What happens when supercooling is terminated due to curvature flipping?

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arXiv: 2412.15864

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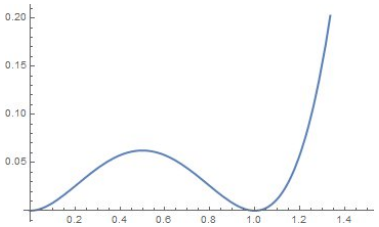
Supercooled FOPTs

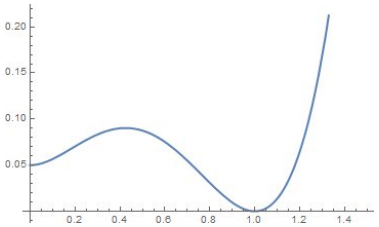
General features of FOPTs

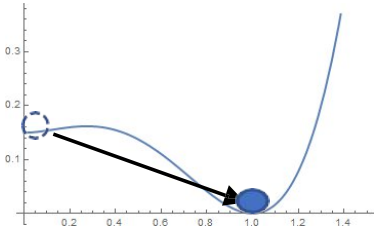
T

$T_{\rm crit}$

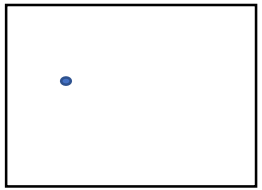
$T_{\rm nucl}$

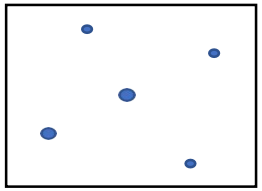










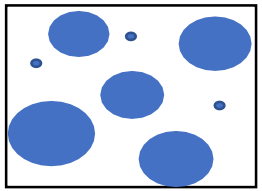


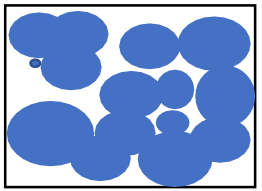
$\Gamma_n \neq 0, \Gamma_n \ll H^4$

Bubble nucleation is possible, but bubbles are diluted away by the spacetime expansion outside bubbles

$\Gamma_n \simeq H^4$

One bubble nucleation per one Hubble patch within one Hubble time





nucleation → **expansion** **collision**

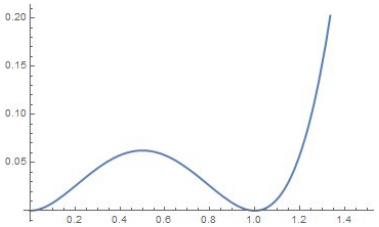
Figures courtesy of Tae Hyun Jung

General features of FOPTs

T

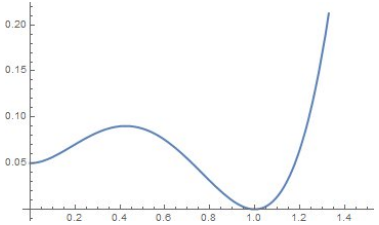
$T_{\rm crit}$

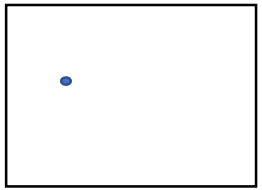
$T_{\rm nucl}$





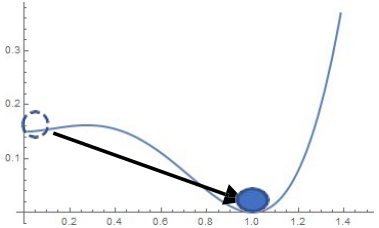
Supercooled: $T_{\rm nucl} \ll T_{\rm crit}$

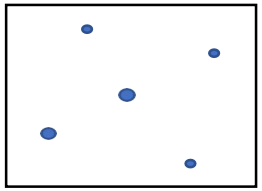




$\Gamma_n \neq 0, \Gamma_n \ll H^4$

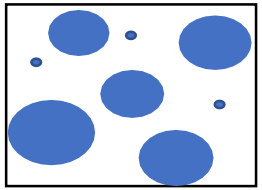
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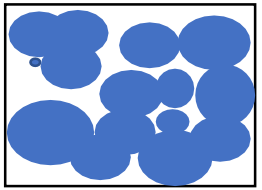




$\Gamma_n \simeq H^4$

One bubble nucleation per one Hubble patch within one Hubble time





nucleation → **expansion** **collision**

Figures courtesy of Tae Hyun Jung

Supercooled First Order Phase Transitions

(Recent) popularity in studying supercooled phase transitions in cosmological contexts.

- Can produce a secondary, smaller period of ‘thermal inflation’ which can help dilute the abundances of some long-lived BSM particles which could spoil the predictions of BBN, e.g. gravitino, moduli… hep-ph/9510204, hep-ph/9602263, 0801.4197, 1412.7814....

- Can produce sizeable gravitational wave signals, through collision and motion of true-vacuum bubbles within the thermal plasma. 1809.08242, 1811.11169, 2007.15586, 2208.11697, 2303.02450....

- Supercooled phase transitions can be used as explanations for other problems in particle cosmology, e.g. PBH production, dark matter filtering, baryogenesis catalysts..
  1912.04238, 2110.04271, 2206.04691, 2206.09923, 2304.00908, 2305.10759....

- Constructing supercooled models is not difficult, just require relatively flat potentials, most commonly classically scale invariant or SUSY models.

Supercooling warmup

Simple Toy Model:

A classically scale-invariant potential

$$V_0(\Phi, \dots) \supset \lambda |\Phi|^4$$

develops a flat direction at some scale $\lambda(\mu_*) = 0$

Coupling to other fields (e.g. gauge boson - Coleman-Weinberg, scalar portal - Gildener-Weinberg) induces radiative corrections

$$V_{CW}(m_i^2(\phi)) = (-1)^{2s_i} g_i \frac{m_i^4(\phi)}{64\pi^2} \left[\log\left(\frac{m_i^2(\phi)}{\mu^2}\right) - c_i \right]$$

A minimum forms radiatively $(\langle\Phi\rangle \sim \mu_*)$ $\Delta V / \langle\Phi\rangle^4 \ll 1$

$$V''(\phi)\Big|_{\phi=0} = 0$$

Thermal corrections

Coupling of field to thermal bath induces corrections

$$V(\phi, T) = V_0(\phi) + \sum_i V_{CW}(m_i^2(\phi)) + \sum_i V_T(m_i^2(\phi))$$

$$V_T(m_i^2(\phi)) = \pm \frac{g_i}{2\pi^2} T^4 J_{\mathrm{B,F}}\left(\frac{m_i^2(\phi)}{T^2}\right) \qquad J_{\mathrm{B,F}}(y^2) = \int_0^\infty dx\ x^2 \log\left[1 \mp \exp\left(-\sqrt{x^2 + y^2}\right)\right]$$

Induces temperature-dependent corrections, particularly for small field values including the extremum at the origin

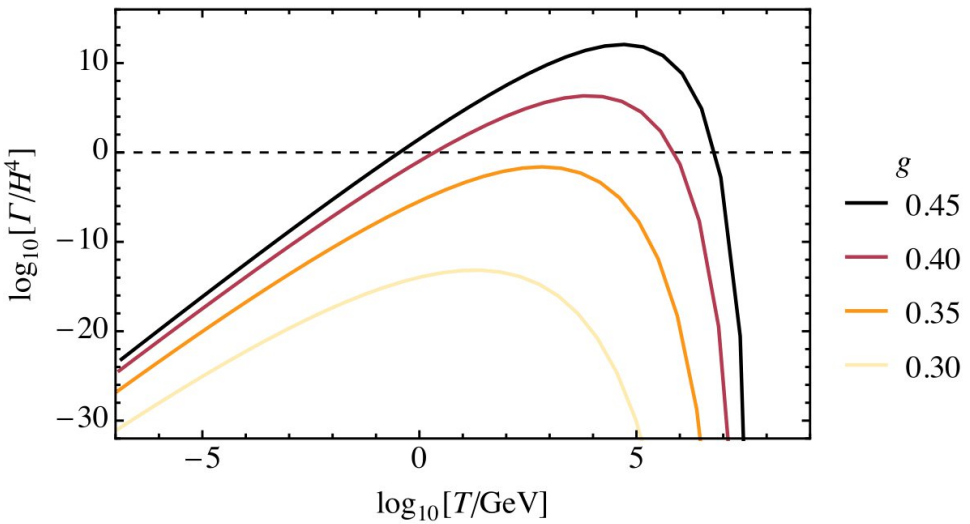
$$V''(\phi)\Big|_{\phi=0} = 0 + \frac{g^2}{12} T^2 > 0$$

A minimum around the origin forms. At some temperature will become meta-stable. Tree-level conformal invariance -> barrier remains for entire thermal history

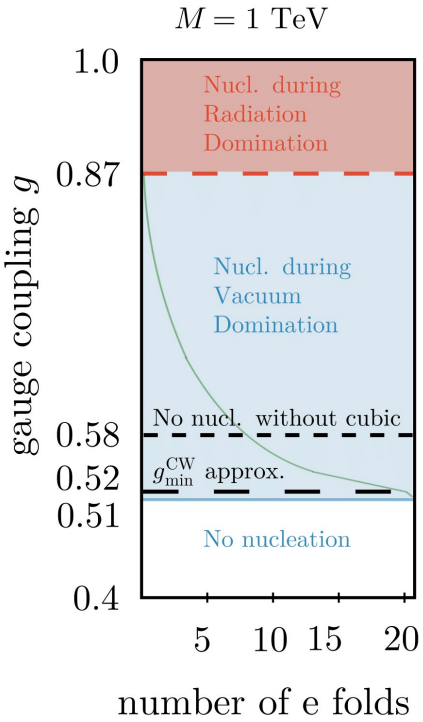
Dangers with ‘scaleless’ supercooling

Can lead to scenarios with ‘eternal inflation’ for not-that-small values of couplings.

$$\text{nucleation} \implies \Gamma_n = H(T)^4$$



2212.08085



2106.09706

Introduce an additional scale

Realistically, completely reasonable to expect the existence of additional scales (e.g. soft breaking terms, QCD phase transition...) which can change the simple picture somewhat

E.g.

$$V_0 = \lambda^2\phi^2 S^2 + M_s^2 S^2$$

$$V_{\rm CW}(\phi) \to \frac{(\lambda^2\phi^2 + M_s^2)^2}{64\pi^2}\left[\log\left(\frac{\lambda^2\phi^2 + M_s^2}{\mu^2}\right) - \frac{3}{2}\right] \qquad V_T(\phi) \to \frac{T^4}{2\pi^2} J_B\left(\frac{\lambda^2\phi^2 + M_s^2}{T^2}\right)$$

Destabilisation effect on the origin at zero temperature

$$V''_{\rm eff}\Big|_{\phi=0,T=0} = \frac{\lambda^2 M_s^2}{16\pi^2}\left(2\log\left(\frac{M_s}{\mu}\right) - 1\right) < 0$$

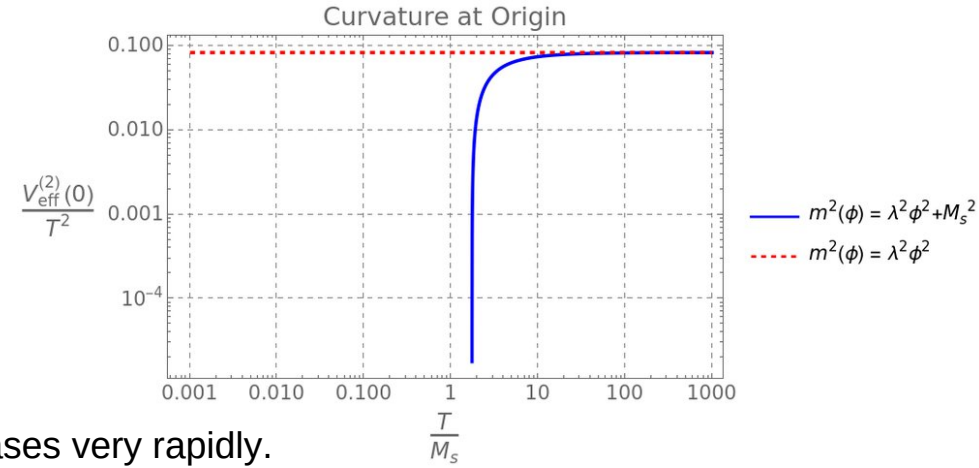
Introduce an additional scale

The thermal corrections to the curvature are (obviously) significant at sufficiently large temperatures

$$V''_{\rm eff}\Big|_{\phi=0} = \frac{\lambda^2}{16 M_s \pi^2}\left[M_s^3\left(2\log\left(\frac{M_s}{\mu}\right)-1\right)+8T^3 J'_B\left(\frac{M_s}{T}\right)\right]$$

At high temperatures behaviour unchanged but needs to be a sign flip at some scale -> phase transition always completes. Terminated Supercooling.

