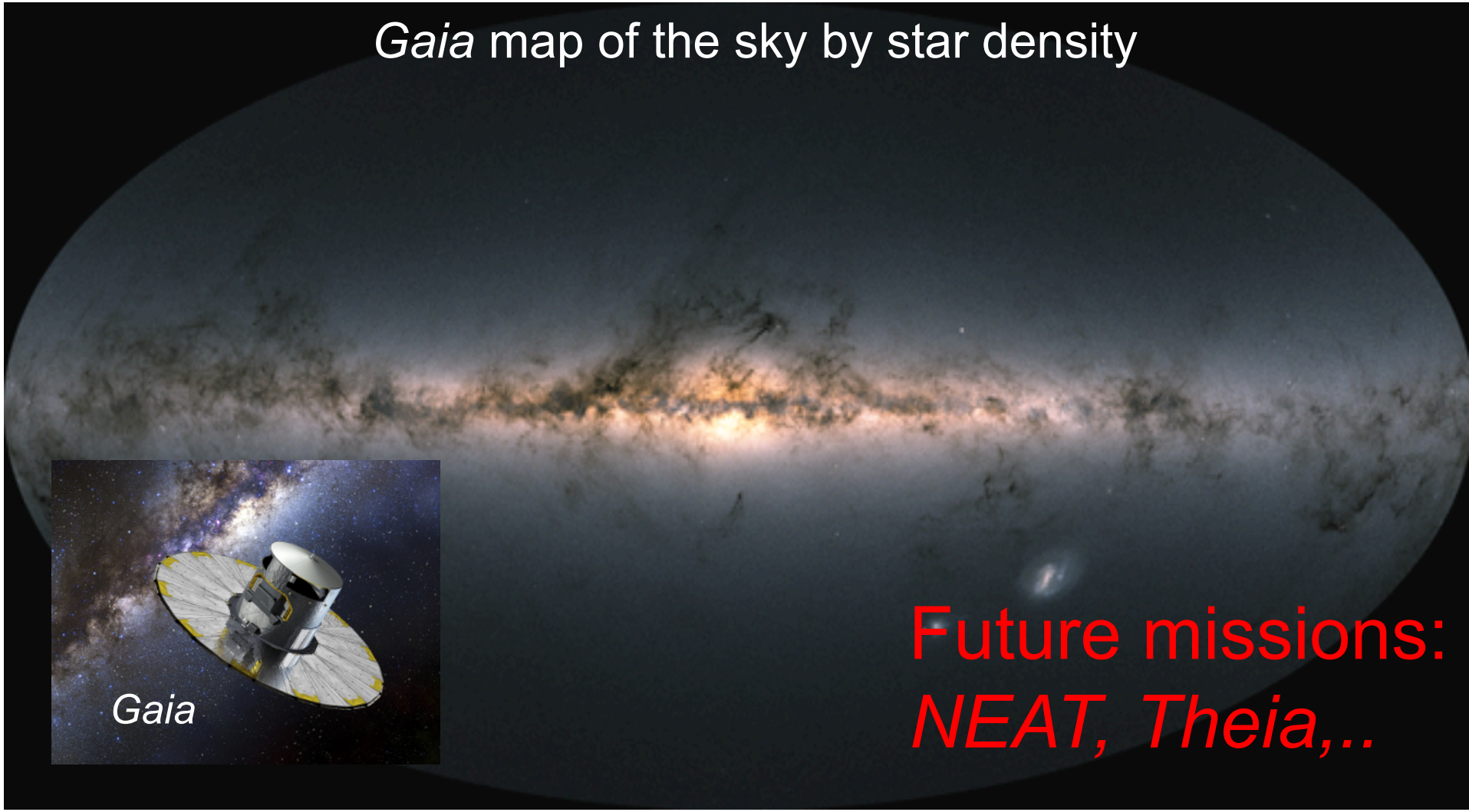
D telescope size

Very Long Baseline Interferometry (LVBI)

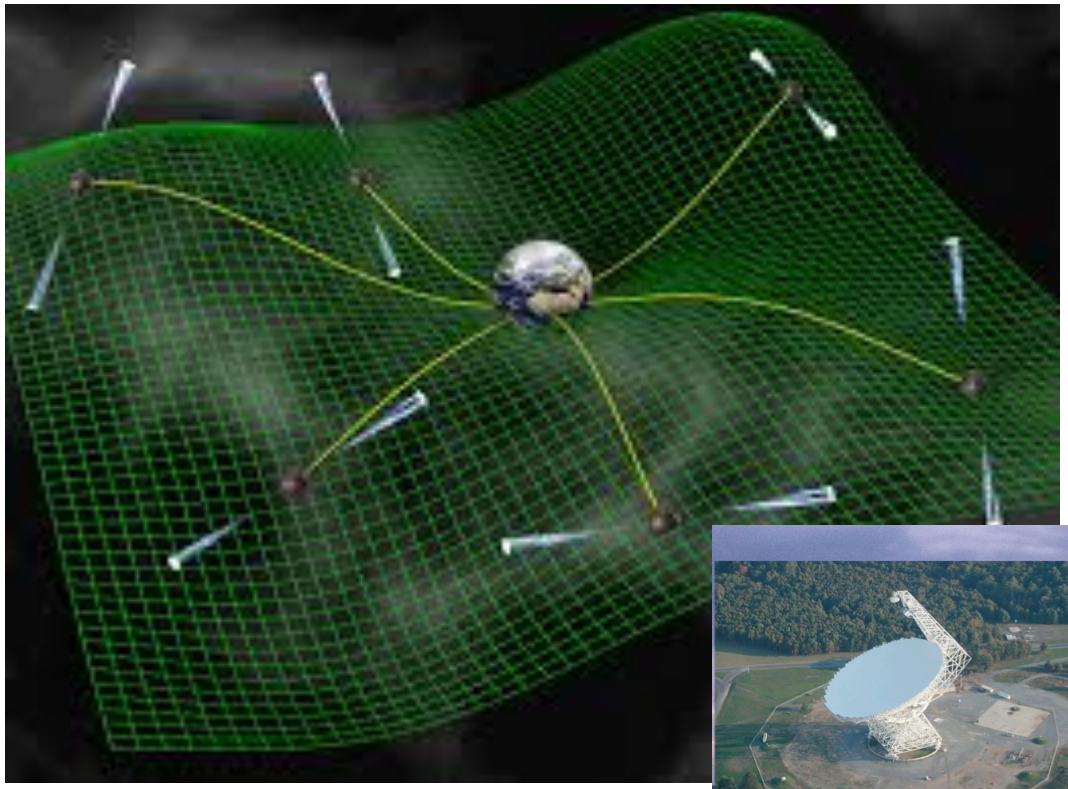


technique can be used. With VLBI, < 0.2-milli-arcsecond (mas) precision has been achieved. A pioneering work of this technique is Gwinn et al. (1997) [10], hereafter G97. Darling et al. (2018) [11], hereafter D18, measure proper motions of 711 objects with the VLBI and set a upper limit on low frequency GWs, $\Omega_{\rm gw} < 0.64 \times 10^{-2}$ for $f < 10^{-9}$ Hz. While the VLBI technique is successful,

Gaia space optical telescope (2013-2022) for astrometry: measuring the positions, distances and motions of a billion stars with micro arcsecond precision.



Current Pulsar Timing Arrays (PTAs)

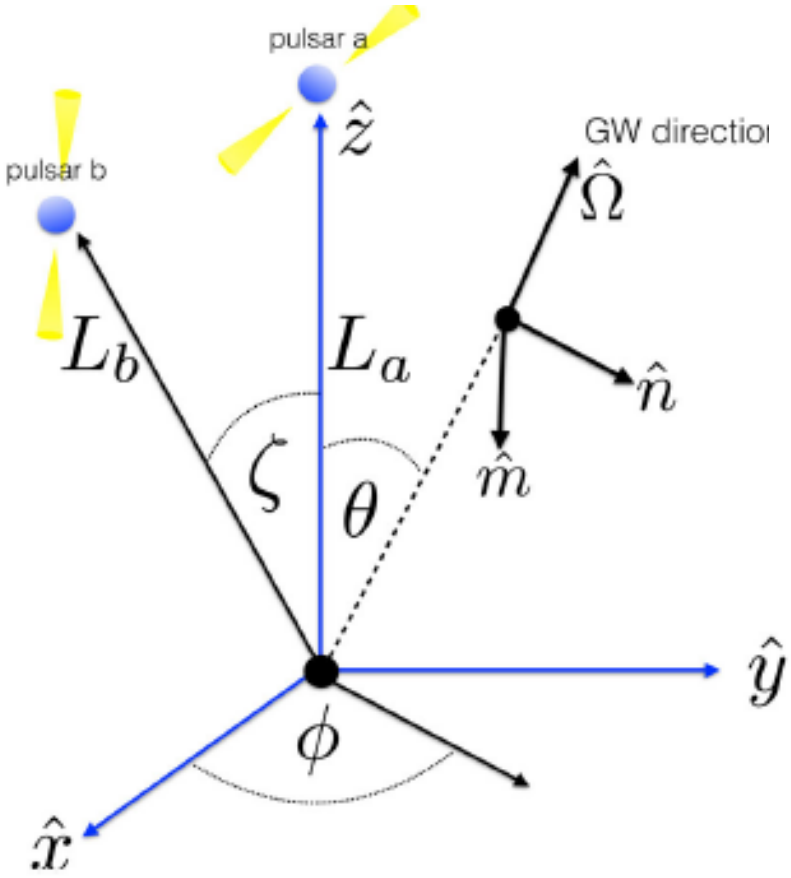


Nano-Hz GWs cause small correlated changes to the times of arrival of radio pulses from millisecond pulsars (MSPs)

