Holography and Defects

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(Look here for detailed references)

Based on arXiv:2405.06014 & 2408.11088, with Igal Arav, Jerome P. Gauntlett, Yusheng Jiao, Christopher Rosen

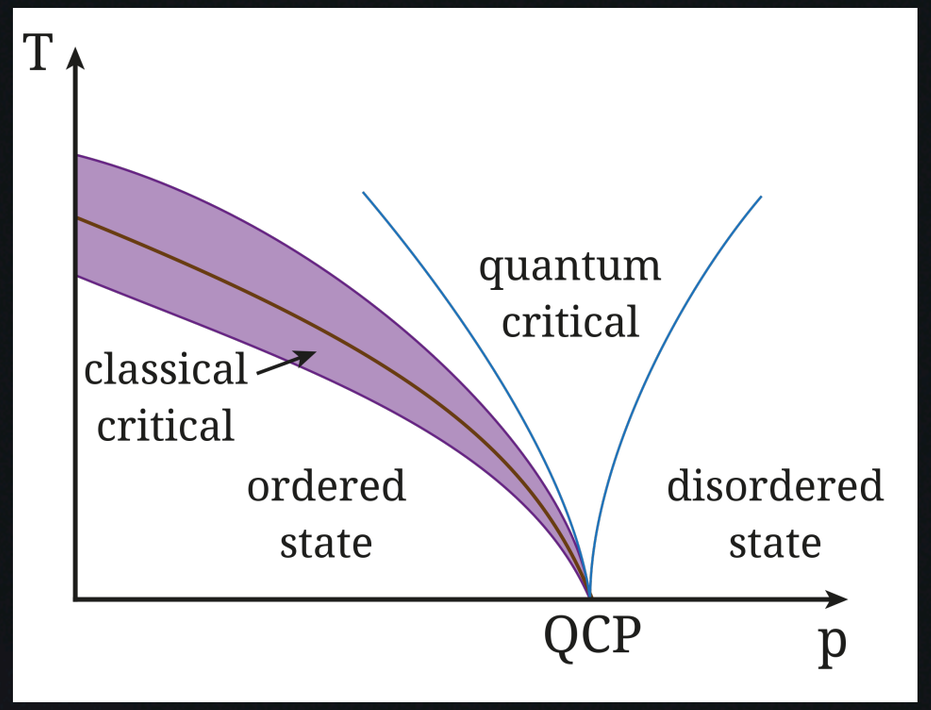
High1 Workshop on Particle, String and Cosmology, January 13, 2025

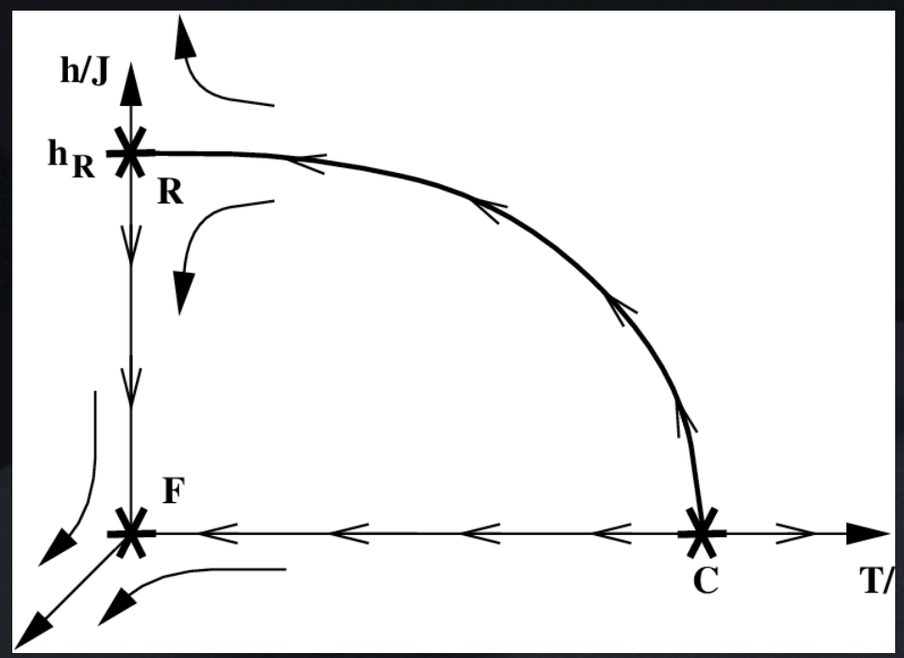
Supersymmetric magnetic defects

- Renormalization Group (RG) Flows and Fixed Points
- Defects and RG flows
- Magnetic defects
  - Supersymmetric magnetic defects
  - Dynamics of the defects
- RG flows between defect theories