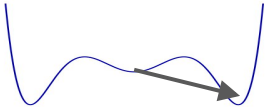
Useful e.g. for thermal inflation



In fact, barrier decreases very rapidly.

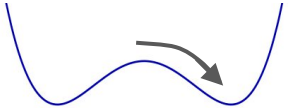
Implications for transition

- (i) $T_{\rm phase} > M_s$



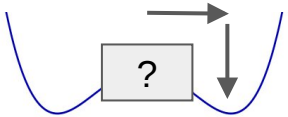
If the phase transition can complete, and completes at large temperatures, usual conformal supercooling scenario -> bubble formation

- (ii) $T_{\rm phase} < M_s$



Field remains trapped until after the barrier disappears. Will roll down to the true vacuum.

- (iii) $T_{\rm phase} \simeq M_s$



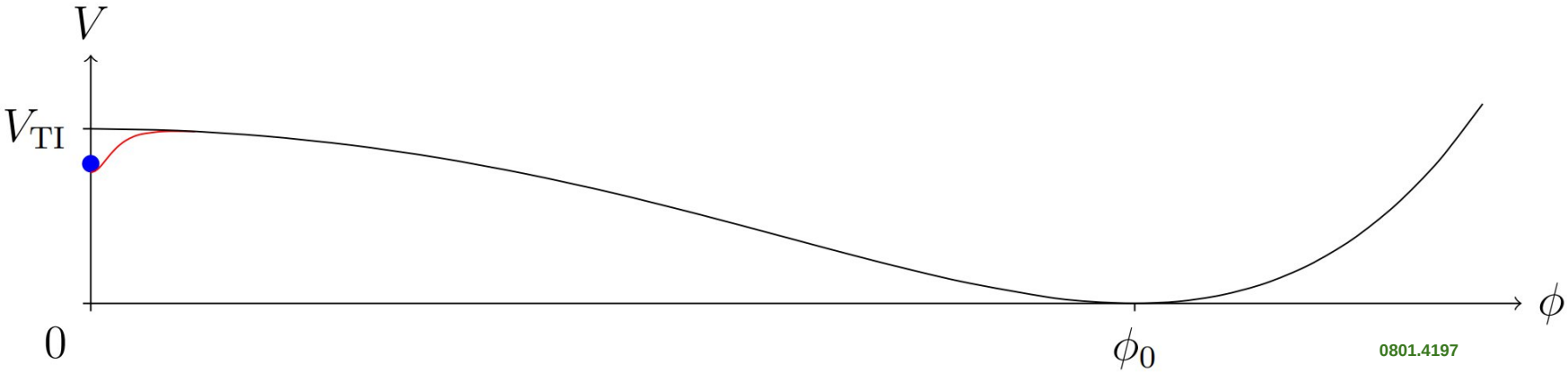
The phase transition proceeds when a potential barrier still exists around the meta-stable origin. **The barrier is rapidly disappearing but thermal fluctuations ∝ T persist.**

Still bubble formation? Prospects for gravitational wave signals?

Considering termination more closely

Case Study:

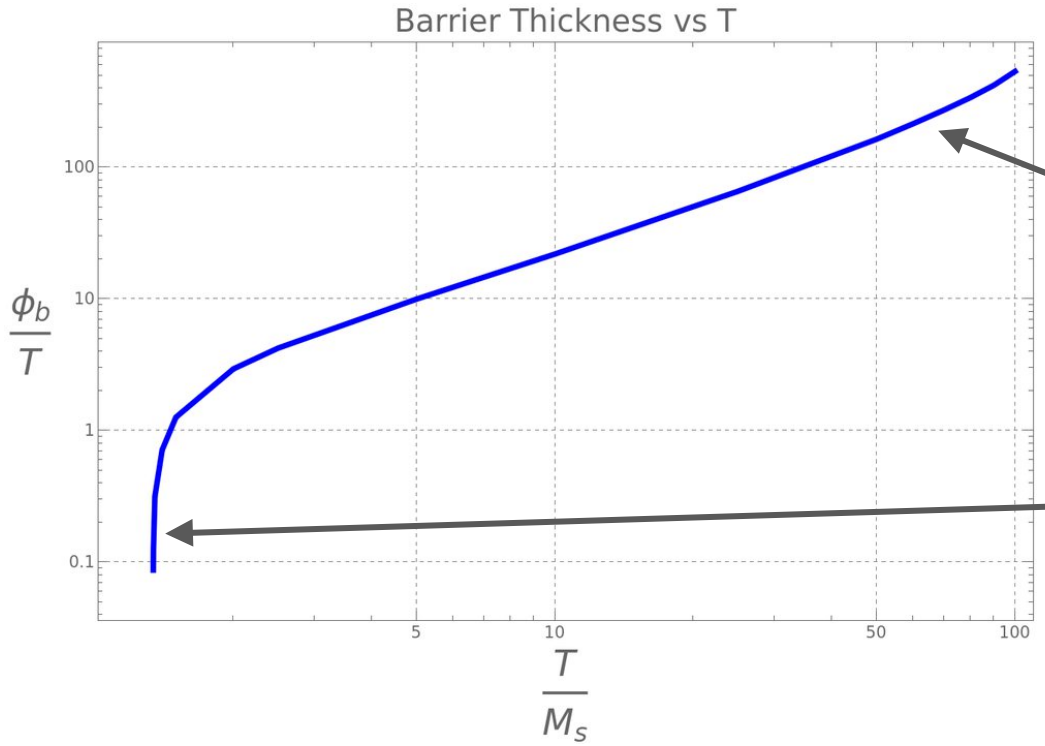
Consider now a SUSY motivated picture of thermal inflation with some soft SUSY breaking scale,



At relatively large temperatures the scalar field is held at the origin (finite T effect) and the potential barrier is very wide.

Barrier size vs temperature

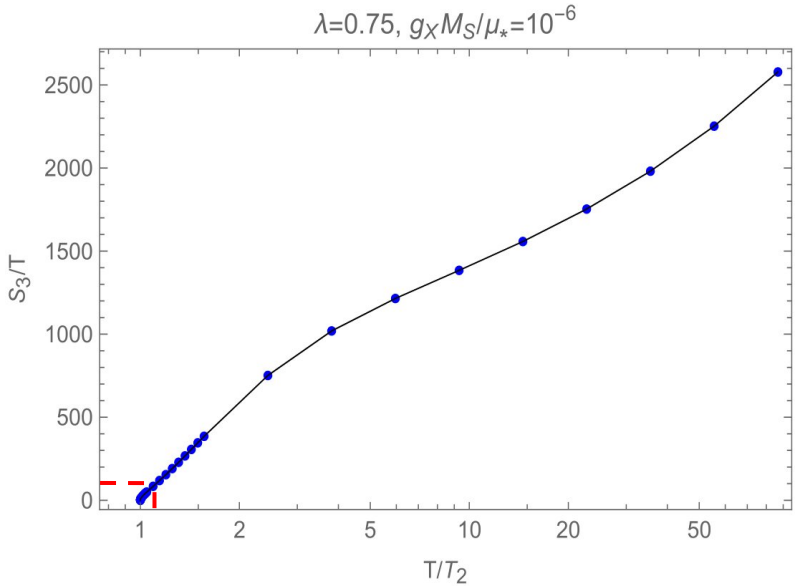
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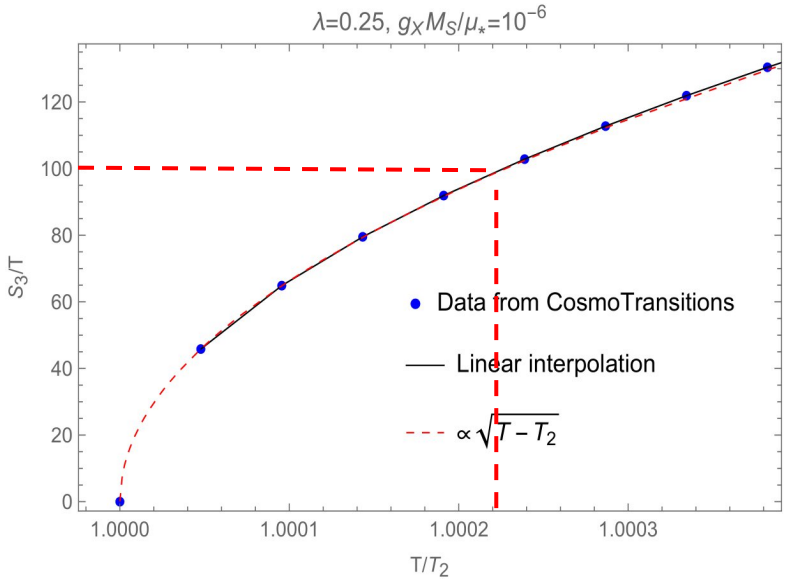


At high temperatures the field is strongly trapped. Good candidate for supercooling. Can generate a number of e-folds.

Barrier disappears so eternal inflation is prevented

When can bubbles nucleate?



