Mutual Information between Current Halo Properties and the Initial Tidal Misalignment

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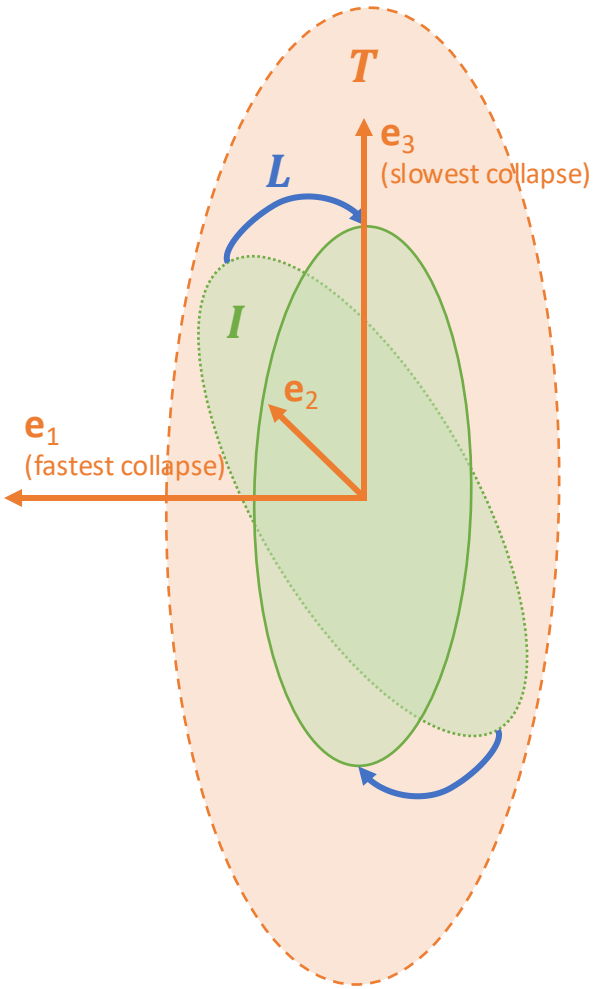
Credit: TNG Collaboration

Motivation: Tidal Torque Theory

- The spin acquisition of galaxies can be explained by the linear tidal torque theory (LTTT; Peebles 1969; Doroshkevich 1970; White 1984), where the tidal field exerts a torquing on protogalactic objects.

- The angular momentum of a protogalaxy **L** can be described as $L_\alpha \propto \epsilon_{\alpha\beta\gamma} I_{\beta\sigma} T_{\sigma\gamma}$, where **I** is the inertia tensor of the protogalaxy and **T** is the tidal tensor (i.e., the hessian of the potential).

- The misalignment between the inertia and tidal tensors is a necessary condition for the spin generation in the tidal torque theory (Lee & Pen 2000; Schäfer 2009).



Primordial Spin Factor (τ)

- The τ parameter quantifies the misalignment between the protogalactic inertia tensor and the initial tidal field.

- Method I (Moon & Lee 2024a): The inertia tensor ($\boldsymbol{I}$) is expressed in the principal frame spanned by three eigenvectors ($\hat{\boldsymbol{t}}$) of the tidal field ($\boldsymbol{T}$), where $\boldsymbol{I} = \boldsymbol{W}^T \boldsymbol{I}' \boldsymbol{W}$ and $\boldsymbol{W}_{\alpha\gamma} = (\hat{\boldsymbol{t}}_\alpha)_\gamma$, then

$$\tau \equiv \left(\frac{\boldsymbol{I}_{12}^2 + \boldsymbol{I}_{23}^2 + \boldsymbol{I}_{31}^2}{\boldsymbol{I}_{11}^2 + \boldsymbol{I}_{22}^2 + \boldsymbol{I}_{33}^2} \right)^{1/2}$$

  - If the inertia tensor is perfectly aligned with the tidal tensor, the off-diagonal components of $\boldsymbol{I}$ are zero and τ = 0.
  - The more the principal axes of the two tensors are misaligned with each other, the larger the value of tau.