Renormalization group flow

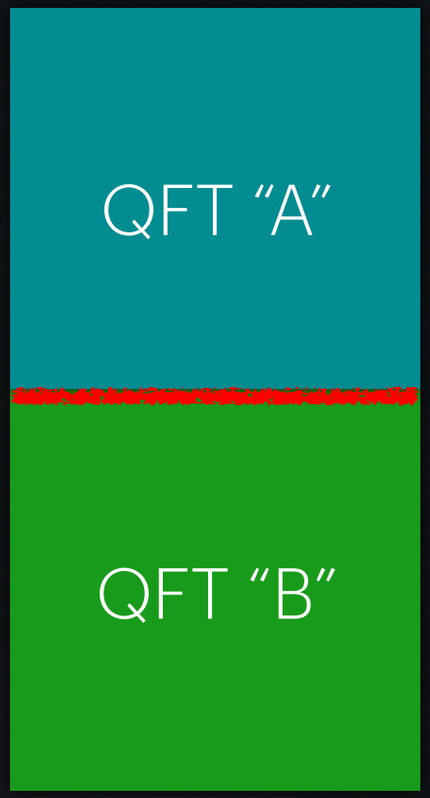
- Consider interacting d.o.f. with various scales (interaction energies, masses, etc)
- In deep IR limit, we are left with a scale invariant theory: Conformal field theory
- Given a CFT, one can consider deforming the theory by a relevant operator (adding mass, transverse B field, ...)
- $$\Delta H = g \int d^d x \mathcal{O}(x)$$
- $$[D, \mathcal{O}] = i\Delta\mathcal{O}, \qquad \frac{d-2}{2} \leq \Delta \leq d,$$
- $$[g] = d - \Delta, \qquad 0 \leq [g] \leq \frac{d+2}{2}$$
- δ introduces a scale, and drives an "RG flow" leading to new fixed point in deep infrared