International Pulsar Timing Arrays



Pulsar Timing – MSPs are precise clocks



$$z(t,\hat{\Omega}) \equiv \frac{\nu(t)-\nu_0}{\nu_0} = \frac{1}{2}\frac{\hat{p}^i\hat{p}^j}{1+\hat{\Omega}\cdot\hat{p}}\Delta h_{ij}(t,\hat{\Omega}),$$

Shapiro time delay

$$\Delta h_{ij}(t,\hat{\Omega}) \equiv h_{ij}(t,\hat{\Omega}) - h_{ij}(t_p,\hat{\Omega})$$

$$z(t) = \int d\hat{\Omega} z(t,\hat{\Omega})$$

pulsar residual $\quad r(t) = \int^t dt' z(t')$

detector tensor

$d^{ij} = p^i p^j$

$$\langle r_a^*(t_j) r_b(t_k)\rangle = \left\langle \int^{t_j} dt' \int^{t_k} dt'' z_a^*(t') z_b(t'')\right\rangle$$

$$\langle r_a^*(t_j) r_b(t_k) \rangle = \int^{t_j} dt' \int^{t_k} dt'' \int_{-\infty}^{+\infty} df e^{-i2\pi f(t'-t'')} H(f)^{(ab)}\Gamma(f)$$

$$\langle h_A^*(f,\hat{\Omega}) h_{A'}(f',\hat{\Omega}') \rangle = \delta^2(\hat{\Omega},\hat{\Omega}')\delta_{AA'}\delta(f-f')H(f)P(\hat{\Omega})$$

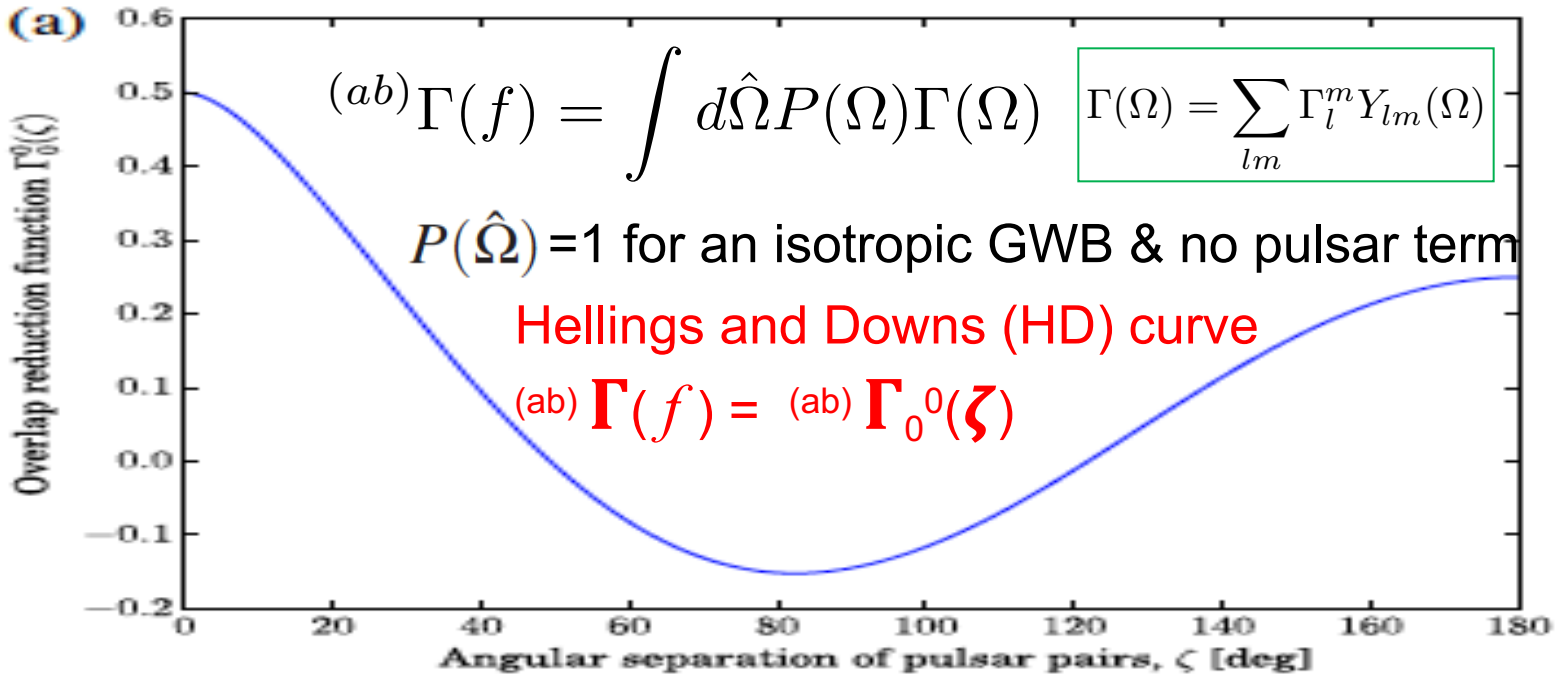
Overlap Reduction Function

$$^{(ab)}\Gamma(f) \equiv \int d\hat{\Omega} P(\hat{\Omega}) \kappa_{ab}(f,\hat{\Omega}) \left[\sum_A F_a^A(\hat{\Omega}) F_b^A(\hat{\Omega}) \right],$$

$$\kappa_{ab}(f,\hat{\Omega}) \equiv [1 - e^{i2\pi f L_a(1+\hat{\Omega}\cdot\hat{p}_a)}][1 - e^{-i2\pi f L_b(1+\hat{\Omega}\cdot\hat{p}_b)}]. \qquad F^A(\hat{\Omega}) = \left[\frac{1}{2} \frac{\hat{p}^i \hat{p}^j}{1+\hat{\Omega}\cdot\hat{p}} e_{ij}^A(\hat{\Omega}) \right]$$

Earth term

pulsar term $fL >> 1$



$$^{(ab)}\Gamma(f) = \int d\hat{\Omega} P(\Omega)\Gamma(\Omega) \qquad \Gamma(\Omega) = \sum_{lm} \Gamma_l^m Y_{lm}(\Omega)$$

$P(\hat{\Omega})$ =1 for an isotropic GWB & no pulsar term

Hellings and Downs (HD) curve

$^{(ab)}\boldsymbol{\Gamma}(f) = {}^{(ab)}\boldsymbol{\Gamma}_0^0(\boldsymbol{\zeta})$

Signal

supermassive black hole binaries

$$h_c(f) = A_{\rm GWB}\left(\frac{f}{f_{\rm yr}}\right)^{\alpha} \qquad \alpha = -2/3 \text{ for a population of inspiraling SMBHBs}$$

Pulsar residual

$$\langle r_a^*(t_j) r_b(t_k)\rangle = S_{ab}(f) = \Gamma_{ab}\frac{{A_{\rm GWB}}^2}{12\pi^2}\left(\frac{f}{f_{\rm yr}}\right)^{-\gamma} f_{\rm yr}^{-3} \qquad \gamma = 3 - 2\alpha$$

- Monopolar ORF Γ_{ab}=1 (*due to clock error*)
- Dipolar ORF Γ_{ab}= cos ζ (*due to error in solar system ephemeris*)
- Quadupolar ORF Γ_{ab} = HD curve (*genuine GWB signal*)

Noise

white noises (instrumental) + pulsar intrinsic red noise
(including pulsar spin noise, pulsar profile changes, dispersion measure variations,…)

$$N_{aa}(f) = A_{\rm red}^2 \, (f / f_{\rm yr})^{-\gamma} \, f_{\rm yr}^{-3}$$

North American Nanohertz Observatory for Gravitational Waves

A National Science Foundation Physics Frontiers Center

The NANOGrav 12.5-year Data Set: Search For An Isotropic Stochastic Gravitational-Wave Background

arXiv:2009.04496 9 Sep 2020

We search for an isotropic stochastic gravitational-wave background (GWB) in the 12.5-year pulsar timing data set collected by the North American Nanohertz Observatory for Gravitational Waves (NANOGrav). Our analysis finds strong evidence of a stochastic process, modeled as a power-law, with common amplitude and spectral slope across pulsars. The Bayesian posterior of the amplitude for a $f^{-2/3}$ power-law spectrum, expressed as characteristic GW strain, has median 1.92×10^{-15} and 5%–95% quantiles of 1.37–2.67×10^{-15} at a reference frequency of $f_{\rm yr} = 1\ {\rm yr}^{-1}$. The Bayes factor in favor of the common-spectrum process versus independent red-noise processes in each pulsar exceeds 10,000. However, we find no statistically significant evidence that this process has quadrupolar spatial correlations, which we would consider necessary to claim a GWB detection consistent with General Relativity. We find that the process has neither monopolar nor dipolar correlations, which may arise from, for example, reference clock or solar-system ephemeris systematics, respectively. The amplitude posterior has significant support above previously reported upper limits; we explain this in terms of the Bayesian priors assumed for intrinsic pulsar red noise. We examine potential implications for the supermassive black hole binary population under the hypothesis that the signal is indeed astrophysical in nature.