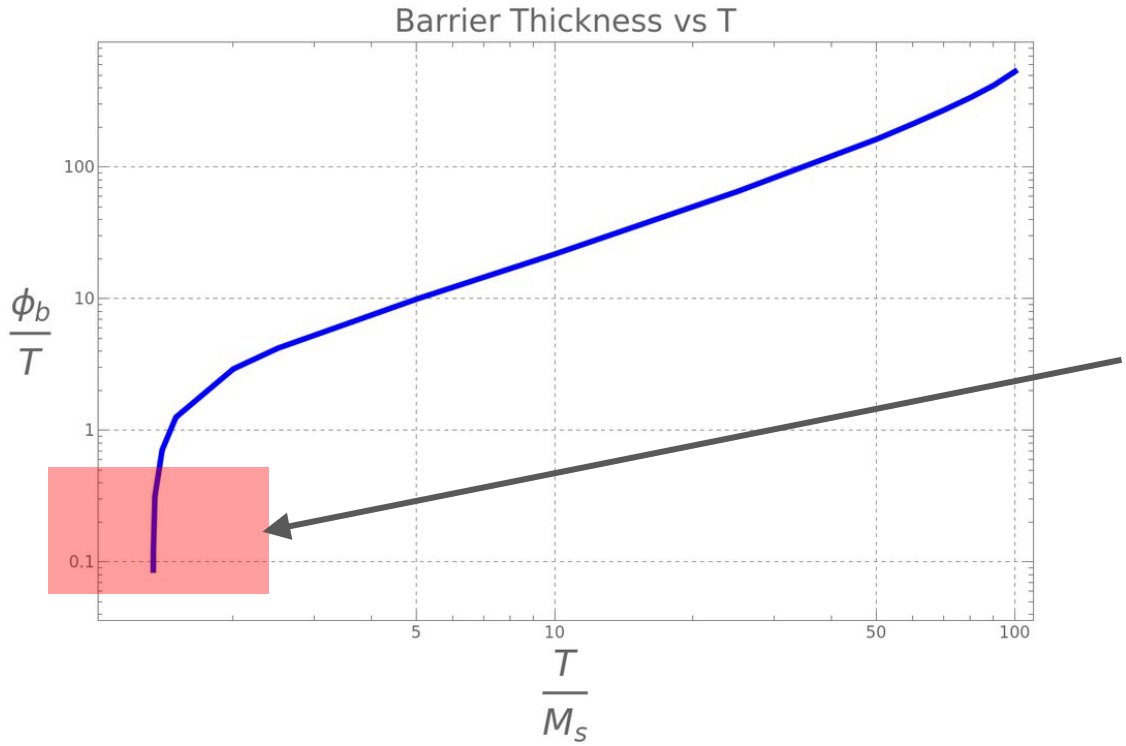


Roughly speaking $\frac{S_3}{T} \simeq \mathcal{O}(100)$ is required for sizeable bubble nucleation.

In such models, this coincides with the period where the thermal barrier has a thickness comparable to temperature (or less).

Barrier size vs temperature

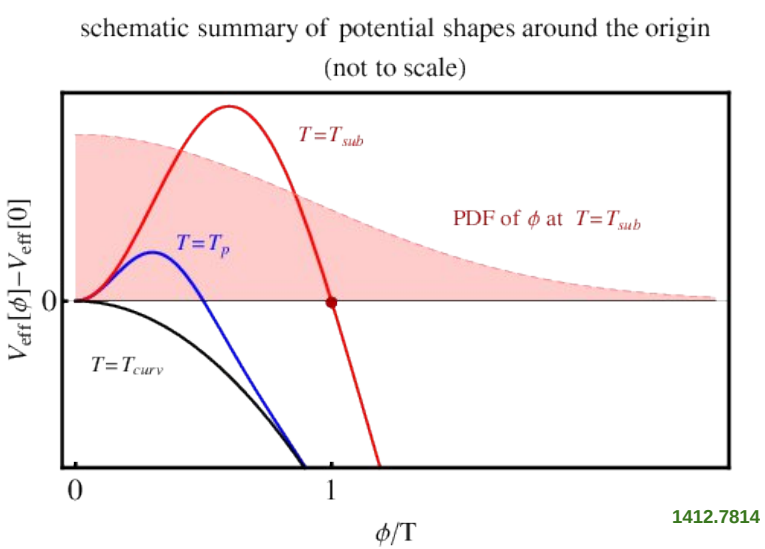
Returning to the barrier size at the epoch of the phase transition.



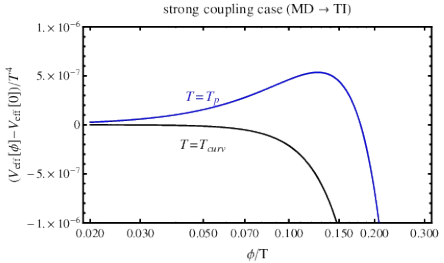
Potential period when bubbles can nucleate

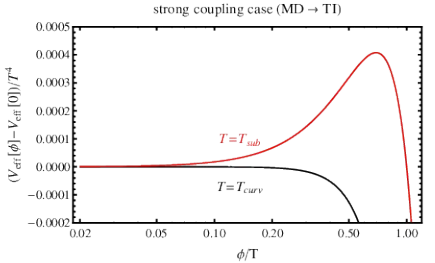
Thermal fluctuations vs bubble formation

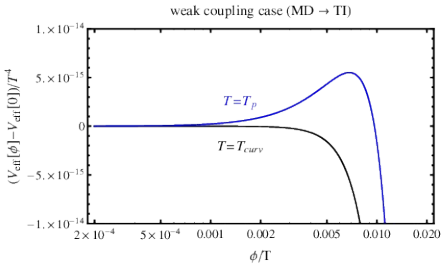
As thermal fluctuations will lead to field-value distributions beyond the thermal barrier, it has been conjectured that bubble formation will not occur.

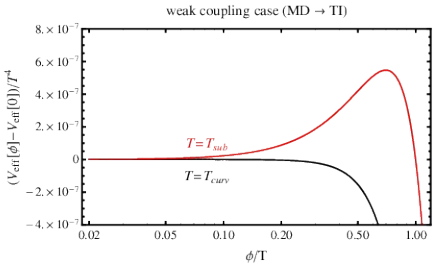


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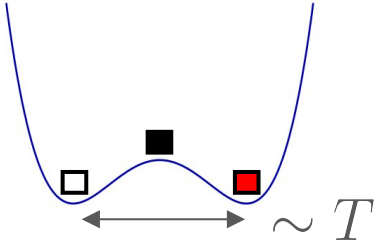


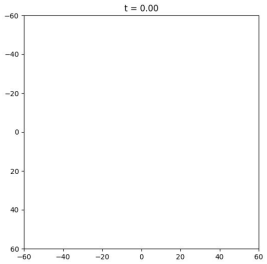


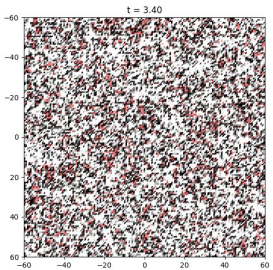


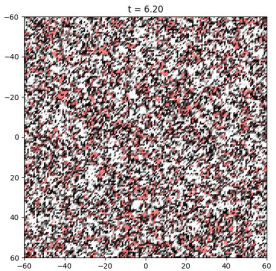
Instead this might lead to something like ‘phase-mixing’ where the field can overcome the barrier without forming critical bubbles.

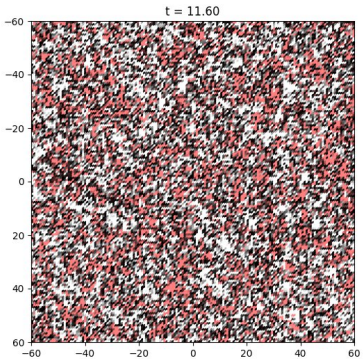
Phase-mixing: An Example

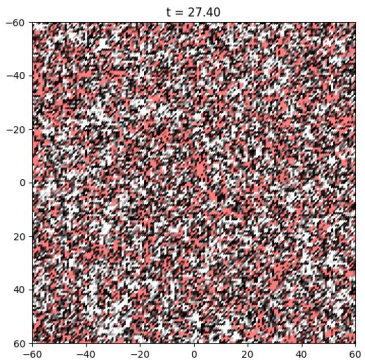








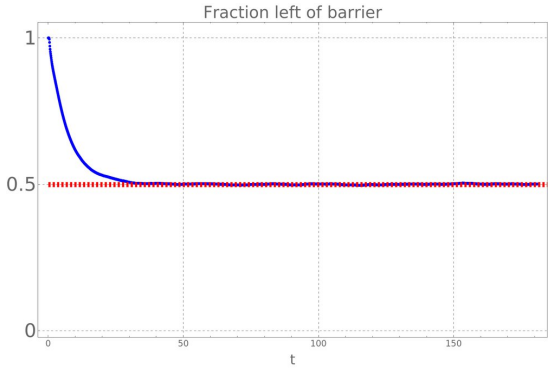




To demonstrate qualitative behaviour of a scenario with phase mixing.

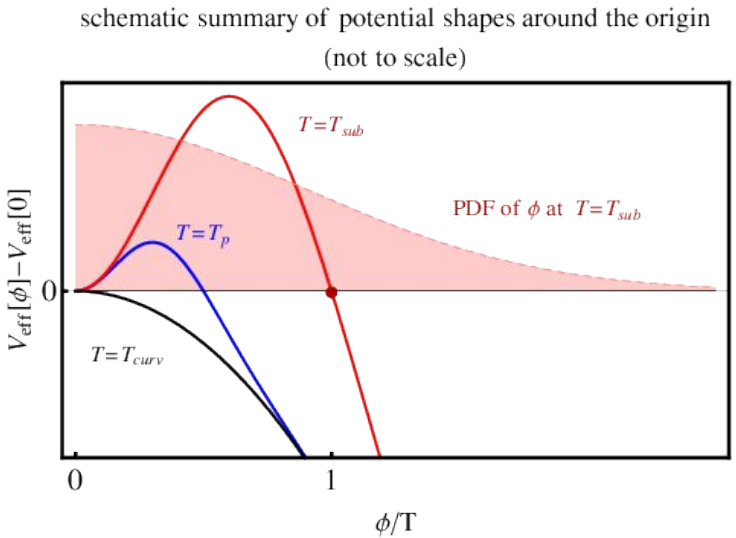
Potential has two degenerate minima with a short (in temperature) distance between them.

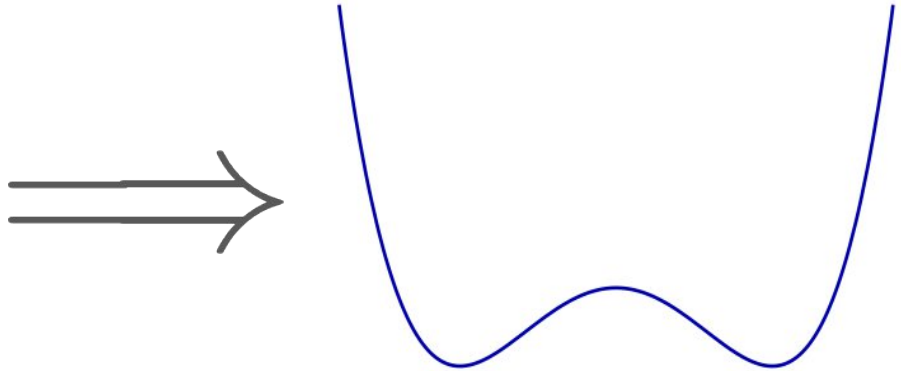
Previous candidate for the EW phase transition. **hep-ph/0410235**



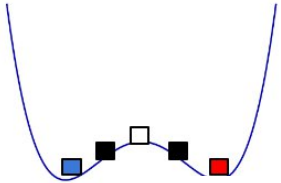
What if the barrier is ignorable?

If the PDF of the field has a significant fraction beyond the barrier, perhaps the barrier can be ignored around the temperature of the phase transition?