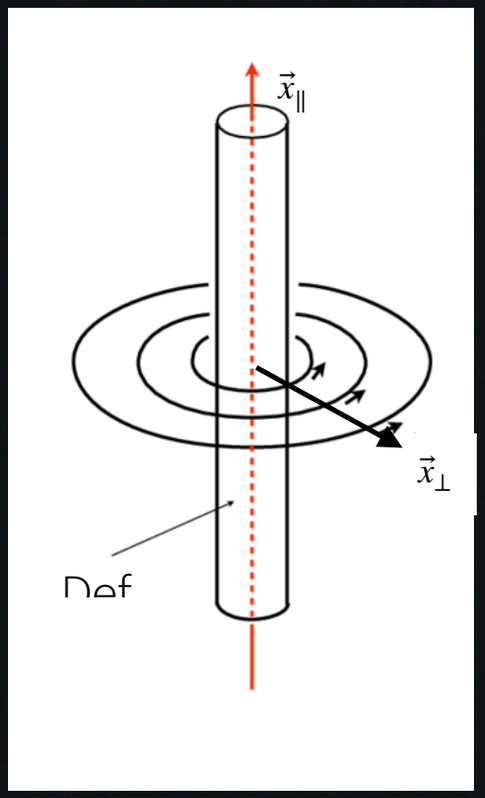


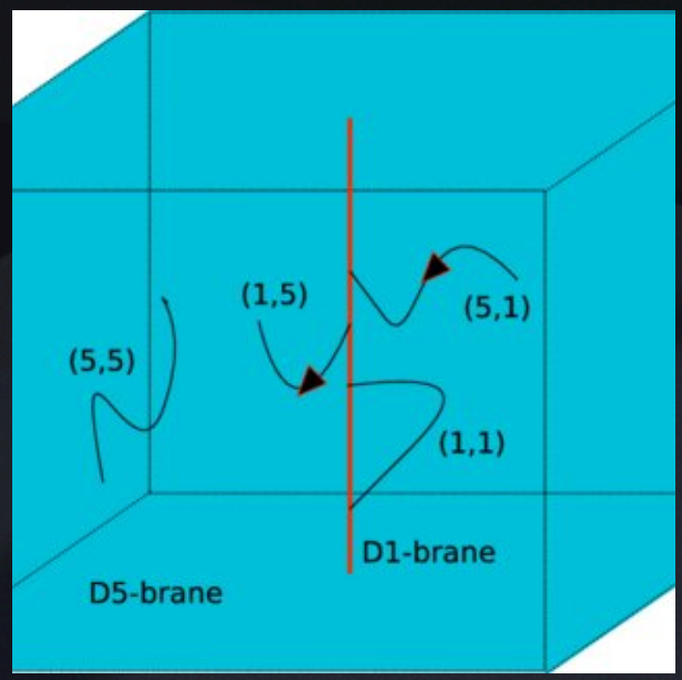


Renormalization group flow

- We know a lot about RG flows when things are Poincare invariant - what if they *aren't*?
  - Gluing two different systems together along a shared boundary
  - Lower dimensional system interacting with bulk ambient theory
  - Line defect inserted into larger theory
- How do RG flows behave here? What types of scale invariant fixed points?
- Can we generalize $c/F/a$ theorems to mixed dimensional systems? (monotonic measures of # d.o.f.)

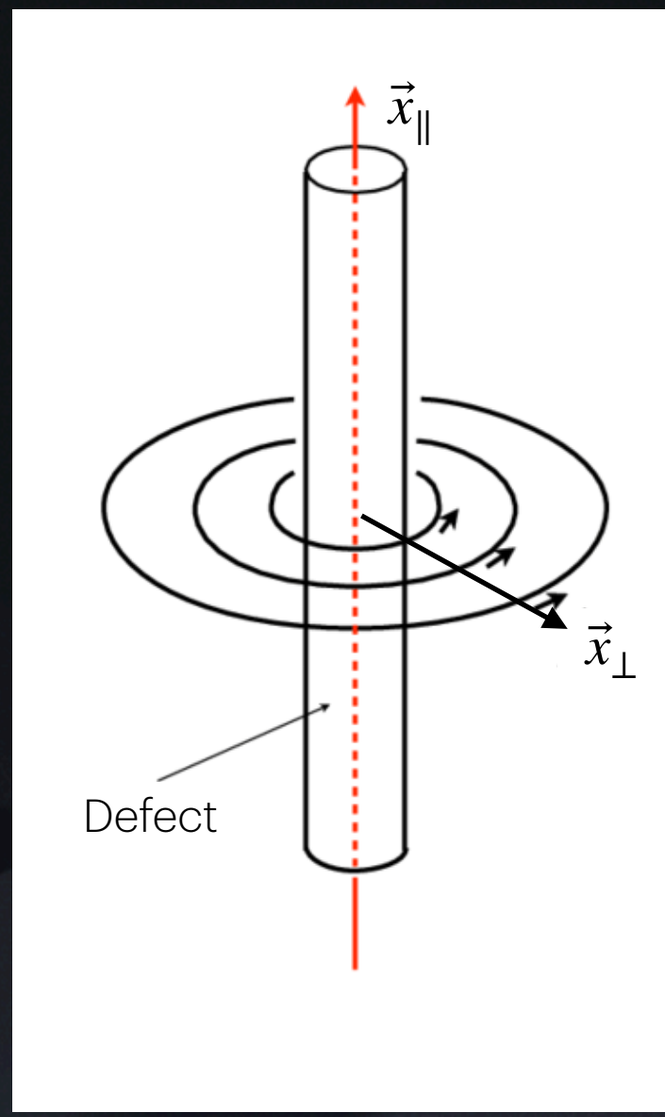






Renormalization group flow

- Add defect (b.c., new d.o.f., ...) to $d_{\|}$-dim'l subspace Σ in a QFT
- Take this whole system and flow to the infrared, resulting in a new mixed-dimensional scale invariant system
- If ambient space and defect are both flat, then
  - $SO(d,2) \to SO(d_{\|},2) \times SO(d_{\perp}), \qquad d_{\|} + d_{\perp} = d$
- Lower dimensional system is not standard CFT, as there is no conserved stress tensor, $\nabla_{\mu} T^{\mu i} = \delta(\Sigma) \hat{D}^{i} \neq 0$
- New "displacement" operator $\hat{D}^{i}$ (literally, moving the defect)
- One-point functions can be nonzero, $\langle T \rangle \sim \frac{h_D}{|x_{\perp}|^d}, \langle \mathcal{O}_{\Delta} \rangle \sim \frac{\alpha}{|x_{\perp}|^{\Delta}},$
- Mixed system, with ambient (bulk) and defect operators