$h_c = A = 1.92 \times 10^{-15}$

$$\Omega_{\rm GW} h^2 = \frac{2\pi^2}{3} h_c^2 \left(\frac{f_{\rm yr}}{100\,{\rm km\,s^{-1}\,Mpc^{-1}}} \right)^2 \simeq 2.3 \times 10^{-9}$$

Common-spectrum Process supported by
PPTA arXiv:2107.12112
EPTA arXiv:2110.13184
IPTA arXiv:2201.03980

The NANOGrav 12.5 year data set: 45 MSPs

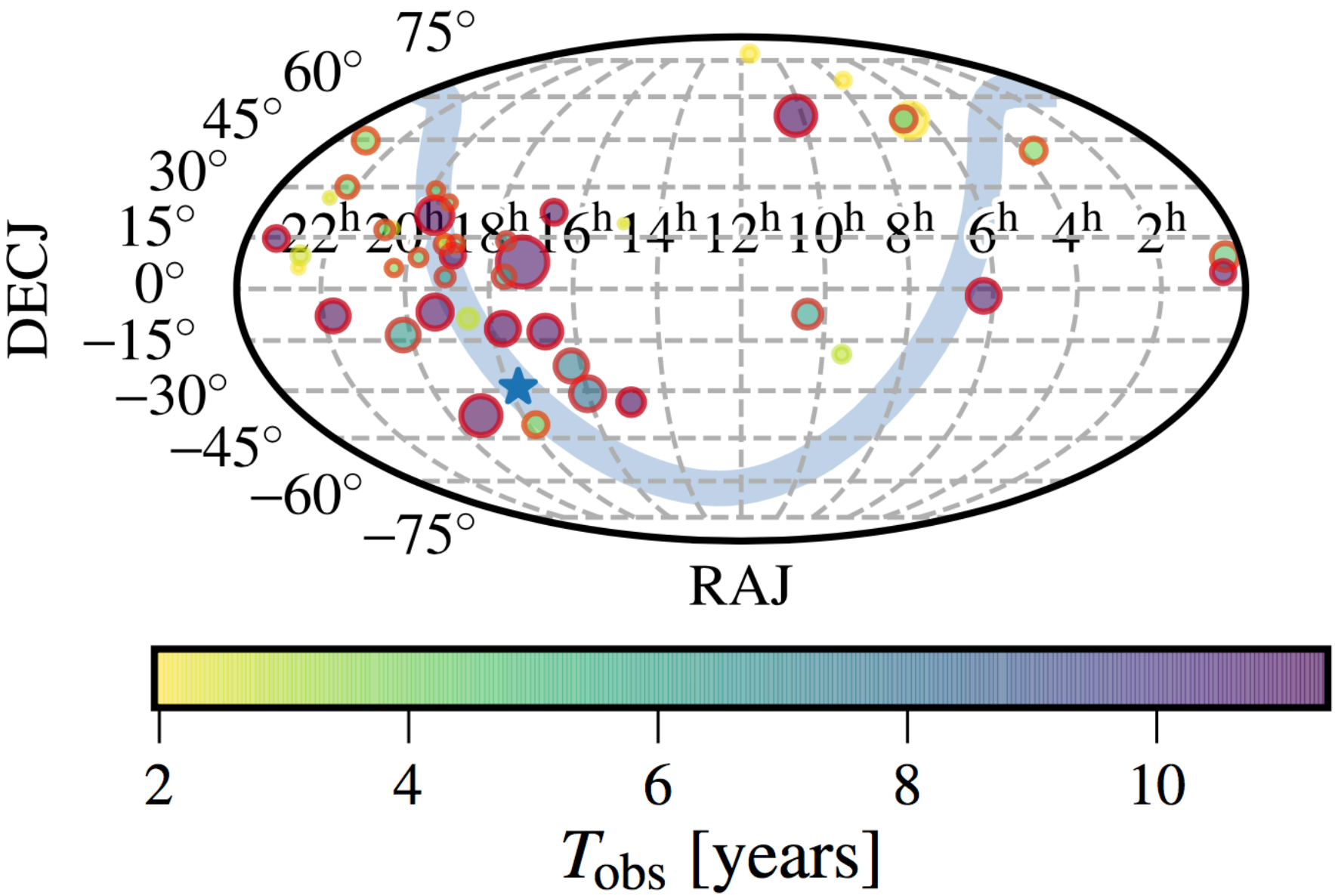
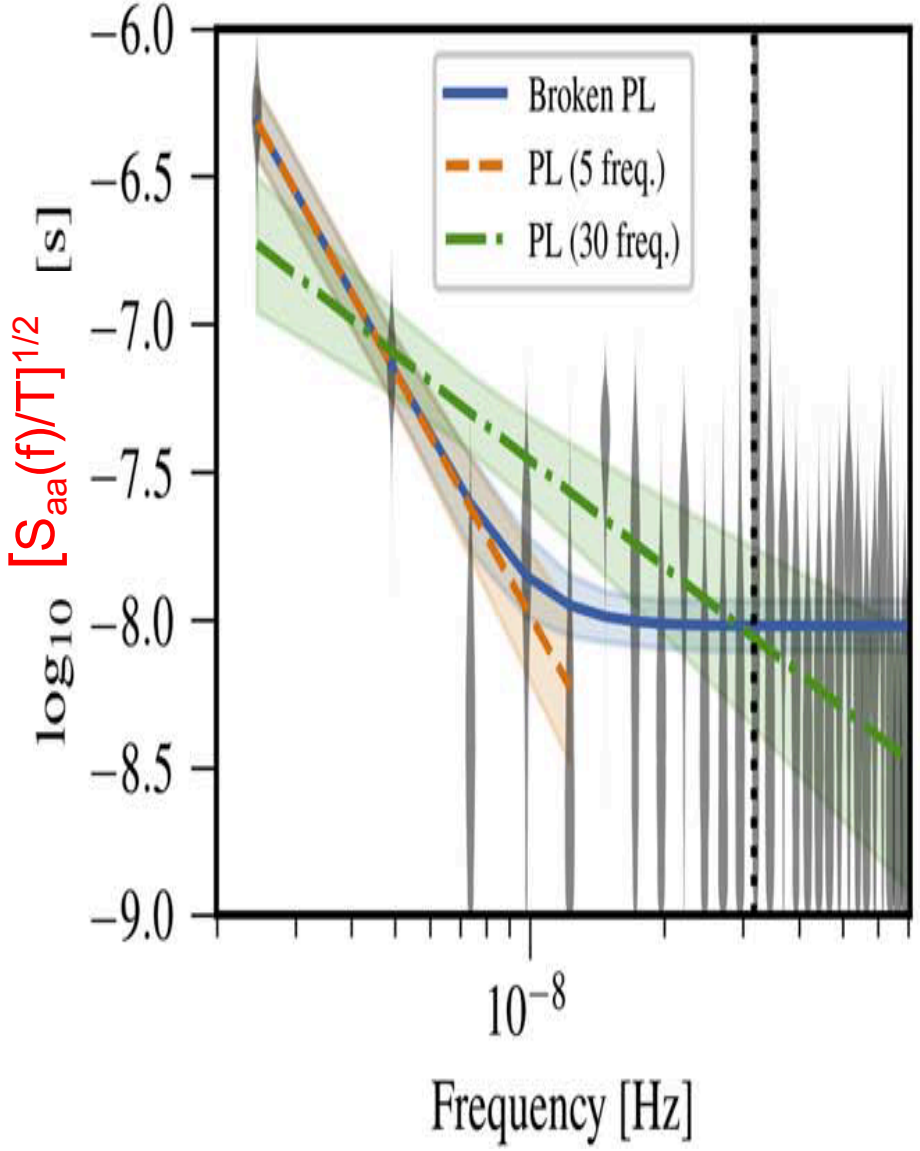
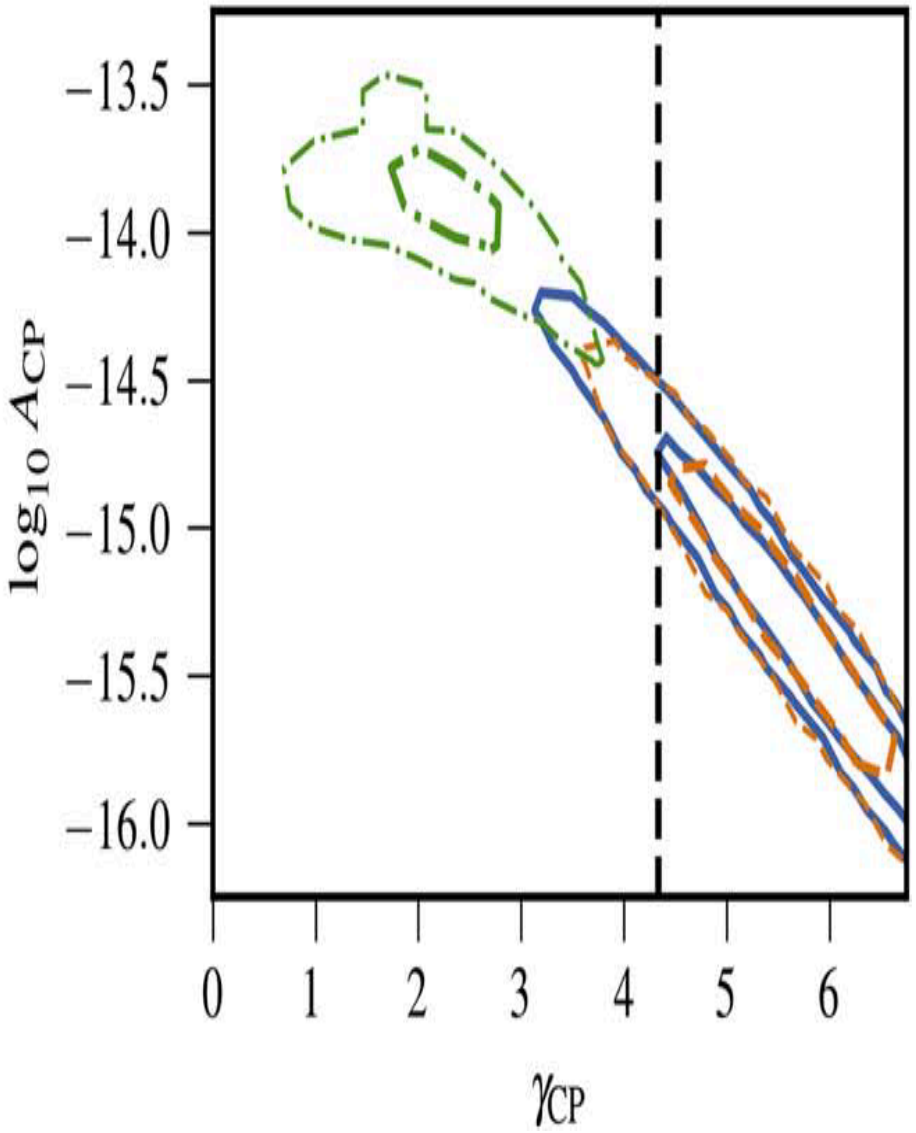