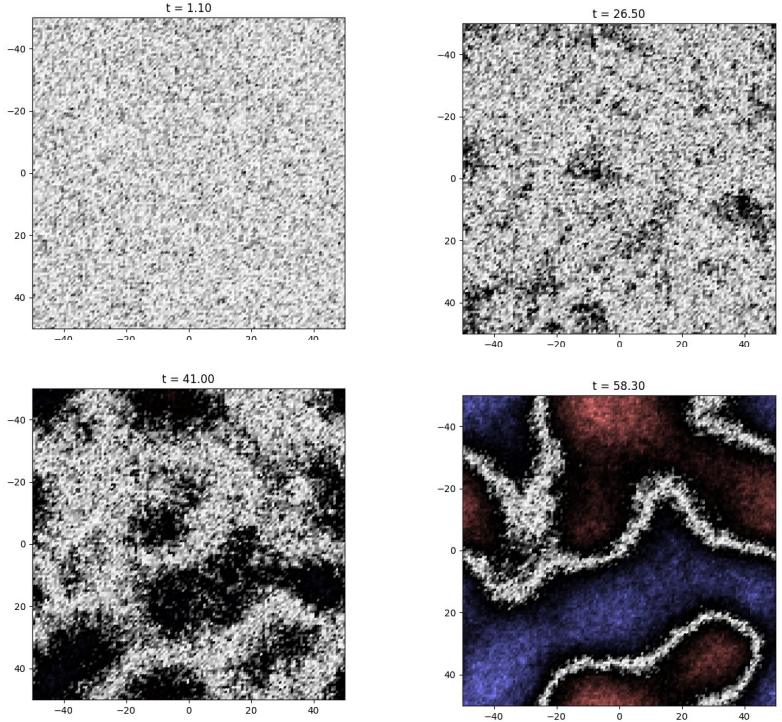




To first approximation, perhaps we can ignore the barrier and we therefore expect a roll-down transition (with some thermal perturbations)..



What if the barrier is ignorable? An example simulation



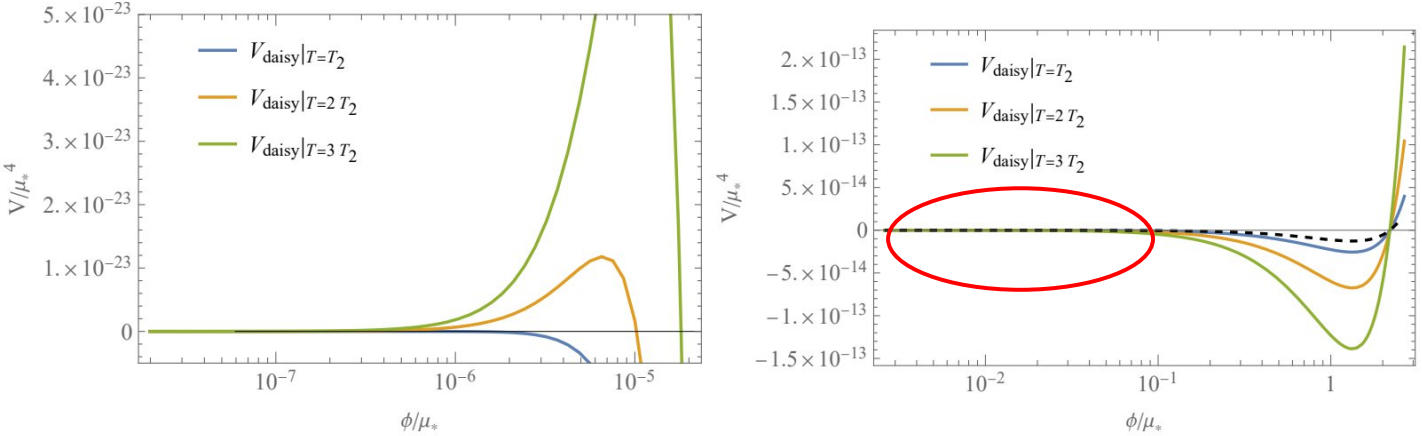
- Simulation of a Mexican Hat pot. **not** the flat pot. of interest
- No bubbles (obviously)
- Homogeneous-like evolution of the field from the initial state at the meta-stable min.
- No bubbles to collide -> suppressed gravitational wave signals

The case for bubble formation

A phase transition proceeding by bubble formation may still be a likely explanation:

- The barrier is not the full story, i.e. Potential is very flat $|V'(\phi)| < T^3\Big|_{\phi<\mathcal{O}(10)T}$

  Field can reasonably be trapped around the origin (even though field value is beyond the barrier thickness), due to thermal random walk, until a critical bubble can form.



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  Field can reasonably be trapped around the origin (even though field value is beyond the barrier thickness), due to thermal random walk, until a critical bubble can form.

- The three-dimensional Euclidean action of the O(3) symmetric bounce solution (critical bubble) remains large e.g.

$$\frac{S_3}{T}\Big|_{\phi_b\simeq 0.1T} > \mathcal{O}(10)$$

  Might naïvely expect that the action should reflect that critical bubble formation becomes ill-defined but still might expect deviations from this prediction.

Simulating thin barriers

Lattice simulation with thin barriers

We wish to address this question by simulating a potential with similar characteristics numerically on a lattice and determine the nature of the phase transition.

Within the simulation we assume we have a scalar field coupled to a thermal bath with temperature T and a potential

$$V(\Phi) = \frac{1}{2}m^2\Phi^2 - \frac{\lambda}{4!}\Phi^4 + \frac{\epsilon}{6!}\Phi^6$$

subject to the constraint

$$m^2 = \frac{\lambda}{12}\phi_b^2 \quad , \quad \phi_b = aT$$

such that the potential barrier width and height is fixed:

$$V(0) = V(\phi_b)$$

.

Lattice simulation with thin barriers

Although this potential has similar characteristics to potentials appearing in thermal inflation the main advantage for lattice simulations is a accurate semi-analytic expression for the bounce action

$$V(\Phi) = \frac{1}{2}\frac{\lambda}{12}\phi_b^2\Phi^2 - \frac{\lambda}{4!}\Phi^4 + \frac{\epsilon}{6!}\Phi^6 \qquad \left(\epsilon T^2 < 1\right)$$

This can allow for (ideally) a controllable and predictable rate of bubble nucleation to compare with the results of the simulation: Linde, 1983

$$\frac{S_3}{T} \simeq 19\sqrt{3}\frac{\phi_b}{\sqrt{\lambda}T}$$

Simulate dynamics through Langevin equation:

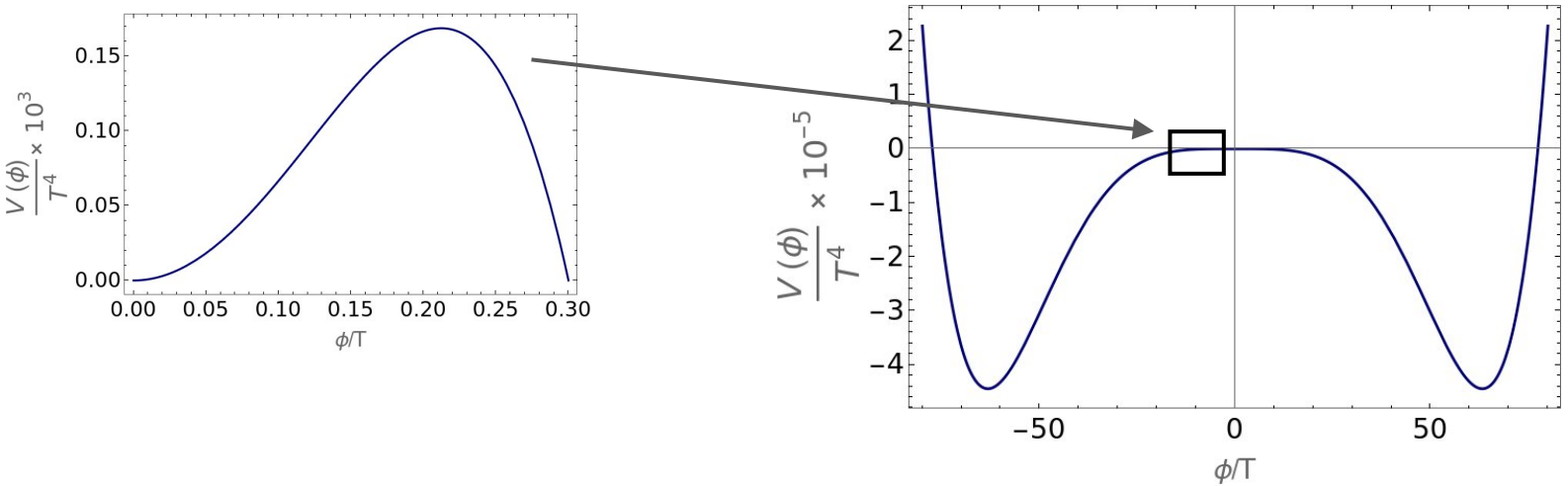
$$\partial_t^2\Phi(\mathbf{x},t) + \eta\partial_t\Phi(\mathbf{x},t) - \nabla^2\Phi(\mathbf{x},t) + \partial_\Phi V(\Phi) = \xi(\mathbf{x},t)$$

Some simulation results

Supercool inspired potential on the lattice

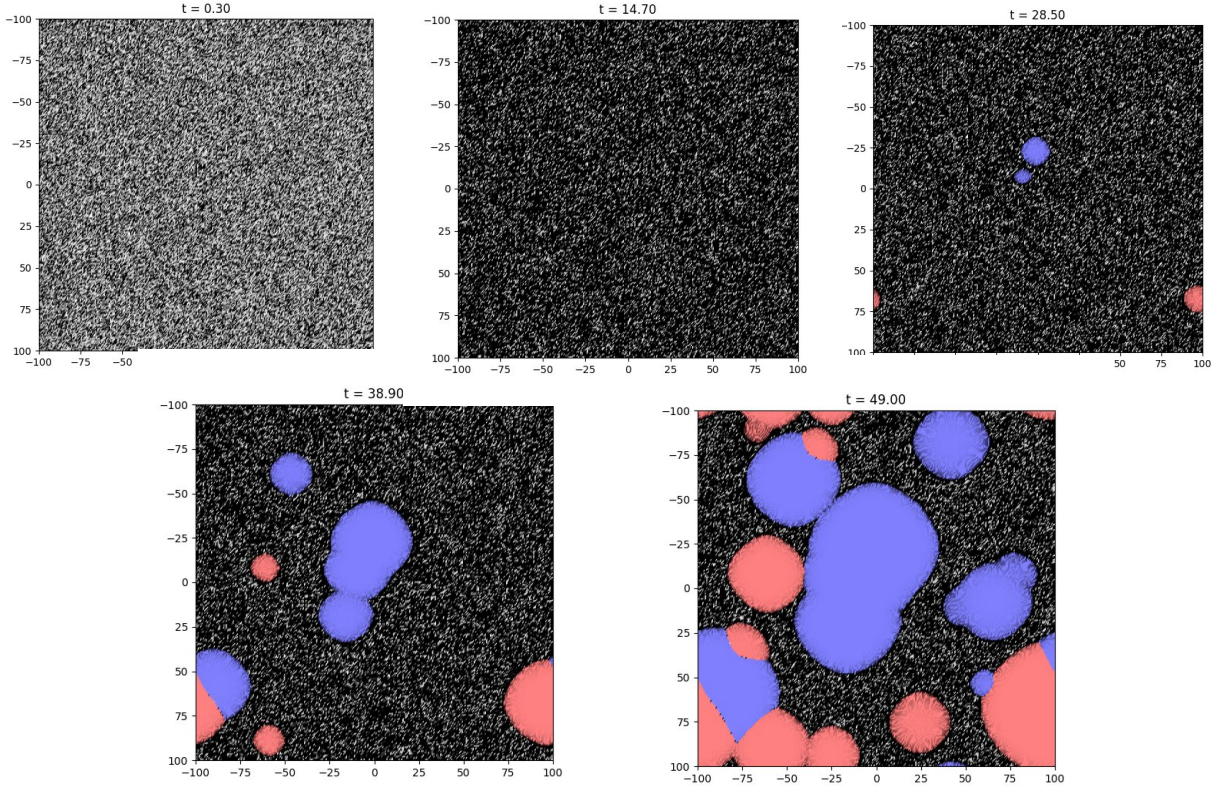
We performed a number of simulations of our toy-model potential taking the potential barrier to be an order of magnitude smaller than the temperature scale of the thermal bath - for different cases