Magnetic defects

- Magnetic defects in 3+1d system: Consider magnetic flux for a background global symmetry

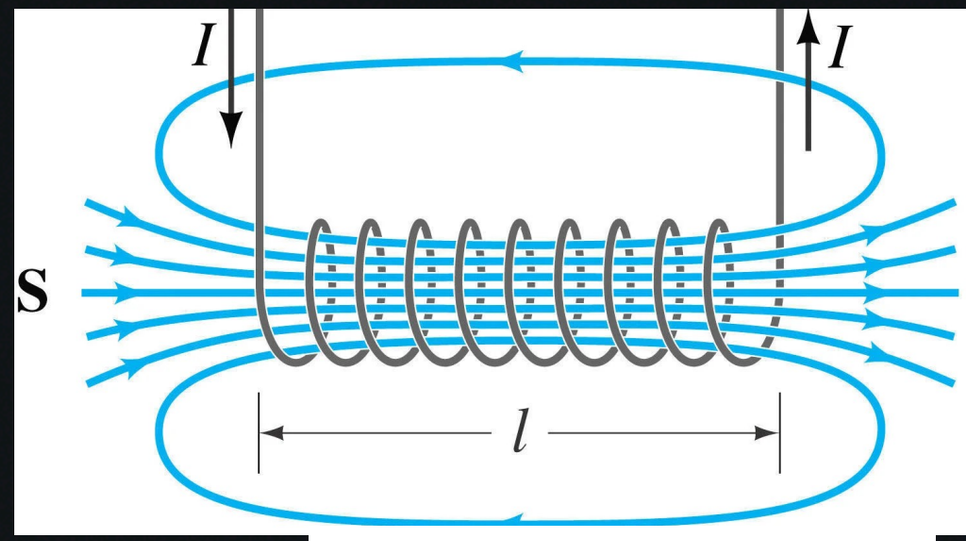
- In the limit of infinitely thin solenoid, $(\hbar = e = 1)$

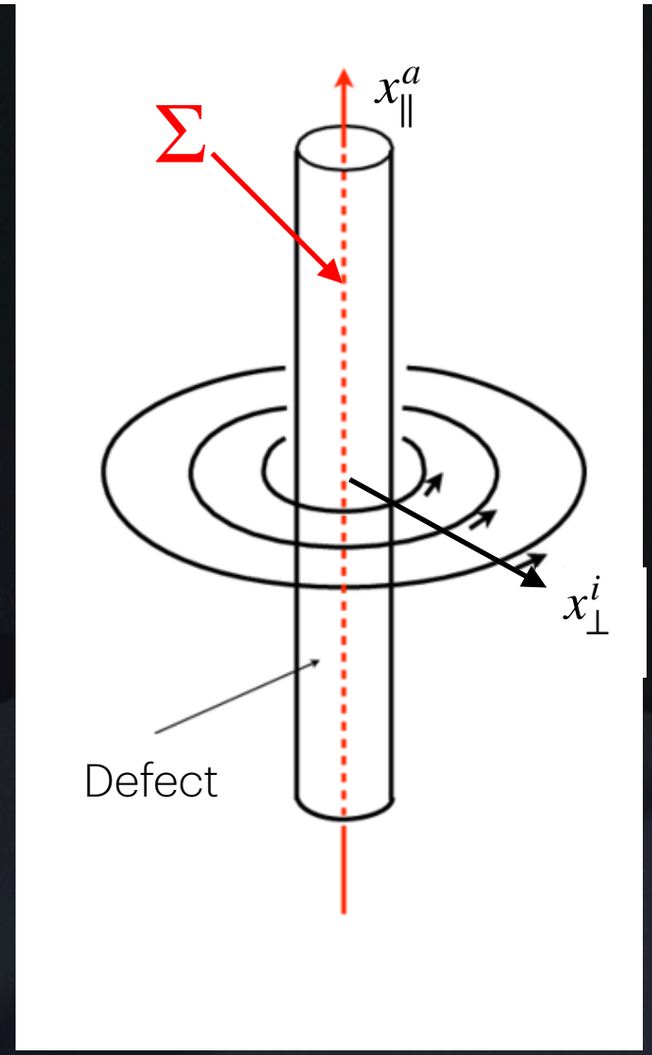
- $A = \mu d\theta,\ F = 2\pi\ \mu\ \delta(\Sigma) dx \wedge dy$