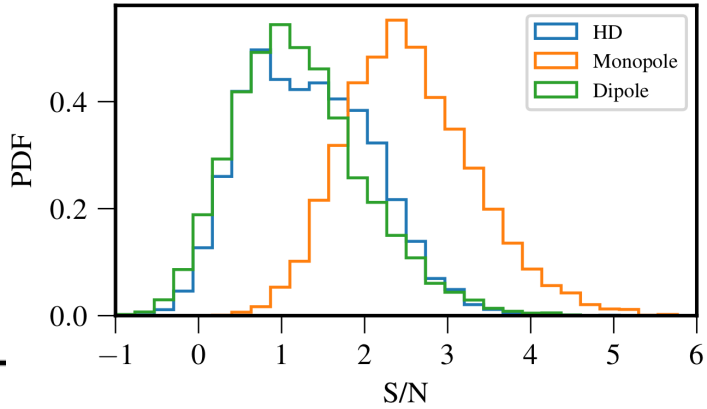
Image: sky map of NANOGrav pulsars in the 11-yr data set

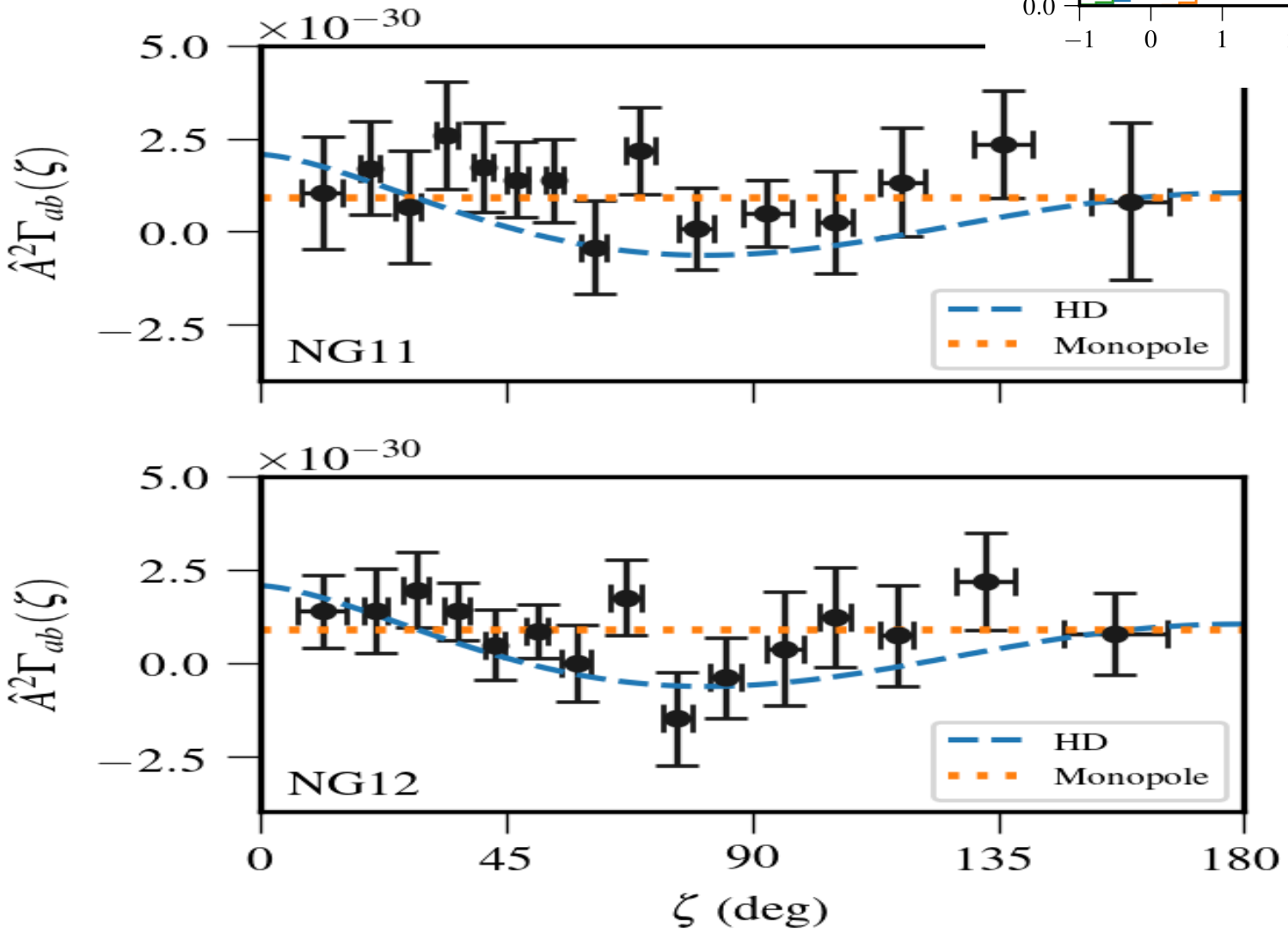
common-spectrum process across MSPs $S_{aa}(f)$