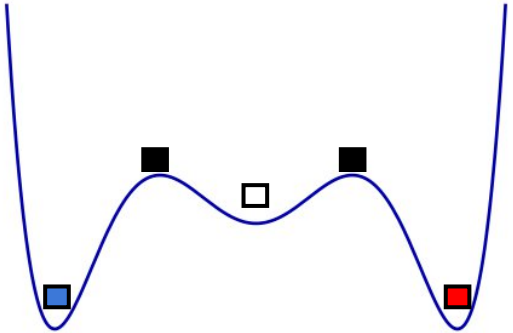
$$\phi_b \sim (0.1 - 0.5)T$$

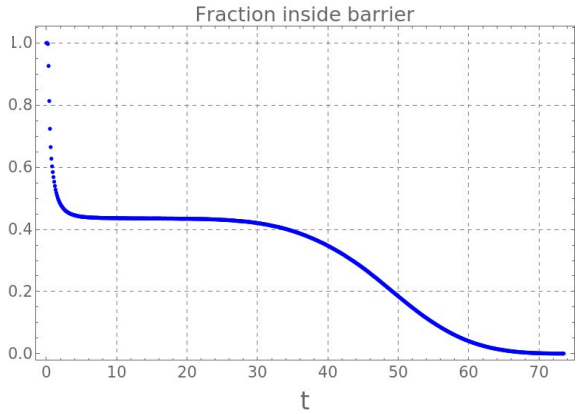


Aside: This toy-potential features two degenerate minima, realistic models should avoid this, e.g. U(1) complex scalar, etc.

Supercool inspired potential on the lattice







Simulation supports bubble nucleation prediction

Supercool inspired potential on the lattice

We performed a number of simulations for different values of the potential parameters with different lattice volumes, spacings, time steps etc

- Although only one simulation result is shown here, they all seem to indicate a phase transition through bubble nucleation

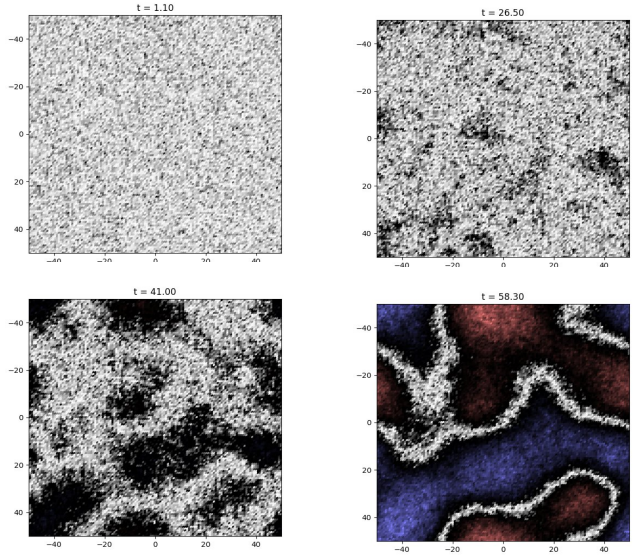
- *Despite* the fact that the barrier is thin and the field has a significant distribution outside of the barrier for large time periods

- For now we are only taking this simulations as a qualitative test of the phase transition

- Good idea to see how the simulation behaves under different deformations of the potential…

Sanity checks: changing simulation

- **No barrier:** Already confirmed simulation in case of no barrier (negative curvature at the origin):



No bubble formation as expected

Sanity checks: changing simulation

- **Different rate:** For given potential we have

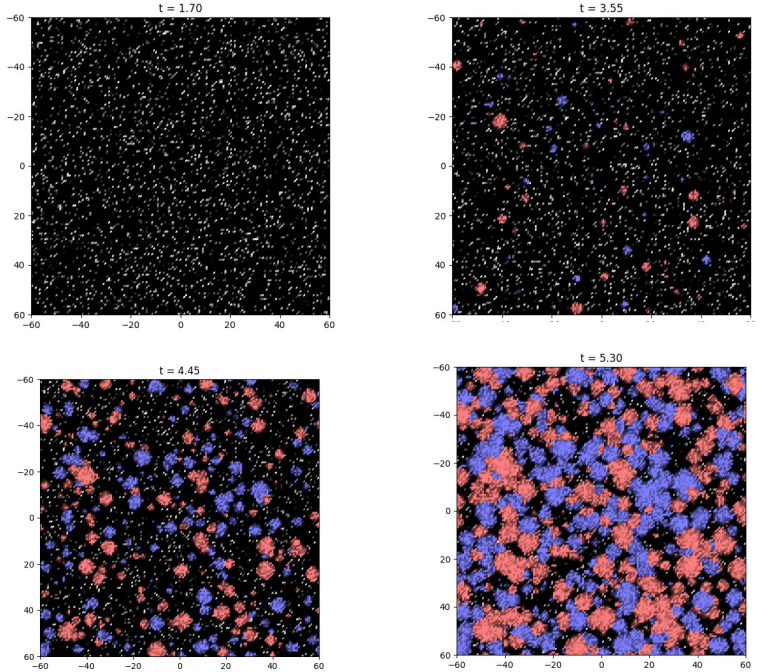
$$\frac{S_3}{T} \propto \frac{1}{\sqrt{\lambda}}$$

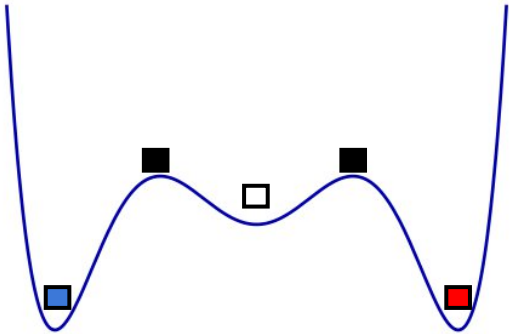
So, e.g. significantly increasing the quartic coupling (O(10)) of this lattice toy-model should significantly decrease simulation time scale (exponentially)

$$\Gamma_n \propto \exp\left(-S_3/T\right)$$

Sanity checks: changing simulation

- **Different rate:**





Significantly faster phase transition, more bubbles, smaller bubbles at collision…

Conclusion

- Our lattice simulations indicate that supercooling with termination have a phase transition proceeded by bubble formation, even with a thin barrier.

- Robust simulation results when lattice dimensions and parameters in the scalar potential are varied.

- Simulation exhibits very good qualitative behaviour compared to the predictions of the bubble nucleation rate for the given potential. Ongoing work to extract accurate numerical predictions from the simulation vs the predictions.

- In future would like to extend these simulations to realistic potentials to obtain information about phase-transition parameters, e.g. bubble radius at collision, no. of bubbles.

Fin.

Backup

Particle Cosmology using Supercooling

Including thermal fluctuations

Introduce thermal fluctuations by incorporating random-walk effects to the equation of motion for the scalar field (Langevin dynamics)

$$\partial_t^2 \Phi(\mathbf{x}, t) + \eta \partial_t \Phi(\mathbf{x}, t) - \nabla^2 \Phi(\mathbf{x}, t) + \partial_\Phi V(\Phi) = \xi(\mathbf{x}, t)$$

with a friction and stochastic noise term:

$$\langle \xi(\mathbf{x}, t) \xi(\mathbf{x}', t') \rangle = D \delta(t - t') \delta^3(\mathbf{x} - \mathbf{x}')$$

related through the fluctuation dissipation theorem

$$D = 2T\eta$$

$$N_{\text{lattice}}^3 = \left(\frac{L}{A} \right)^3 \sim (200 - 400)^3$$

Sanity checks: changing simulation

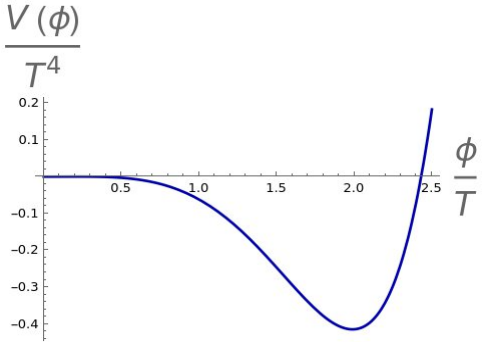
- **Different rate:** For given potential we have

$$\frac{S_3}{T} \propto \frac{1}{\sqrt{\lambda}}$$

So, e.g. significantly increasing the quartic coupling (O(10)) of this lattice toy-model should significantly decrease simulation time scale (exponentially)

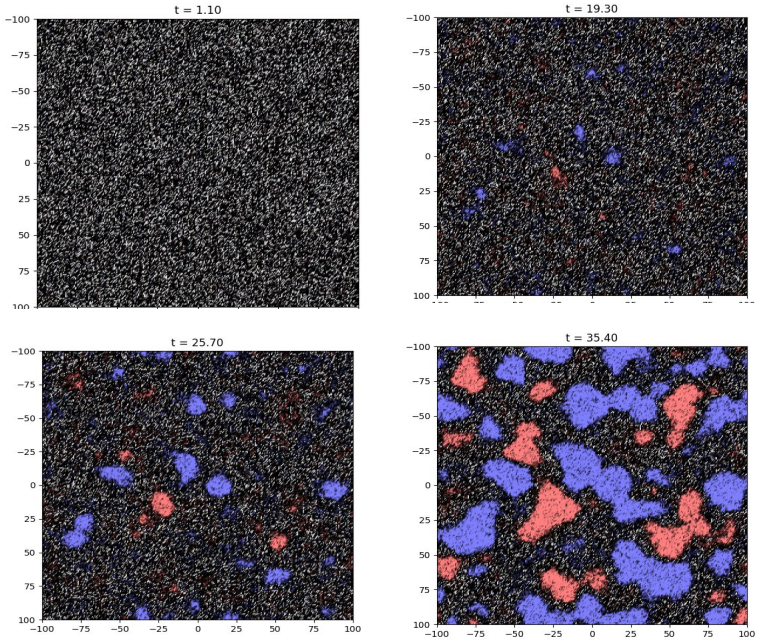
$$\Gamma_n \propto \exp\left(-S_3/T\right)$$

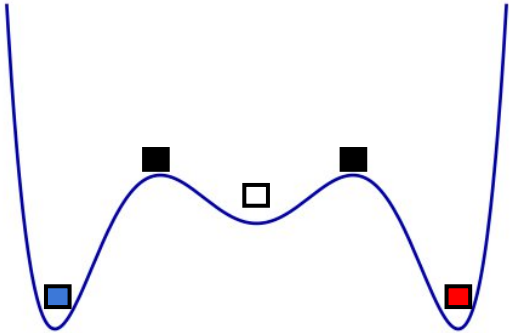
- **Different shape:** What about bringing the meta-stable and stable minimum closer together (and decreasing the pot. difference b/w the two)?