- Method II (Moon & Lee 2024c): The inertia tensor ($\boldsymbol{I}$) is approximated by the Hessian matrix ($\boldsymbol{H}$) of the density field (Codis et al. 2015; Motloch et al. 2021). For $\boldsymbol{H}$ in the tidal principal frame, where $\boldsymbol{H} = \boldsymbol{W}^T \boldsymbol{H}' \boldsymbol{W}$ and $\boldsymbol{W}_{\alpha\gamma} = (\hat{\boldsymbol{t}}_\alpha)_\gamma$,

$$\tau' \equiv \left(\frac{\boldsymbol{H}_{12}^2 + \boldsymbol{H}_{23}^2 + \boldsymbol{H}_{31}^2}{\boldsymbol{H}_{11}^2 + \boldsymbol{H}_{22}^2 + \boldsymbol{H}_{33}^2} \right)^{1/2}$$

  - τ' is generally easier to obtain than τ, but they have the same statistical properties for a large sample.

Measurement of τ in IllustrisTNG

- We utilize the TNG300 cosmological simulations in the IllustrisTNG project (Nelson et al. 2019).
- The constituent DM particles from each subhalo at z = 0 are traced back to the initial condition of the simulation.

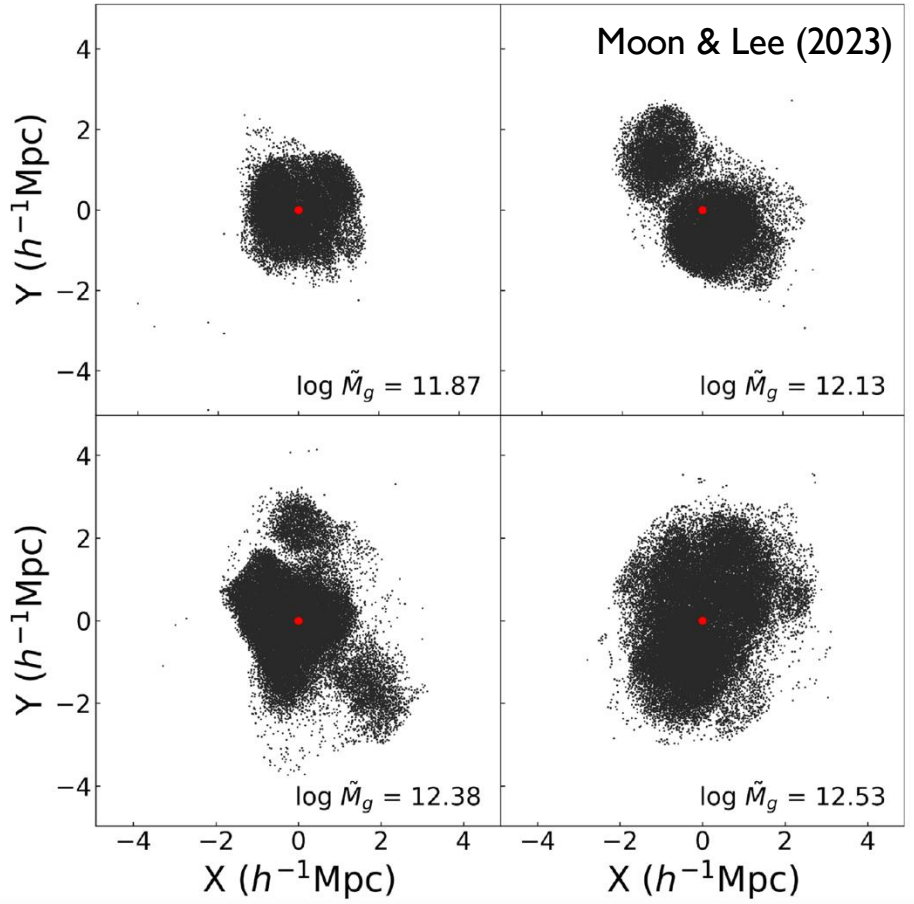
- The inertia tensor in the Cartesian frame ($\boldsymbol{I}'$) is calculated from the initial comoving positions ($\boldsymbol{x}$) of the constituent DM particles with respect to their center of mass ($\overline{\boldsymbol{x}}$).

$$\boldsymbol{I}'_{\alpha\beta} = \sum_i m_i(\boldsymbol{x}_\alpha - \overline{\boldsymbol{x}}_\alpha)(\boldsymbol{x}_\beta - \overline{\boldsymbol{x}}_\beta)$$

- The tidal field ($\boldsymbol{T}$) and the density Hessian field ($\boldsymbol{H}$) are obtained at $\overline{\boldsymbol{x}}$ from the density contrast field (δ) with a certain smoothing scale (R_f).