- $$\frac{1}{2\pi}\int F = \frac{1}{2\pi}\oint A = \mu$$

- A.B. phase: $g = e^{2\pi i \mu}, \qquad \mu \sim \mu + 1$

- Scale invariant: μ is a pure number

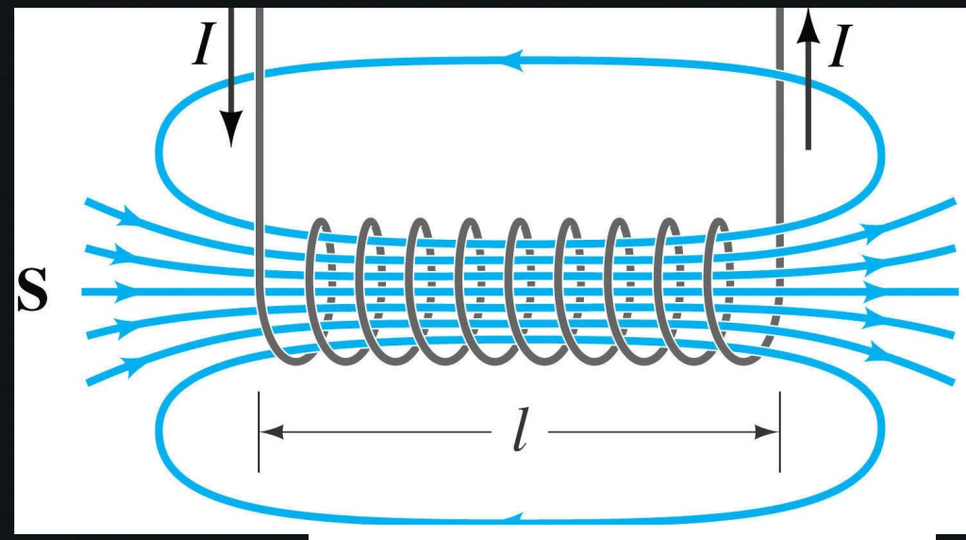


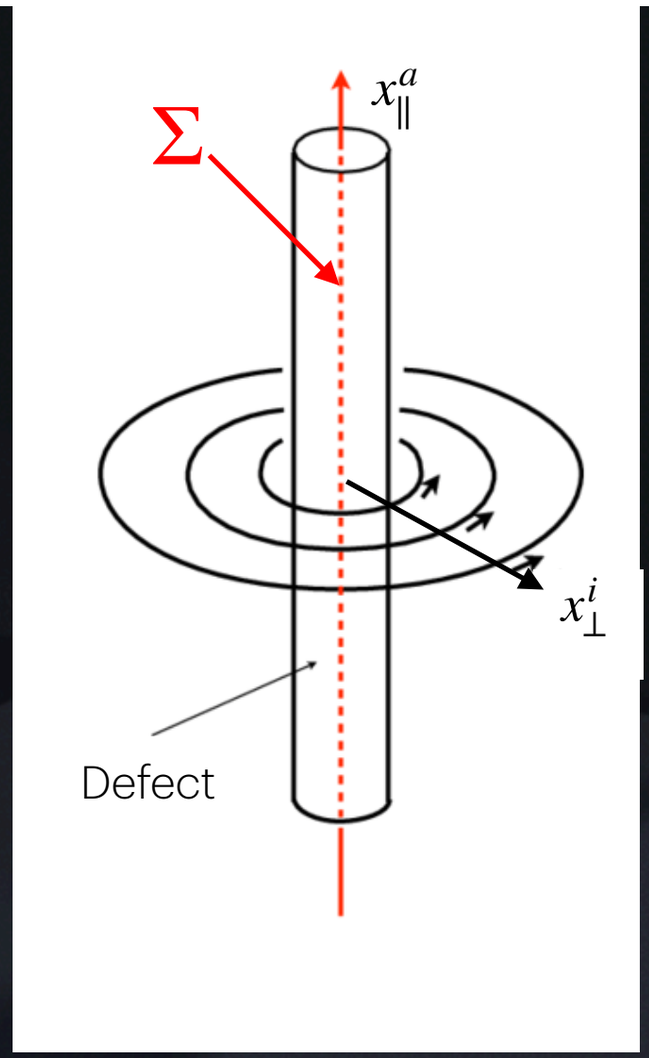


Magnetic defects

- Let's identify the displacement operator
  - External field for global symmetry
  - Ward Identity: $\nabla_\mu T^{\mu\nu} = J^\mu F_\mu{}^\nu$

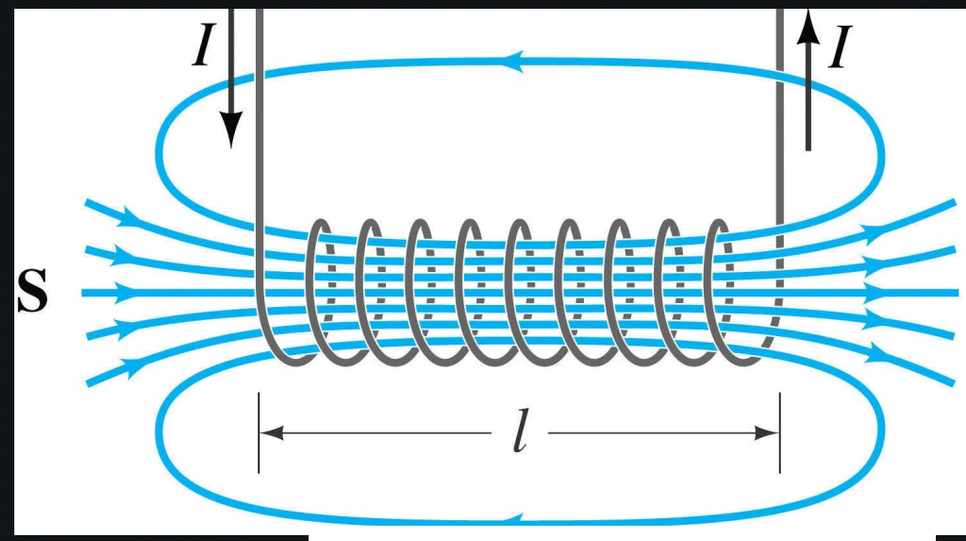
- $F = 2\pi\ \mu\ \delta(\Sigma) dx \wedge dy$
- Displacement is the current operator on the defect
- $\hat{D}^i = -2\pi\mu\epsilon^{ij} J^j|_\Sigma$