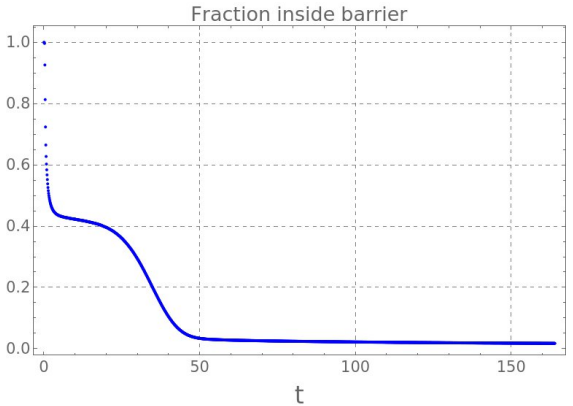


Sanity checks: perturbation of sim.

3. Different shape:





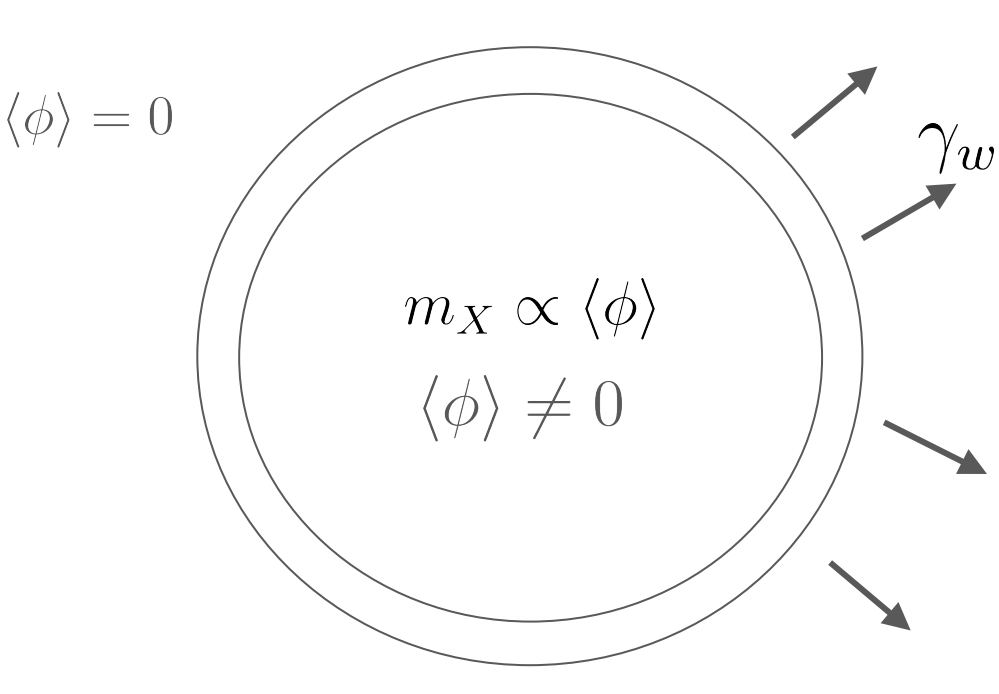


Still bubbles but shape appears affected by thermal effects:

$$\Delta V/T < 1$$

Particle Physics and Supercooling

Define a number of relevant parameters involved in cases where FOPT bubbles interact with plasma particles:



$$T_{\rm crit}$$

$$\downarrow$$

$$T_{\rm perc}/T_{\rm nuc}$$

$$\downarrow$$

$$T_{\rm reh}$$

$$\sim \left(\Delta V + \rho_{\rm plasma}(T_{\rm nuc})\right)^{1/4}$$

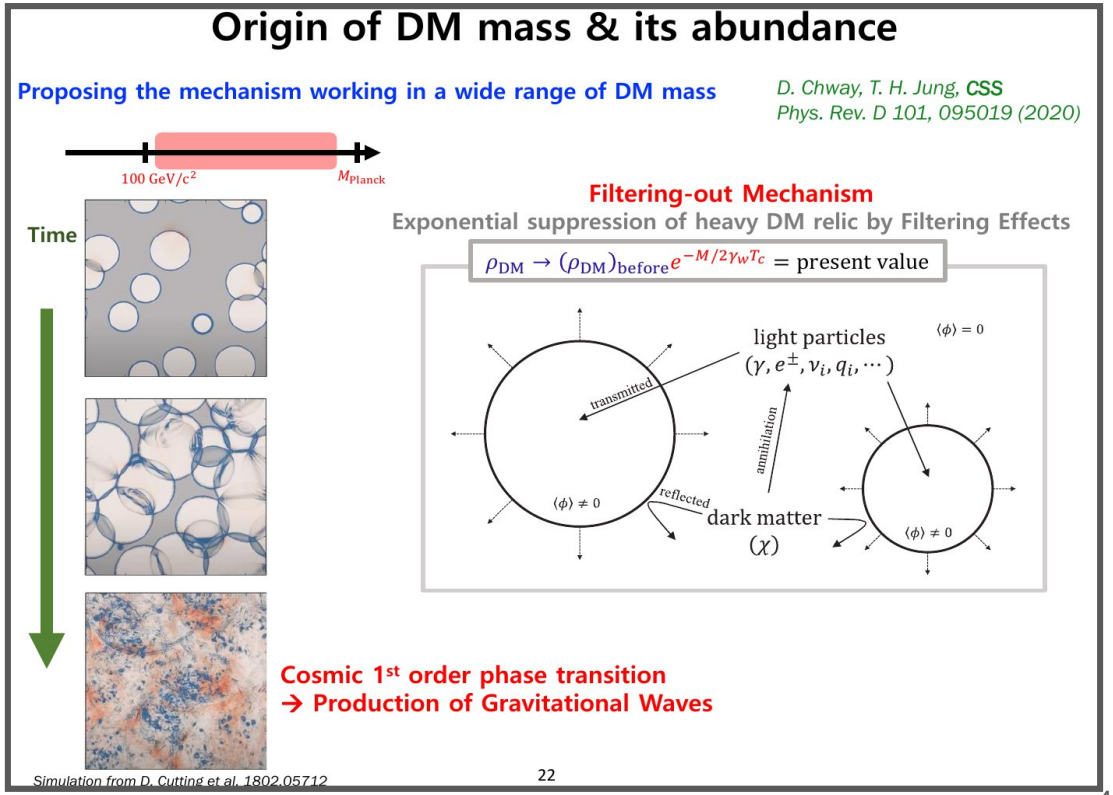
Particle Physics and Supercooling

"**Squeezing**" scenario

$$T_{\rm nuc} < \gamma_w T_{\rm nuc} \ll m_X$$

Only a few DM particles can penetrate the bubble, from the Boltzmann tail.

The remaining one are reflected (squeezed) into false vacuum pockets and eventually disappear.



Slide taken from talk by Chang Sub Shin yesterday

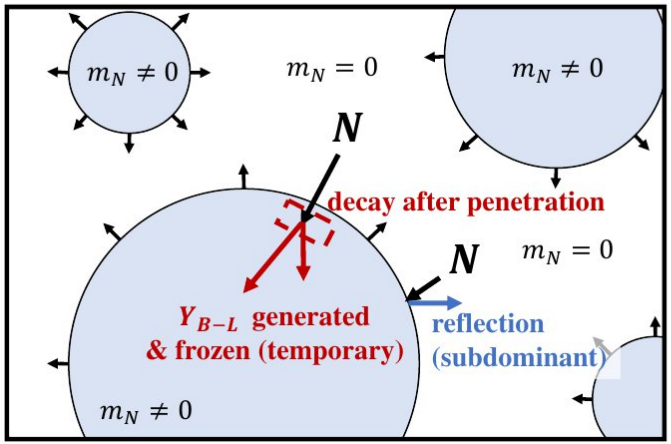
Particle Physics and Supercooling

"**Mass-gain**" scenario

$$T_{\rm perc} < m_X < \gamma_w T_{\rm perc}$$

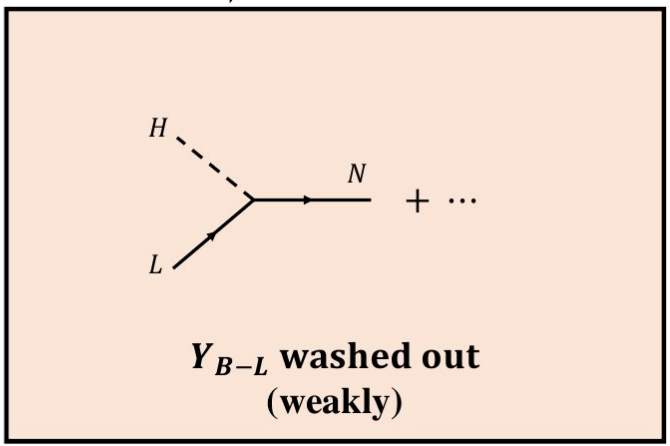
$$Y_X \propto \left(\frac{T_{\rm nuc}}{T_{\rm reh}}\right)^3$$

$T = T_{\rm nuc}$



Bubble collisions

$T = T_{\rm reh}$ dilution $= (T_{\rm nuc}/T_{\rm reh})^3$



Chun, TPD, Jung, Nagels, Vanvlasselaer

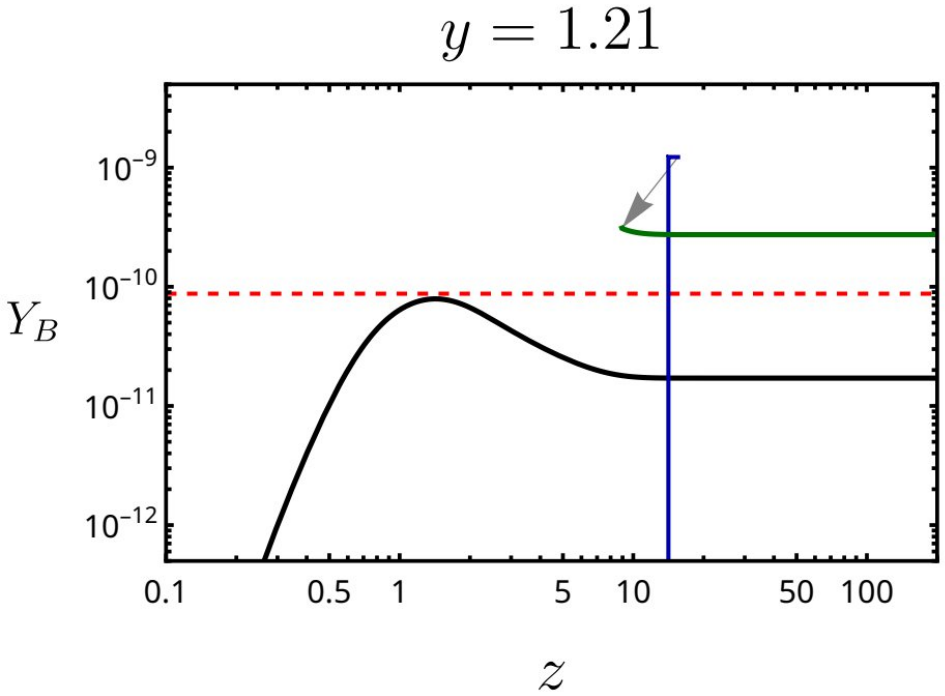
Particle Physics and Supercooling

“Mass-gain” scenario

RHNs can decay without wash out, different to typical scenario in strong-washout leptogenesis.