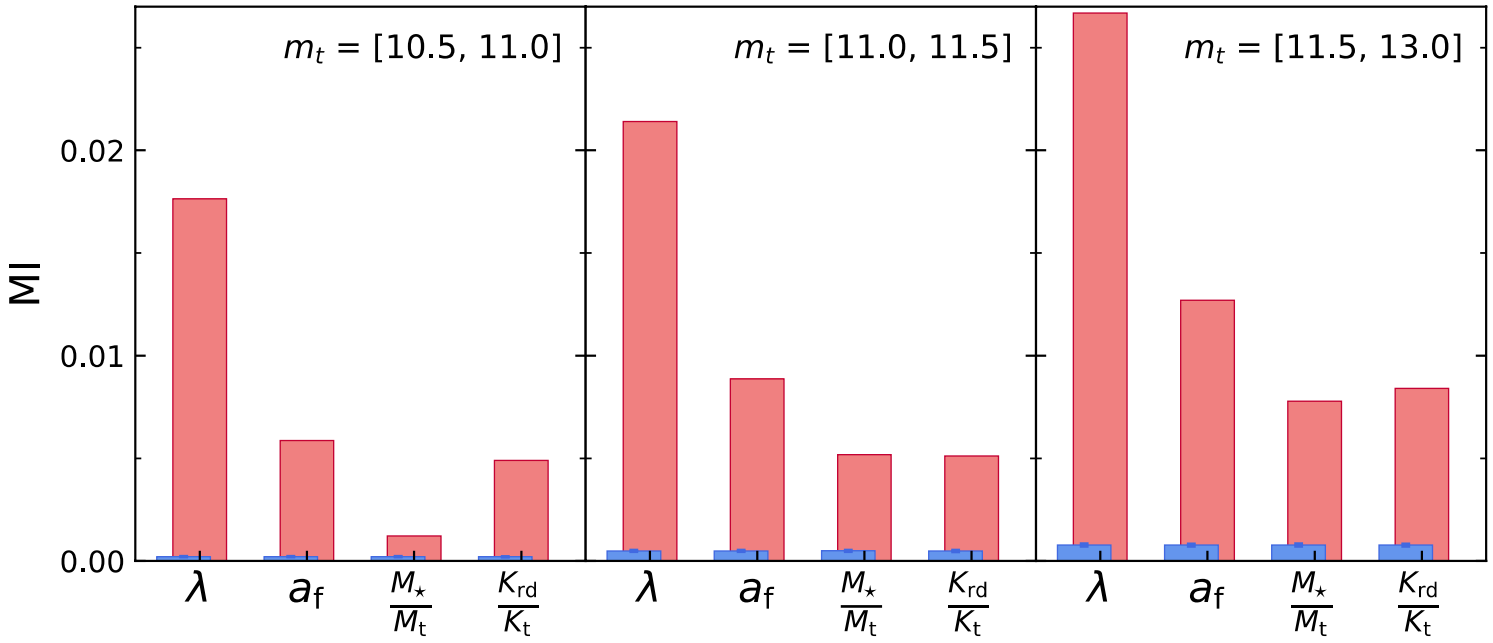
$$T_{\alpha\beta}(\boldsymbol{k}) = k_\alpha k_\beta \delta(\boldsymbol{k}) \exp\left(-|\boldsymbol{k}|^2 R_f^2/2\right)/|\boldsymbol{k}|^2$$

$$H_{\alpha\beta}(\boldsymbol{k}) = k_\alpha k_\beta \delta(\boldsymbol{k}) \exp\left(-|\boldsymbol{k}|^2 R_f^2/2\right)$$



Mutual Information between τ and Halo Properties

$$\mathrm{MI}(\tau,\eta) = \int d\tau \int d\eta\, p(\tau,\eta) \log\left[\frac{p(\tau,\eta)}{p(\tau)p(\eta)}\right]$$