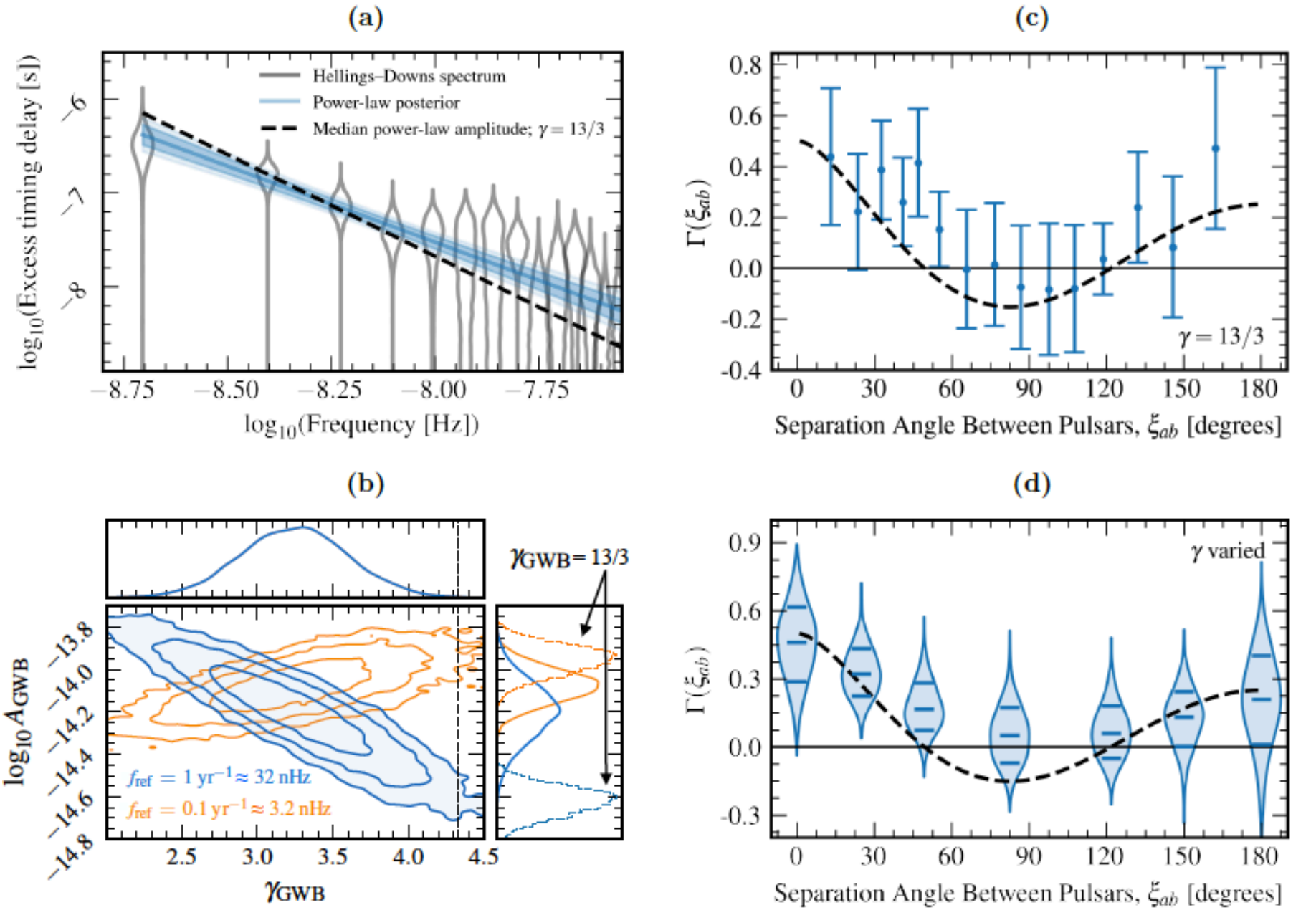


Spatial correlation $S_{ab}(f)$





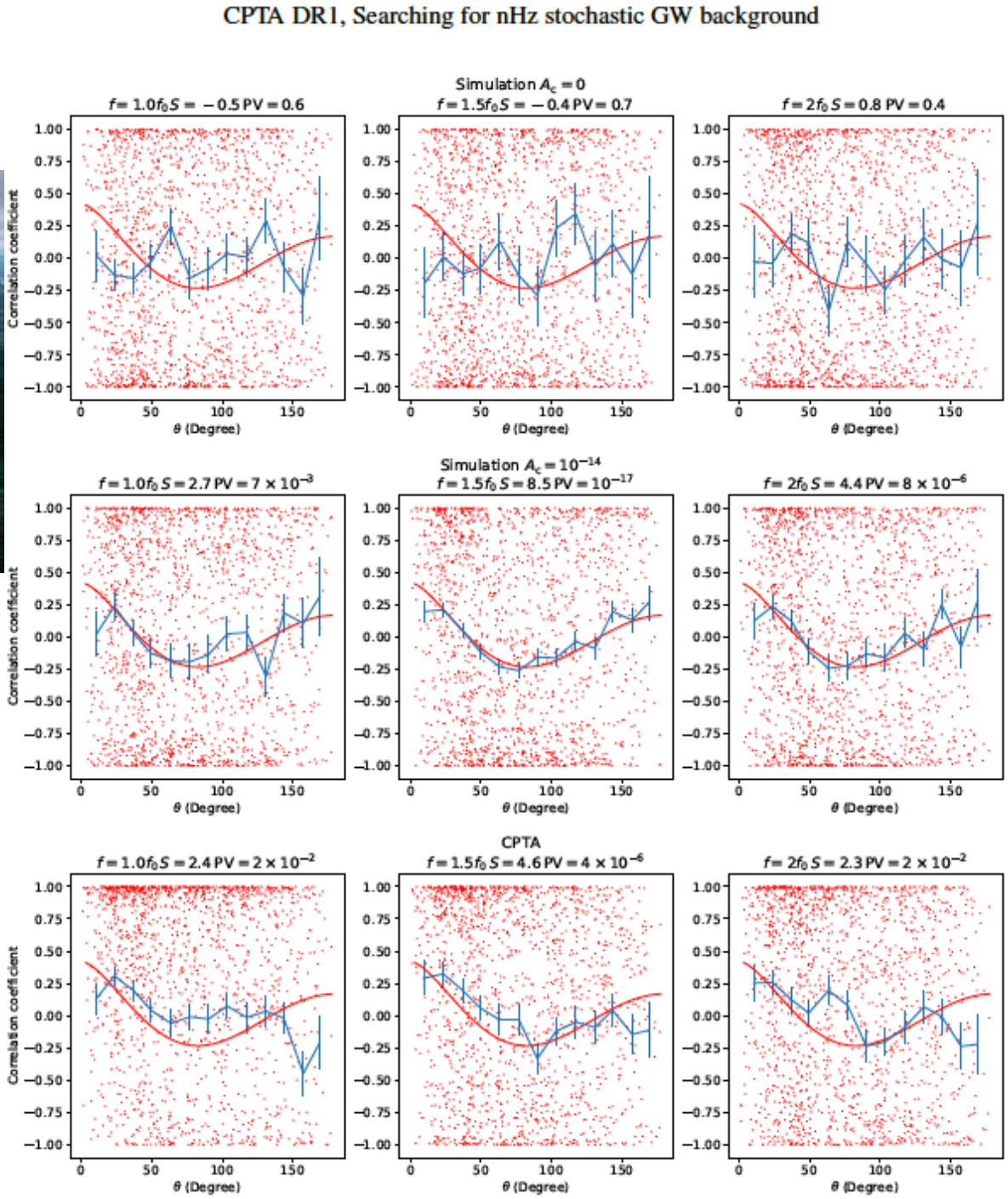
On June 29 2023, four pulsar timing arrays (NanoGrav, EPTA (+InPTA), PPTA, and CPTA) announced the detection of a nanohertz GWB


(a)
Hellings–Downs spectrum
Power-law posterior
Median power-law amplitude; γ = 13/3
log10(Excess timing delay [s])
log10(Frequency [Hz])
(b)
γGWB = 13/3
log10AGWB
fref = 1 yr−1 ≈ 32 nHz
fref = 0.1 yr−1 ≈ 3.2 nHz
γGWB
(c)
Γ(ξab)
γ = 13/3
Separation Angle Between Pulsars, ξab [degrees]
(d)
γ varied
Γ(ξab)
Separation Angle Between Pulsars, ξab [degrees]

Chinese PTA



Five-hundred-meter Aperture Spherical radio Telescope



Future SKA will observe thousands of millisecond pulsars



Future data pipelines

A fast algorithm to generate overlap reduction functions (ORFs - responses of a detector pair) to anisotropic and polarized GWs, based on spherical harmonics analysis