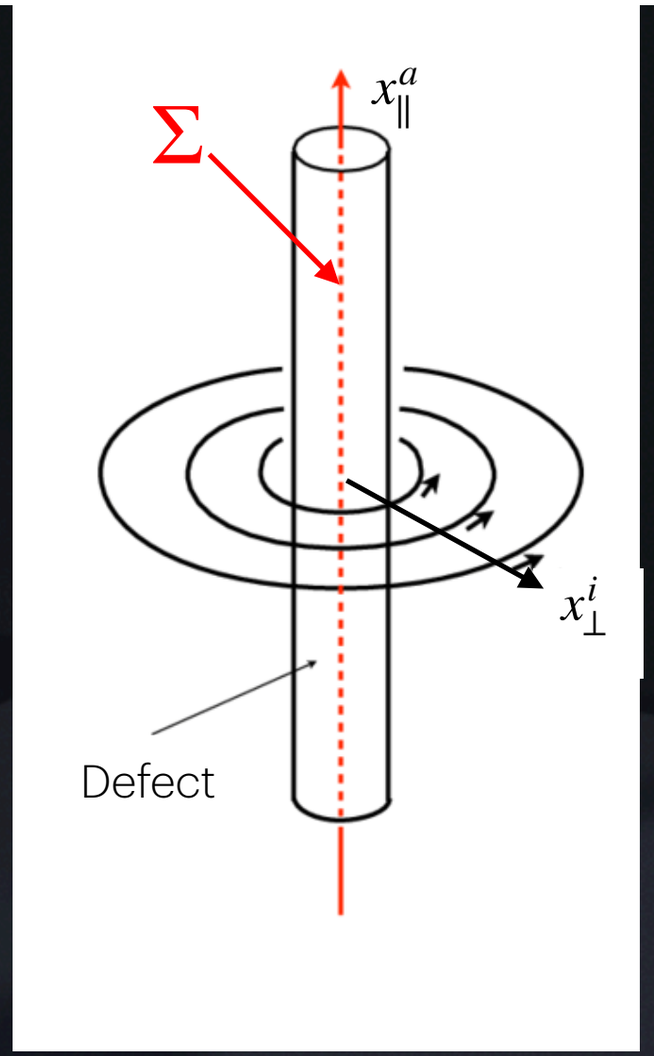




Magnetic defects

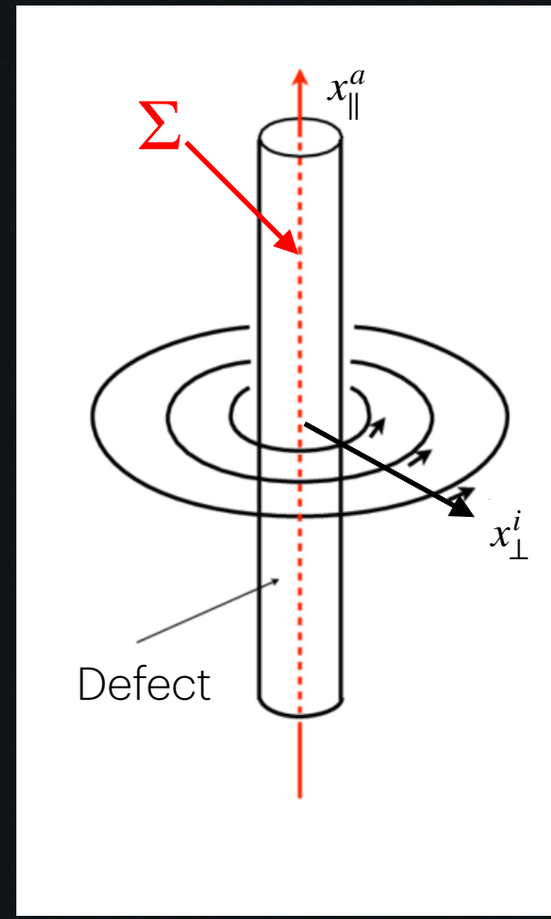
- What dynamics do we expect from a magnetic solenoid?
- A charged particle passing through the solenoid will be deflected as it begins a cyclotron orbit within the solenoid
- If we create virtual pairs near the solenoid, one may scatter off it and be deflected, generating a circulating current
  - Magnetic field breaks time reversal symmetry
  - We then expect nontrivial $\langle J_\mu \rangle$
- General structure of 1pt.s fixed by remaining conformal symmetry