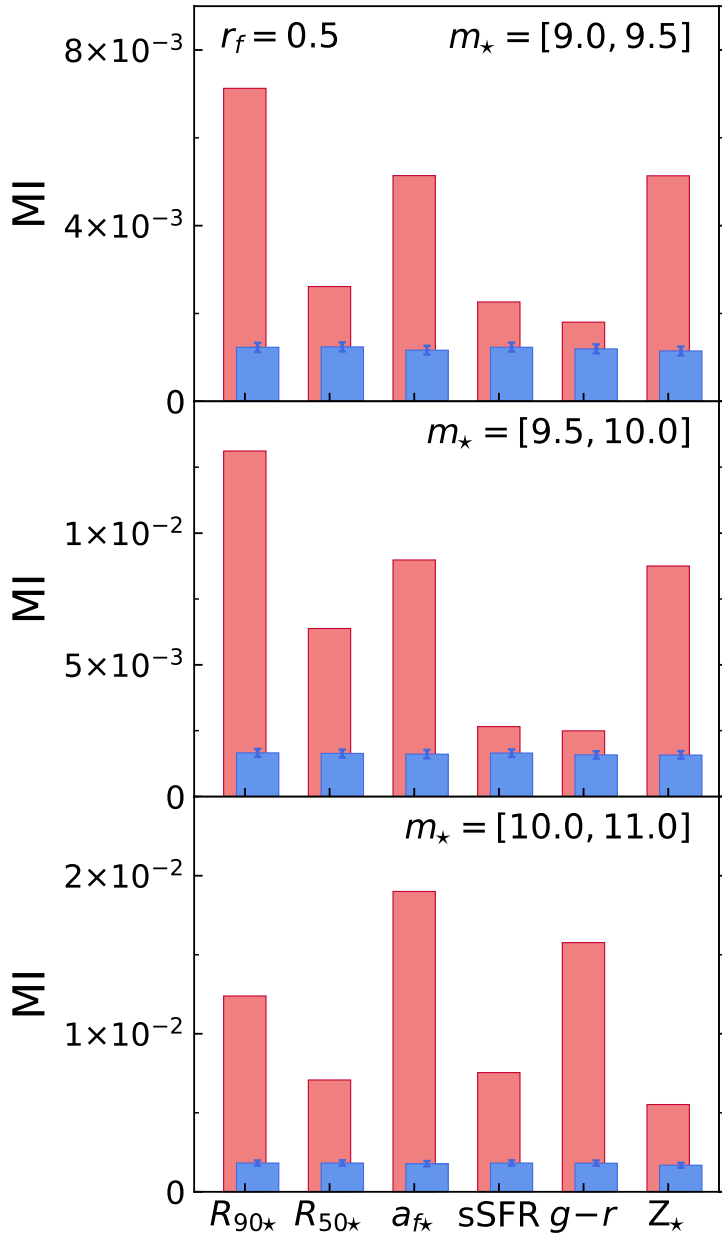
- λ: Bullock's spin parameter
- a_f: formation epoch at which the mass of the main progenitor exceeds half the final mass
- $M_\star/M_t$: stellar mass to total mass ratio
- K_{rd}/K_t: the fraction of kinetic energy not in ordered rotation

Moon & Lee (2024a)

- The mutual information (MI) is a measure of the dependence between two variables, which quantifies the amount of information that the two variable share (Shannon 1948; Sarkar & Pandey 2020; Sarkar et al. 2021).
- The largest amount of MI about τ is found to be possessed by the spin parameter (λ).

* On average, the increment of τ leads λ and a_f to increase, while $M_\star/M_t$ and K_{rd}/K_t to decrease.