Gravitational Waves

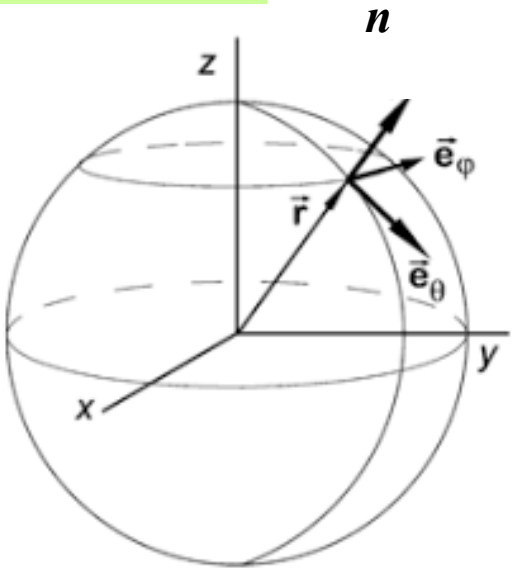
$$h_{ij}(t,\boldsymbol{x}) = \sum_{P=+,\times} \int_{-\infty}^{\infty} df \int_{S^2} d\boldsymbol{n} h_P(f,\boldsymbol{n}) e^{2\pi i f(-t+\boldsymbol{n}\cdot\boldsymbol{x})} e^P_{ij}(\boldsymbol{n}). \quad (1)$$

Here, the bases for transverse-traceless tensor e^P ($P = +, \times$) are given as

$$e^+ = \hat{e}_\theta \otimes \hat{e}_\theta - \hat{e}_\phi \otimes \hat{e}_\phi, \qquad e^\times = \hat{e}_\theta \otimes \hat{e}_\phi + \hat{e}_\phi \otimes \hat{e}_\theta,$$

$$\begin{pmatrix} \langle h_+(f,\boldsymbol{n}) h_+^*(f',\boldsymbol{n}')\rangle & \langle h_+(f,\boldsymbol{n}) h_\times^*(f',\boldsymbol{n}')\rangle \\ \langle h_\times(f,\boldsymbol{n}) h_+^*(f',\boldsymbol{n}')\rangle & \langle h_\times(f,\boldsymbol{n}) h_\times^*(f',\boldsymbol{n}')\rangle \end{pmatrix}$$

$$= \frac{1}{2}\delta_{\mathrm{D}}^2(\boldsymbol{n}-\boldsymbol{n}')\delta_{\mathrm{D}}(f-f') \times \begin{pmatrix} I(f,\boldsymbol{n}) + Q(f,\boldsymbol{n}) & U(f,\boldsymbol{n}) - iV(f,\boldsymbol{n}) \\ U(f,\boldsymbol{n}) + iV(f,\boldsymbol{n}) & I(f,\boldsymbol{n}) - Q(f,\boldsymbol{n}) \end{pmatrix},$$



Q= <++> - <xx>
U is the same as in a frame rotated by π/8

$$e^R = \frac{(e^+ + ie^\times)}{\sqrt{2}}, \qquad e^L = \frac{(e^+ - ie^\times)}{\sqrt{2}}$$

$$h_R = \frac{(h_+ - ih_\times)}{\sqrt{2}}, \qquad h_L = \frac{(h_+ + ih_\times)}{\sqrt{2}}$$

$$\begin{pmatrix} \langle h_R(f,\boldsymbol{n}) h_R(f',\boldsymbol{n}')^*\rangle & \langle h_L(f,\boldsymbol{n}) h_R(f',\boldsymbol{n}')^*\rangle \\ \langle h_R(f,\boldsymbol{n}) h_L(f',\boldsymbol{n}')^*\rangle & \langle h_L(f,\boldsymbol{n}) h_L(f',\boldsymbol{n}')^*\rangle \end{pmatrix}$$

$$= \frac{1}{2}\delta_{\mathrm{D}}(\boldsymbol{n}-\boldsymbol{n}')^2\delta_{\mathrm{D}}(f-f') \times \begin{pmatrix} I(f,\boldsymbol{n}) + V(f,\boldsymbol{n}) & Q(f,\boldsymbol{n}) - iU(f,\boldsymbol{n}) \\ Q(f,\boldsymbol{n}) + iU(f,\boldsymbol{n}) & I(f,\boldsymbol{n}) - V(f,\boldsymbol{n}) \end{pmatrix}.$$

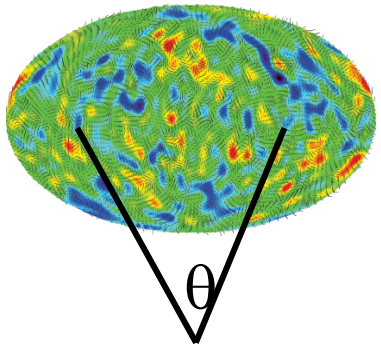
GWB Anisotropy and Polarization Angular Power Spectra

Decompose the GWB sky into a sum of spherical harmonics:

$$T(\theta,\varphi) = \Sigma_{lm}\ a_{lm}\ Y_{lm}\ (\theta,\varphi),\quad V(\theta,\varphi) = \Sigma_{lm}\ b_{lm}\ Y_{lm}\ (\theta,\varphi)$$

$$(Q - iU)\ (\theta,\varphi) = \Sigma_{lm}\ a_{4,lm}\ {}_4Y_{lm}\ (\theta,\varphi)$$

$$(Q + iU)\ (\theta,\varphi) = \Sigma_{lm}\ a_{-4,lm}\ {}_{-4}Y_{lm}\ (\theta,\varphi)$$



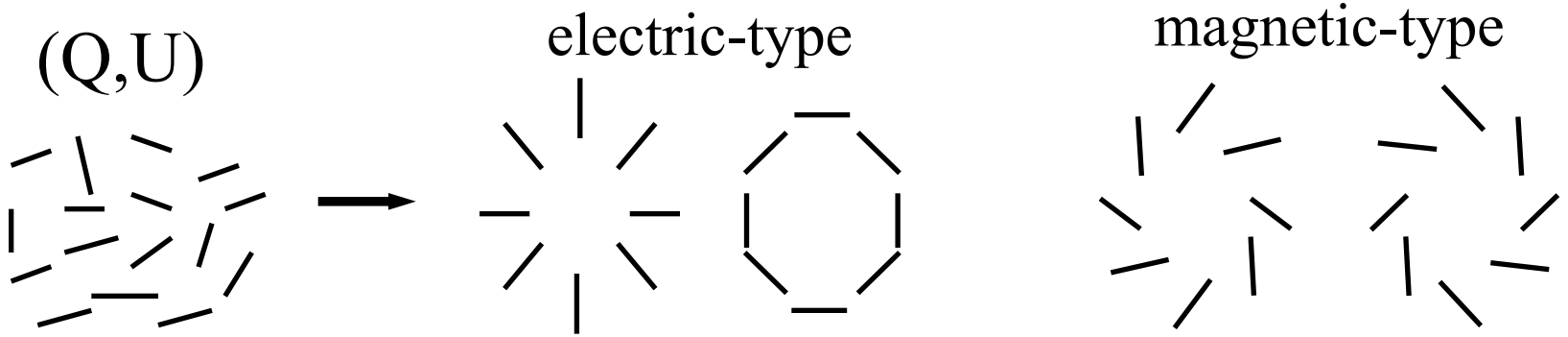
l = 180 degrees/ θ

$C^T_l = \Sigma_m\ (a^*_{lm}\ a_{lm})$ anisotropy power spectrum

$C^V_l = \Sigma_m\ (b^*_{lm}\ b_{lm})$ circular polarization power spectrum

$C^E_l = \Sigma_m\ (a^*_{4,lm}\ a_{4,lm} + a^*_{4,lm}\ a_{-4,lm})$ E-polarization power spectrum

$C^B_l = \Sigma_m\ (a^*_{4,lm}\ a_{4,lm} - a^*_{4,lm}\ a_{-4,lm})$ B-polarization power spectrum



Beyond Einstein's general relativity (phenomenologically)

$$h_{ij}\left(\eta,\vec{x}\right)=\sum_{A}\int_{-\infty}^{\infty}df\int_{S^2}d\hat{k}\;h_A\left(f,\hat{k}\right)\varepsilon_{ij}^{A}e^{-2\pi if\left(\eta-v\hat{k}\cdot\vec{x}\right)}$$

group velocity
v=dω/dk

luminal v=c, sub-luminal v<c, super-luminal v>c

6 degrees of freedom or polarizations

For a GW propagating along the $\hat{\Omega}$ direction, the polarization basis for tensor, vector, and scalar GWs can be expressed as [89]