Different parameter space for leptogenesis + potential of gravitational wave observations of this FOPT.

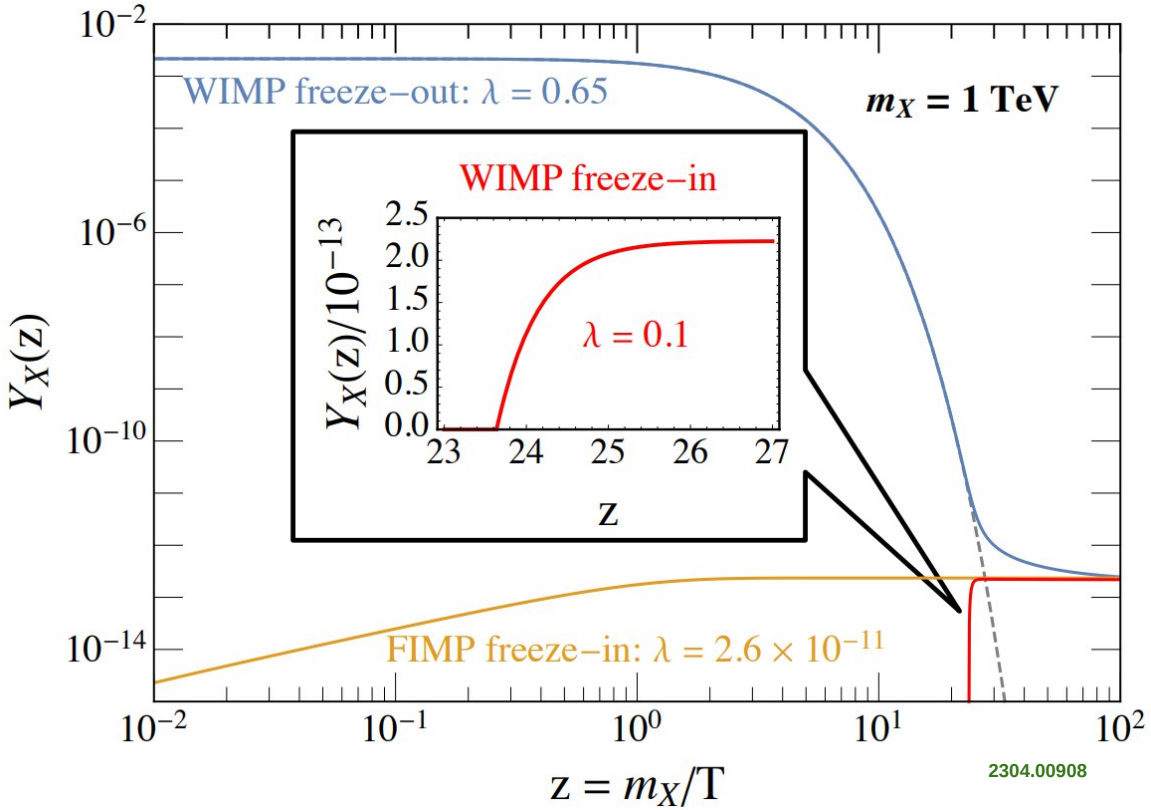
Not a smoking gun of leptogenesis but interesting nonetheless.



Chun, TPD, Jung, Nagels, Vanvlasselaer

Particle Physics and Supercooling

Dilution scenario (beyond the usual e-folding situation)



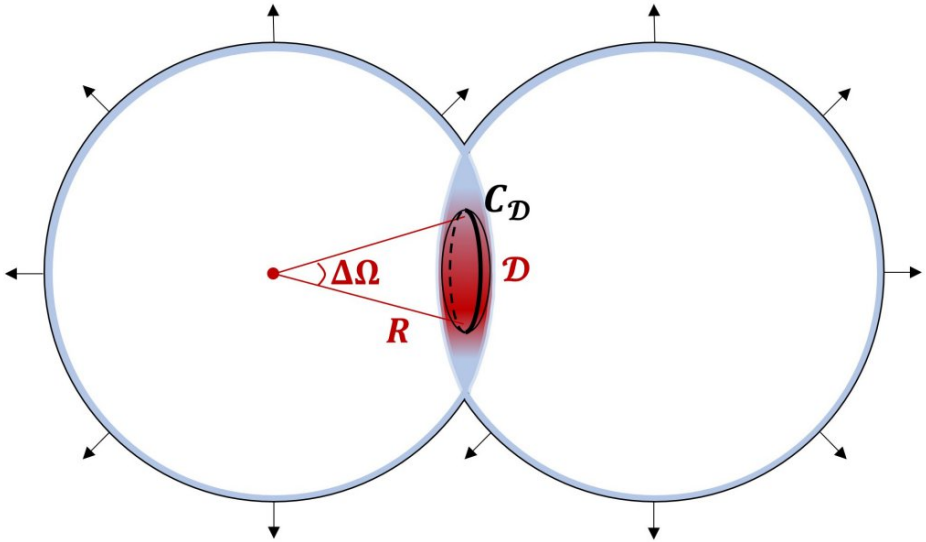
Freeze-in DM model with strong couplings:

Strong FOPT removes any pre-existing thermal DM density

reheating: $Y_X \propto \left(\frac{T_{\rm nuc}}{T_{\rm reh}}\right)^3$

Particle Physics and Supercooling

PBH formation: a number of proposals



2110.04271

Collapse of overdense regions related to inhomogeneous vacuum decay

DM candidate with a motivated production mechanism

2106.05637, 2210.14094, 2212.14037

Case Study:

Consider now, as an example, a SUSY motivated picture of thermal inflation with some soft SUSY breaking scale,

$$W = \sqrt{2}\lambda\hat{\Phi}\hat{X}\hat{\bar{X}}$$

$$V_{\rm SUSY} = 2\lambda^2\left[|\Phi|^2\left(|X|^2 + \left|\bar{X}\right|^2\right) + |X|^2\left|\bar{X}\right|^2\right]$$

$$V_{\cancel{\rm SUSY}} = M_S\left(\lambda\, A\bar{X}X\Phi + \text{h.c.}\right) + M_S^2\left(g_\Phi|\Phi|^2 + g_X|X|^2 + g_{\bar{X}}\left|\bar{X}\right|^2\right)$$

$$V_{\rm CW}(\phi) = \frac{1}{64\pi^2}\left(4\left(g_X M_S^2 + \lambda^2\phi^2\right)^2\left(\log\left[\frac{g_X M_S^2 + \lambda^2\phi^2}{\mu_*^2}\right] - \frac{3}{2}\right) - 4\left(\lambda^2\phi^2\right)^2\left(\log\left[\frac{\lambda^2\phi^2}{\mu_*^2}\right] - \frac{3}{2}\right)\right)$$

= 0 implies exactly flat potential (SUSY symmetry)

Case Study:

Consider now a SUSY motivated picture of thermal inflation with some soft SUSY breaking scale,

$$V_{\rm CW}(\phi) = \frac{1}{64\pi^2}\left(4\left(g_X M_S^2 + \lambda^2\phi^2\right)^2\left(\log\left[\frac{g_X M_S^2 + \lambda^2\phi^2}{\mu_*^2}\right] - \frac{3}{2}\right) - 4\left(\lambda^2\phi^2\right)^2\left(\log\left[\frac{\lambda^2\phi^2}{\mu_*^2}\right] - \frac{3}{2}\right)\right)$$

$$\downarrow$$

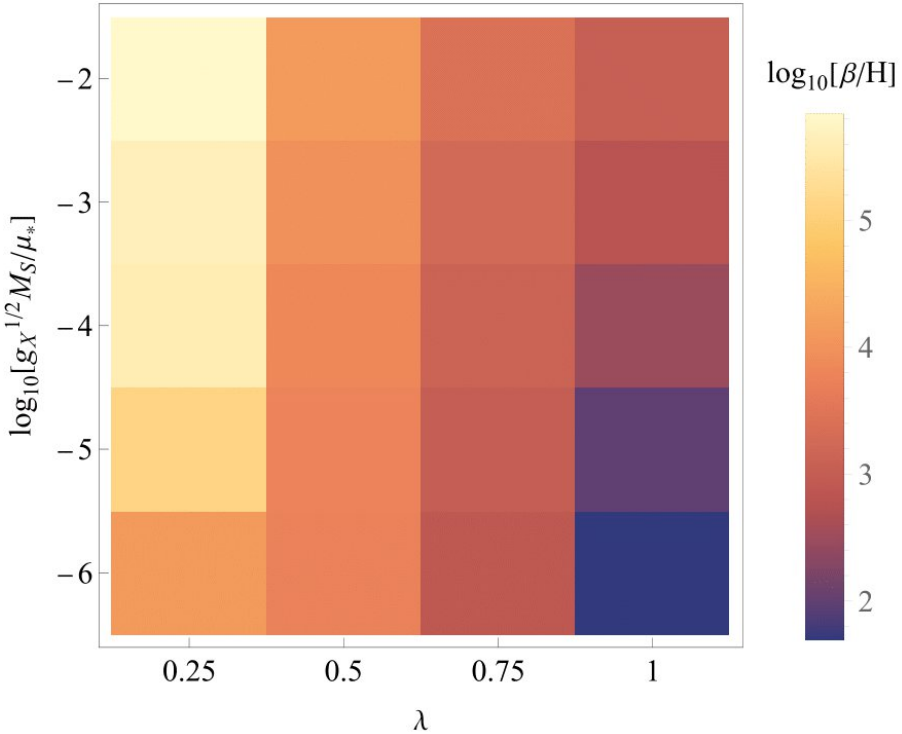
$$v_\phi = \langle\phi\rangle \simeq \frac{e^{1/4}}{\lambda}\mu_* \qquad \Delta V \simeq \frac{e^{1/2}}{8\pi^2} g_X M_S^2 \mu_*^2$$

$$\downarrow$$

$$\frac{\Delta V}{v_\phi^4} \simeq \frac{\lambda}{8\pi^2 e^{1/2}} \frac{g_X M_S^2}{\mu_*^2} \ll 1$$