Mutual Information between τ and Stellar Properties



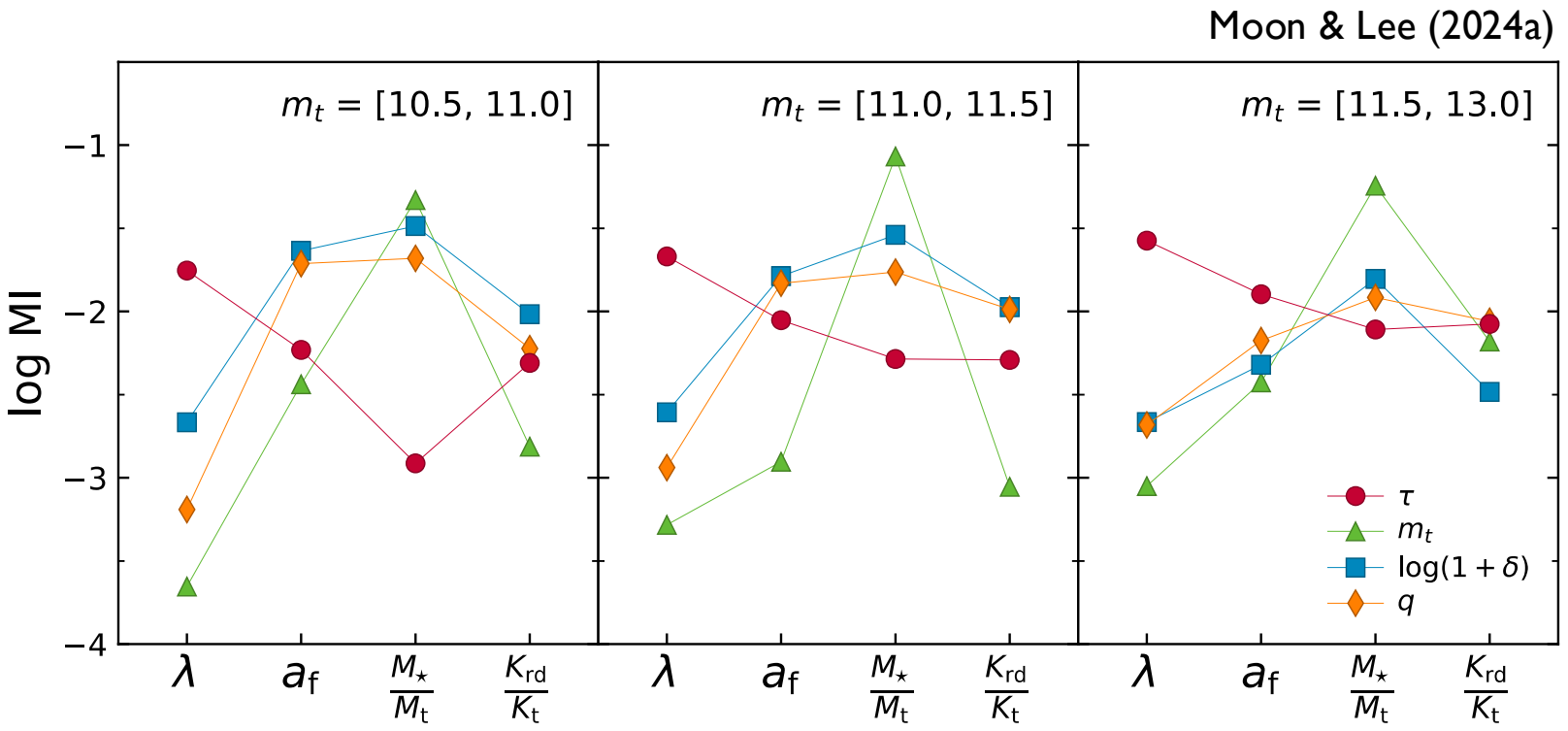
Moon & Lee (2024c)

- $R_{90\star}/R_{50\star}$: galaxy size enclosing 90/50% of stellar mass
- $a_{f\star}$: mean stellar age within 2 × $R_{50\star}$
- sSFR: specific star formation rate within 2 × $R_{50\star}$
- g−r: optical g−r color
- $Z_\star/Z_\odot$: stellar metallicity within 2 × $R_{50\star}$

- Several stellar properties have a statistically meaningful MI with τ, including the galaxy size, age, star formation rate, and metallicity.