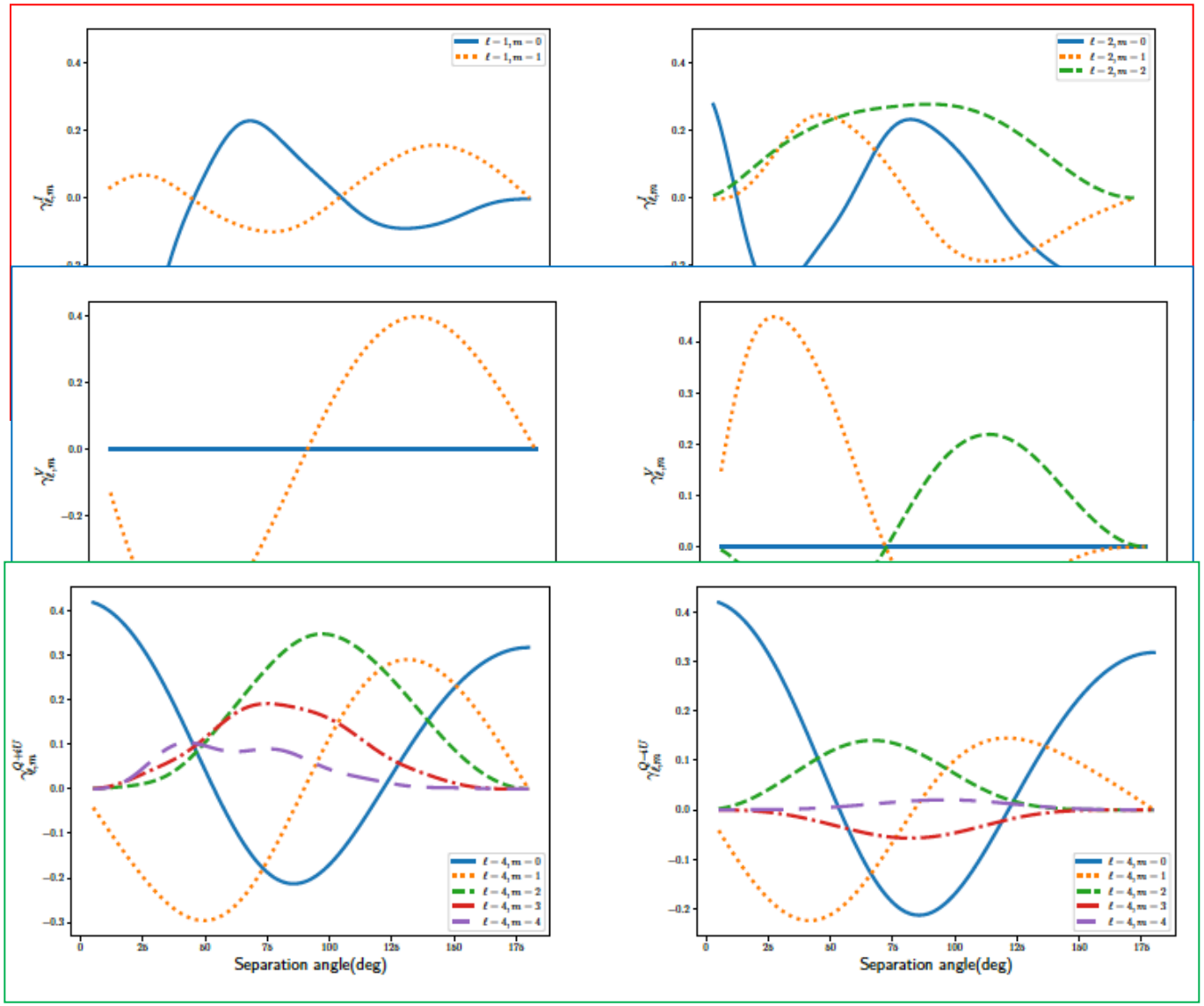
tensor $\quad \varepsilon^{+}=\hat{m}\otimes\hat{m}-\hat{n}\otimes\hat{n} \quad$ and $\quad \varepsilon^{\times}=\hat{m}\otimes\hat{n}+\hat{n}\otimes\hat{m}\,,$ (C1)

vector $\quad \varepsilon^{x}=\hat{m}\otimes\hat{\Omega}+\hat{\Omega}\otimes\hat{m} \quad$ and $\quad \varepsilon^{y}=\hat{n}\otimes\hat{\Omega}+\hat{\Omega}\otimes\hat{n}\,,$ (C2)

scalar $\quad \varepsilon^{b}=\hat{m}\otimes\hat{m}+\hat{n}\otimes\hat{n}$ (breathing mode) $\quad$ and $\quad \varepsilon^{l}=\sqrt{2}\hat{\Omega}\otimes\hat{\Omega}\,,$ (longitudinal mode) (C3)

respectively, where $\left(\hat{m},\hat{n},\hat{\Omega}\right)$ form an orthonormal basis. $(\varepsilon^{+},\varepsilon^{\times})$ are the familiar transverse-traceless tensor GW polarizations, $(\varepsilon^{x},\varepsilon^{y})$ are the corresponding vector GW polarizations, and $(\varepsilon^{b},\varepsilon^{l})$ for scalar GWs.

We have developed an efficient tool for generating ORF $\mathbf{\Gamma}_{lm}(\zeta)$ for any pulsar pair with separation angle ζ