Aside: The importance of bubbles

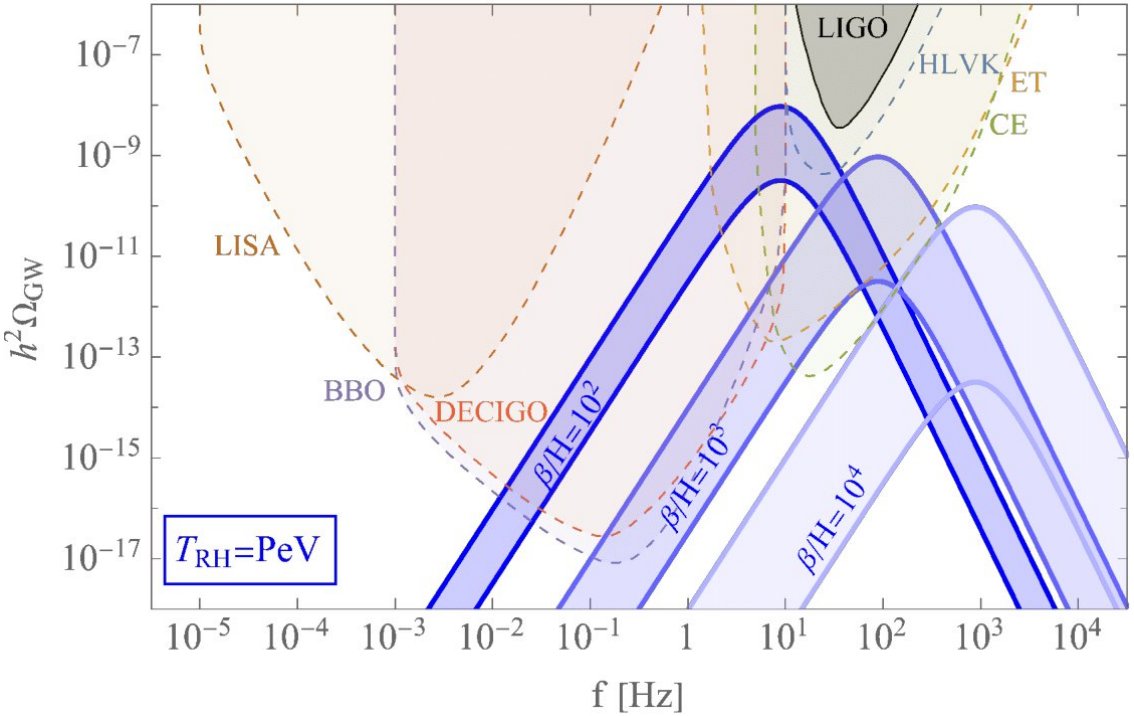
Assuming a bubble nucleation hypothesis, can estimate the gravitational wave signature, (this is really dependent on specific models so should just take this as a indication)



Time scale of transition quite sensitive to changes in the coupling.

Aside: The importance of bubbles

Assuming a bubble nucleation hypothesis, can estimate the gravitational wave signature, (this is really dependent on specific models so should just take this as a indication)



Indicates that detectable gravitational wave signals are feasible **<u>if a bubble nucleation hypothesis is valid</u>.**

So it's important to understand if this should be expected.

Lattice simulation with thin barriers

Although this potential has similar characteristics to potentials appearing in thermal inflation the main advantage for lattice simulations is a accurate semi-analytic expression for the bounce action

$$V(\Phi) = \frac{1}{2}\frac{\lambda}{12}\phi_b^2\Phi^2 - \frac{\lambda}{4!}\Phi^4 + \frac{\epsilon}{6!}\Phi^6 \qquad \left(\epsilon T^2 < 1\right)$$

This can allow for (ideally) a controllable and predictable rate of bubble nucleation to compare with the results of the simulation: Linde, 1983

$$\frac{S_3}{T} \simeq 19\sqrt{3}\frac{\phi_b}{\sqrt{\lambda}T}$$

Aside: There is also a difficult to calculate numerical prefactor entering in the rate which we find using numerical packages 2308.15652

$$\log(\Gamma_{\rm nuc}) = \log(A) - \frac{S_3}{T}$$

Some simulation details

In order to efficiently numerically evolve Φ in time we perform a three-dimensional simulation of a 3-dimensional effective field theory containing the light bosonic field(s) (the zero Matsubara mode), which we write as ϕ with mass-dimension 1, with the remaining heavy fields integrated out

The parameters within the dimensionally reduced theory, from herein denoted as X3, are related to the 3 + 1 dimensional quantum field theory through powers of temperature, e.g.

$$m_3^2 = m^2$$
$$\lambda_3 = \lambda T$$
$$\epsilon_3 = \epsilon T^2$$

The other relevant lattice implementations within the Langevin simulation are

$$\nabla^2\phi(\mathbf{x}) = \frac{1}{12a^2}\sum_i \left(-\phi(\mathbf{x}-2i) + 16\phi(\mathbf{x}-i) - 30\phi(\mathbf{x}) + 16\phi(\mathbf{x}+i) - \phi(\mathbf{x}+2i)\right.$$

$$\langle\xi(\mathbf{x},t)\xi(\mathbf{x}',t')\rangle = D\delta(t-t')\delta^3(\mathbf{x}-\mathbf{x}') \overset{\text{lattice}}{\to} \frac{D}{(\Delta t)a^3}\delta_{x_i,x_j}\delta_{t_a,t_b} \qquad \xi(\mathbf{x}_i,t_a) = \sqrt{\frac{D}{\Delta t a^3}}\mathcal{G}_{i,a}$$

Some 3D-simulation specifics

Potential shape characteristics are related to the temperature of the thermal bath so the lattice values are also chosen as a function of temperature. E.g.

$$\delta t \sim \frac{\mathcal{O}(0.01)}{T} \qquad a \sim \frac{\mathcal{O}(0.5)}{T} \qquad L \sim \frac{\mathcal{O}(100)}{T}$$

In general we perform a number of different simulations with

$$N^3_{\text{lattice}} = \left(\frac{L}{A}\right)^3 \sim (200 - 400)^3$$

with good agreement between different simulations of the same potential.

We start each simulation with the unphysical initial condition

$$\phi(\mathbf{x}, 0) = 0, \dot{\phi}(\mathbf{x}, 0) = 0$$

but we find the time for thermalisation around the origin is always faster than the (eventual) transition.

Fast equilibration

The friction coefficient is relevant for the timescale for the simulation to reach equilibrium which we expect to be related to temperature (a strong enough coupling to thermal bath is required anyway for supercooling).

$$\eta \sim T$$

Implies that the simulation equilibration time scale should be significantly faster than the expansion rate

$$\frac{t_{\text{equil}}}{\text{Hubble}} \sim \frac{\eta^{-1}}{H^{-1}} \ll 1$$

In both a radiation dominated or vacuum dominated scenario. A fairly robust assumption for most choices of friction coefficient.

Therefore we neglect the effect of Hubble expansion within the simulation.

Some simulation details

Mapping the 4D continuum theory to this 3D simulation requires some care due to UV divergences associated with the lattice spacing: a

$$V(\Phi) \to V_3(\phi) = \frac{1}{2} Z_\phi Z_m (m_3^2 + \delta m_3^2)\phi^2 - \frac{1}{4!} Z_\phi^2 (\lambda_3 - \delta\lambda_3)\phi^4 + \frac{1}{6!} Z_\phi^3 (\epsilon_3 + \delta\epsilon_3)\phi^6$$

$$\nabla^2 \Phi \to Z_\phi \nabla^2 \phi$$

$$\delta m_3^2 = -\frac{\Sigma \lambda_3}{8\pi a} + \mathcal{O}(\lambda_3^2 a^0)$$

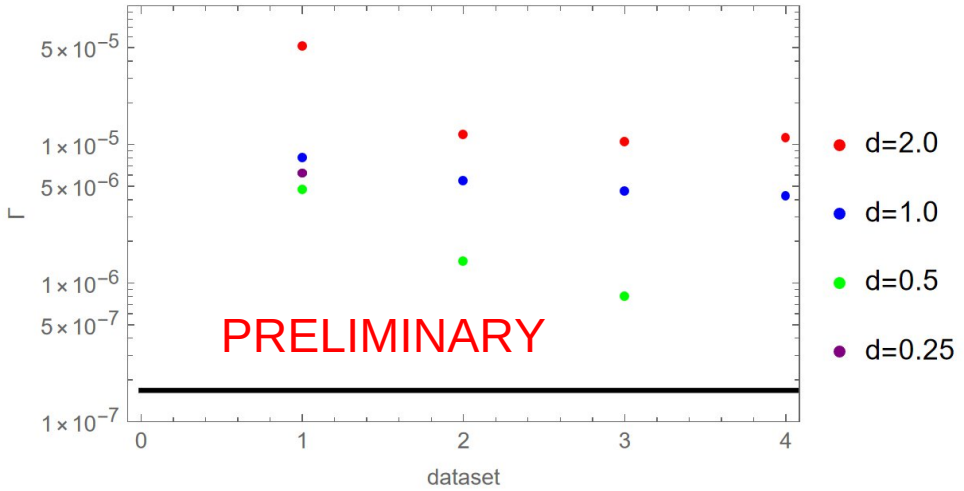
$$\delta\lambda_3 = -\frac{\Sigma \epsilon_3}{8\pi a} + \mathcal{O}(\epsilon_3^2 a^0) + \mathcal{O}(\lambda_3^2 a)$$

$$Z_\phi = 1 + \mathcal{O}(a^2) \qquad Z_m = 1 + \mathcal{O}(a) \qquad \delta\epsilon_3 = 0 + \mathcal{O}(\epsilon_3^2 a)$$

This ensures consistent equilibrium behaviour for the scalar field in the continuum limit (a->0)

Future Aspirations

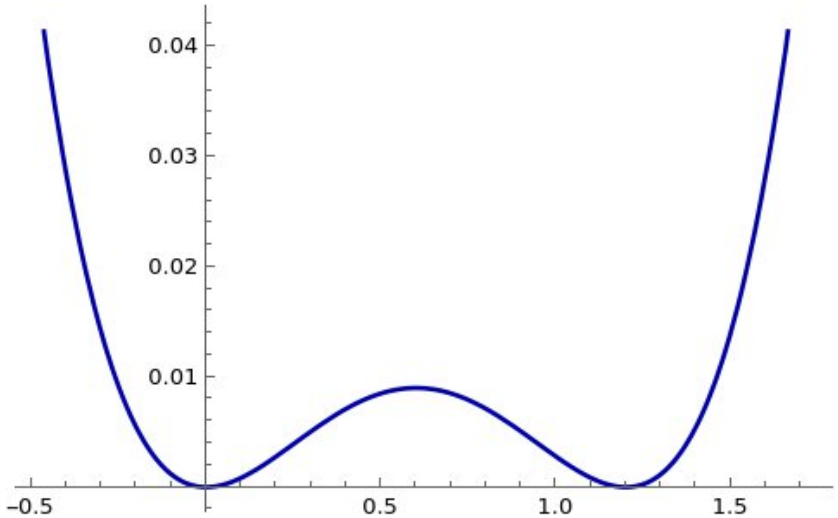
- There is current work in progress to use these simulations to estimate and compare the numerical phase transition dynamics compared to the analytic predictions.
- Currently the simulation has very positive behaviour as a function of the potential shape -> Taking potentials with slower nucleation -> bubble formation takes parametrically longer. Stability of the simulation for different lattice choices, etc.
- Ongoing improvements..



Phase-mixing case

Simulate a ‘Higgs-like’ case at the critical temperature:

$$V(\phi) = \frac{1}{2}m^2\phi^2 - \frac{1}{3!}\phi^3 + \frac{1}{4!}\frac{T^2}{3m^2}\phi^4$$



$$m^2 = \frac{T^2}{5}$$

Lattice results (supercooling case)