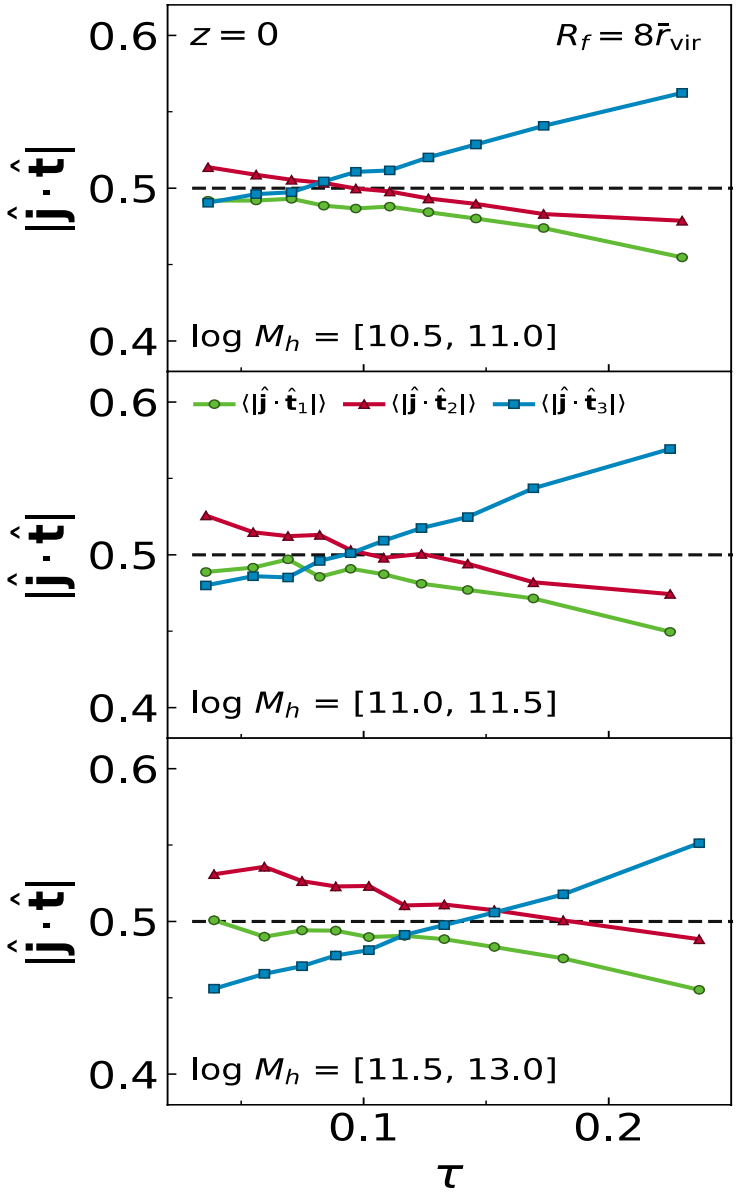
* As τ' increases, galaxies become larger, younger, bluer, and more metal-poor.

Comparison to Dependences on Mass and Environments



- MI(τ, λ) is always the largest over MI(M_t, λ), MI(δ, λ), and MI(q, λ), which indicates that τ has the strongest impact on the halo spin parameters than the total mass and environments.
- MI(τ, a_f) and MI(τ, K_{rd}/K_t) are as large as the MI with M_t, δ, and q, suggesting that τ plays a pivotal role.