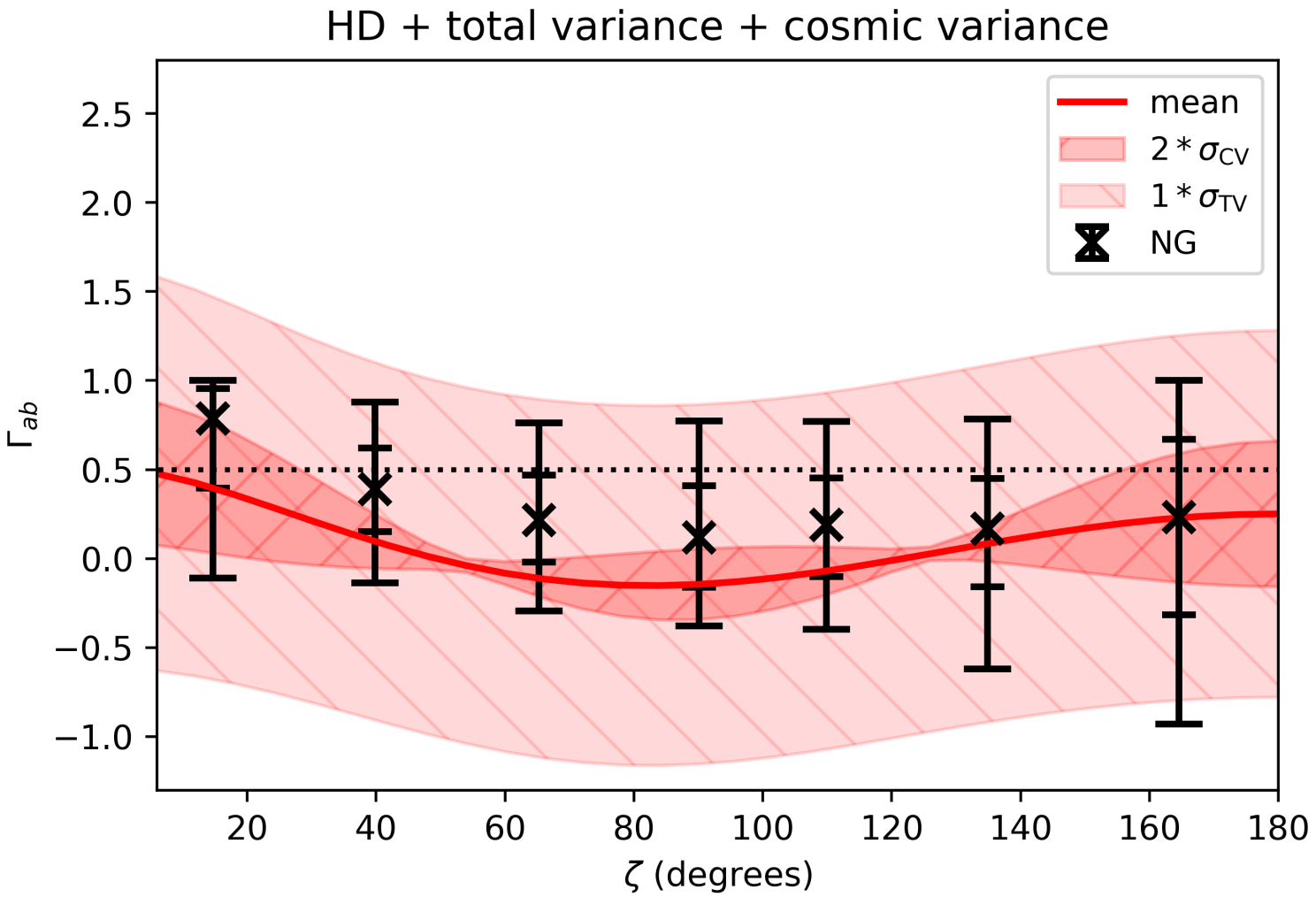
anisotropy

circular polarization

linear polarization

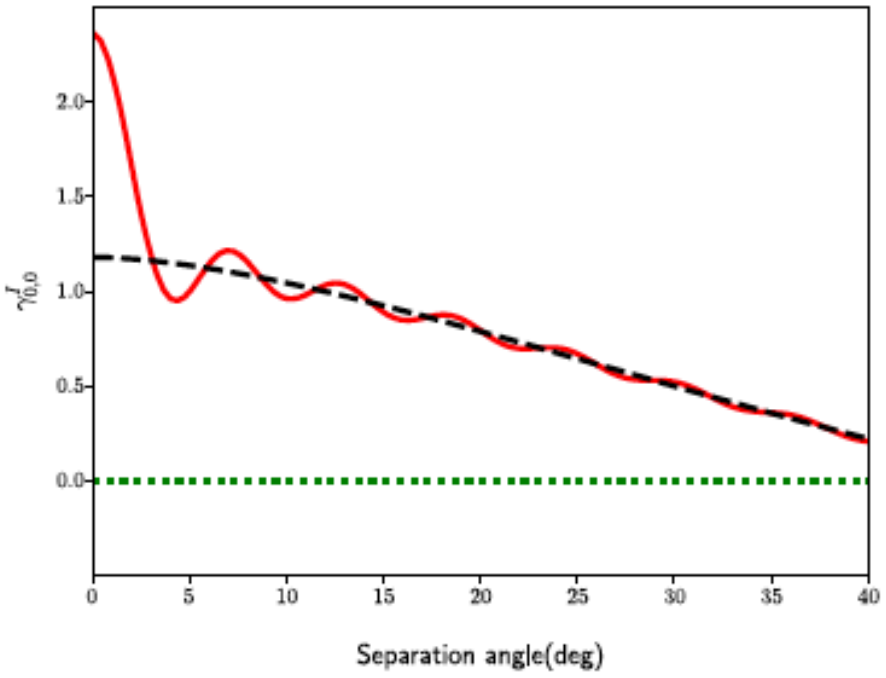
Chu+Liu+Ng 21

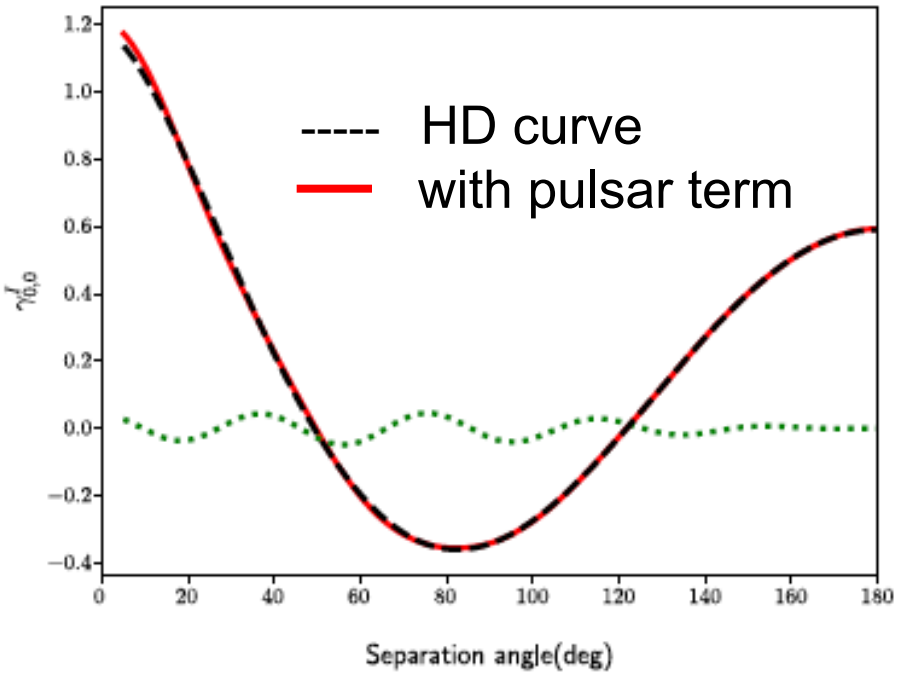
Pulsar and cosmic variances of the Hellings-Downs (HD) curve



Bernardo+Ng 22

Correction to Hellings-Downs curve from the pulsar term

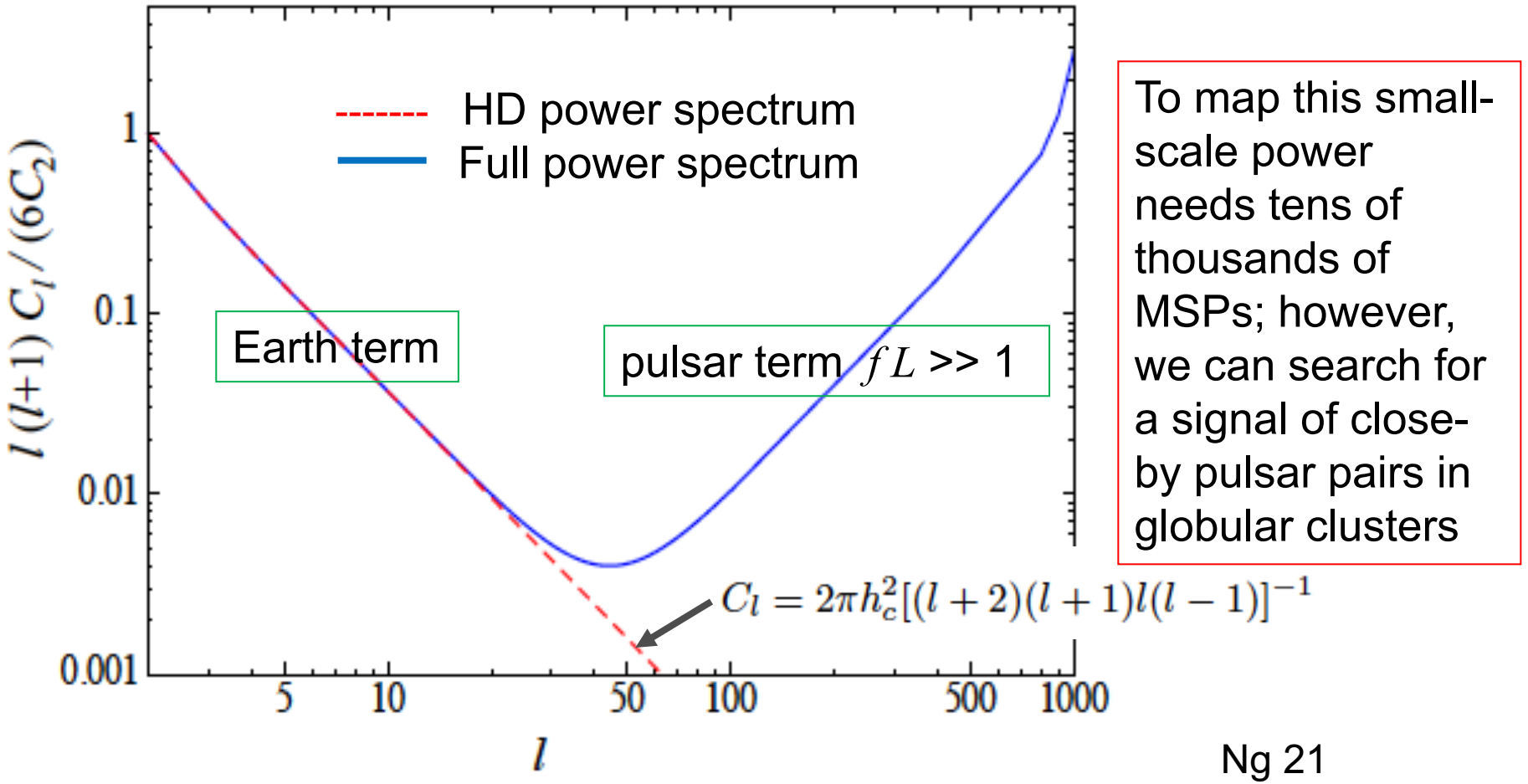




Power spectrum of the pulsar timing residual correlation (l-space)

$$\langle r(t_1)r(t_2)\rangle = \int_0^{t_1} dt' \int_0^{t_2} dt'' \langle z(t')z(t'')\rangle$$
$$= \int_0^{t_1} dt' \int_0^{t_2} dt'' e^{-ik_*(t'-t'')} \langle z(\mathbf{e_1})z(\mathbf{e_2})\rangle$$

$$\langle z(\mathbf{e_1})z(\mathbf{e_2})\rangle = \sum_l \frac{2l+1}{4\pi} C_l P_l(\mathbf{e}_1 \cdot \mathbf{e}_2)$$



To map this small-scale power needs tens of thousands of MSPs; however, we can search for a signal of close-by pulsar pairs in globular clusters

Conclusion

- GWB is a main goal in GW experiments
- GWB is a deep probe into the very early Universe
- Pulsar-timing-arrays saw the nanohertz GWB! What is the origin? Einstein's gravity? Gaussian?
- GWB monopole and Doppler dipole
- GWB anisotropy and polarization
- Correlate with cosmological data such as LSS, CMB
- We construct PTAfast, https://github.com/reggiebernardo/PTAfast

  fast python codes to generate ORFs
- See our review article arXiv:2409.07955 [astro-ph.CO]