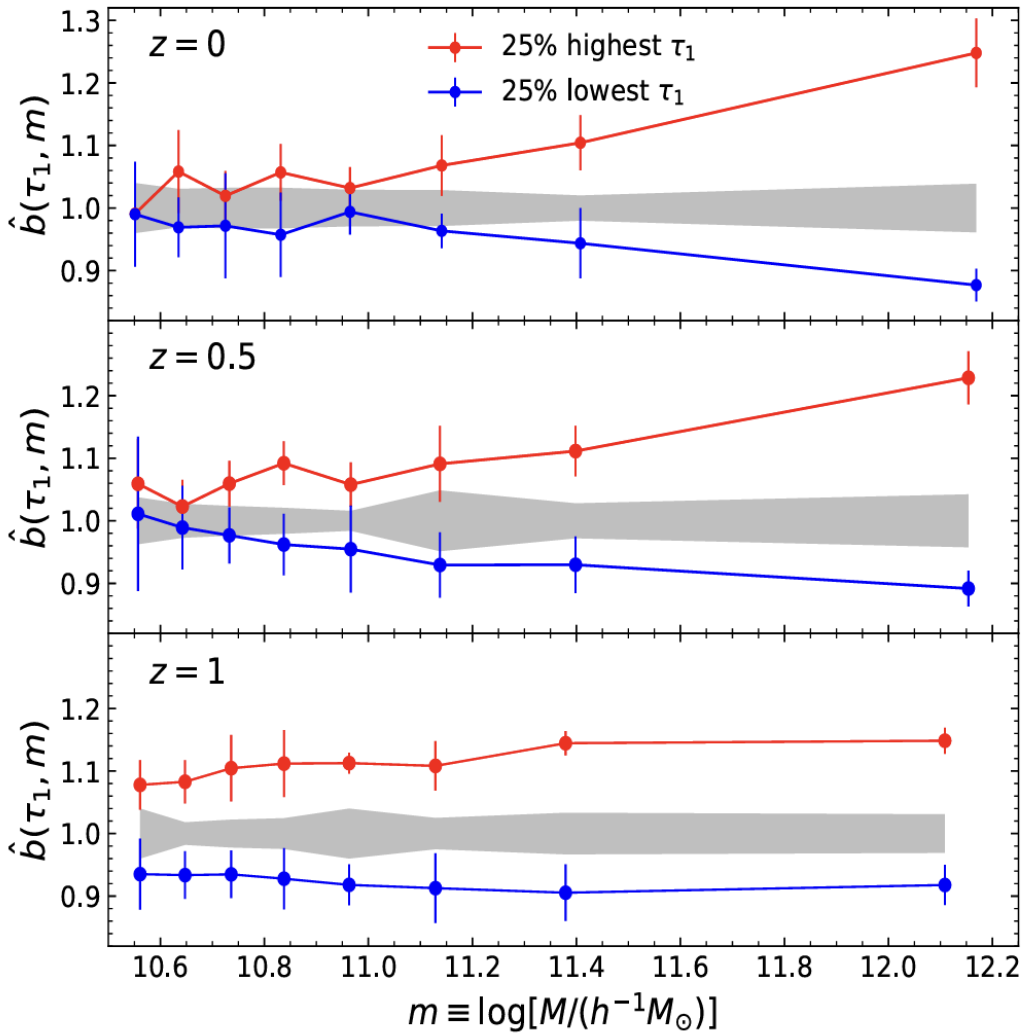
τ-dependent Spin Transition



- Halo spin alignments with the tidal fields depend on τ: the spin transition from minor ($\hat{\boldsymbol{t}}_3$) to intermediate ($\hat{\boldsymbol{t}}_2$) principal axes is witnessed as τ decreases in all mass ranges.
- The transition signal is much stronger as a function of τ rather than of the halo mass, implying that the τ-dependent transition is more fundamental.

Moon & Lee (2024b)

Secondary Halo τ Bias



Lee & Moon (2024)

- A clear signal of τ bias is detected: The halos with 25% highest τ cluster more strongly than those with 25% lowest τ at fixed halo mass.

Please see further results on τ :

Moon, J.-S., & Lee, J. 2024a, JCAP, 2024, 111 - MI on halo properties

Moon, J.-S., & Lee, J. 2024b, ApJ, 966, 100 - τ-dependent spin transition

Lee, J., & Moon, J.-S. 2024, JCAP, in press - secondary halo τ bias

Moon, J.-S., & Lee, J. 2024c, JCAP, submitted - MI on stellar properties

Summary & Conclusion

- We have devised the primordial spin factor, τ, that quantifies **the misalignments between the initial inertia and tidal tensors**.
- Using the IllustrisTNG simulations, we calculated τ for each central subhalos by tracing all member DM particles at z = 0 back to the initial condition of the simulation at z = 127.

- τ shares significant mutual information with basic halo and stellar properties at z = 0.
- **The halo spin parameter** (λ) exhibits the strongest dependence on τ, having even larger MI than mass and environments.

- Our findings suggest that the initial condition can play a significant role in galaxy evolution and that τ seems to be a key initial factor regulating the current galaxy properties.



Moon, J.-S., & Lee, J. 2024a, JCAP, 2024, 111 - MI on halo properties

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