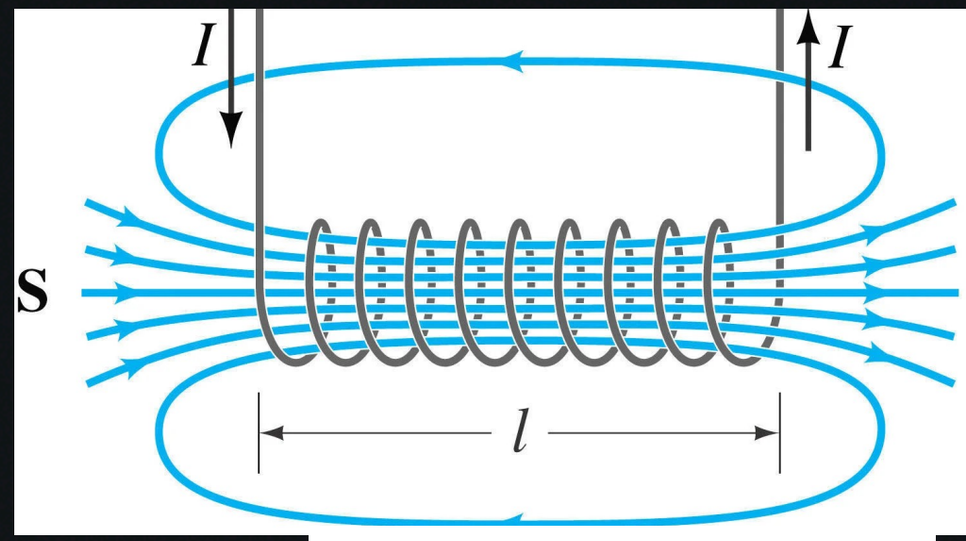
Local response

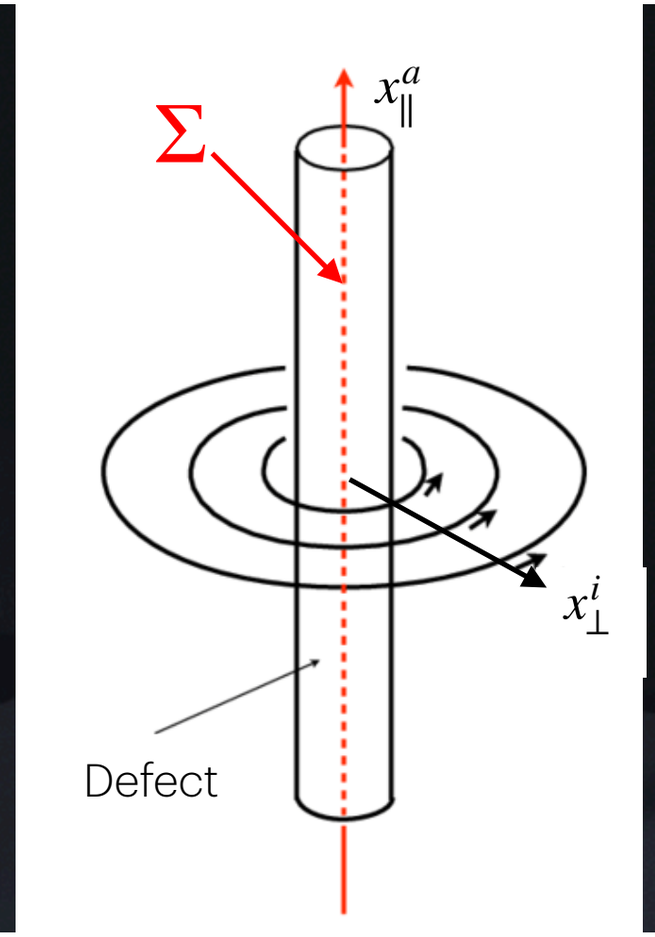
- By symmetries, we can have circulating currents
  - $\langle J_\theta \rangle = \dfrac{C(\mu)}{\rho^2}, \qquad \langle J_\rho \rangle = \langle J_{x_\parallel} \rangle = 0$
- We also have nontrivial stress tensor
  - $\langle T^{ab} \rangle = -\dfrac{h_D(\mu)}{2\pi}\dfrac{\eta^{ab}}{\rho^4}, \langle T^{ij} \rangle = +\dfrac{h_D(\mu)}{2\pi}\dfrac{3\delta^{ij} - 4\frac{x_\perp^i x_\perp^j}{\rho^2}}{\rho^4}$
  - $\langle T^\mu{}_\nu \rangle = -\dfrac{h_D}{2\pi\rho^4}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -3 \end{pmatrix} \qquad x^\mu = (t, x, \rho, \theta)$
- Measure local dynamics via $C(\mu),\ h_D(\mu)$



Magnetic defects

- $SO(4,2) \to SO(2,2) \times SO(2)$
- We can consider more general "flat" backgrounds with conical deficits:
  - $ds^2 = -dt^2 + dx^2 + d\rho^2 + n^2\rho^2 d\theta^2$
  - $\Delta\theta = 2\pi$
- This implies $R \sim (n-1)\delta(\Sigma)$, and may be necessary to cancel $U(1)_R$ monodromy
- Curvature version of solenoid
  - $0 < n < 1$: deficit, $n > 1$: excess





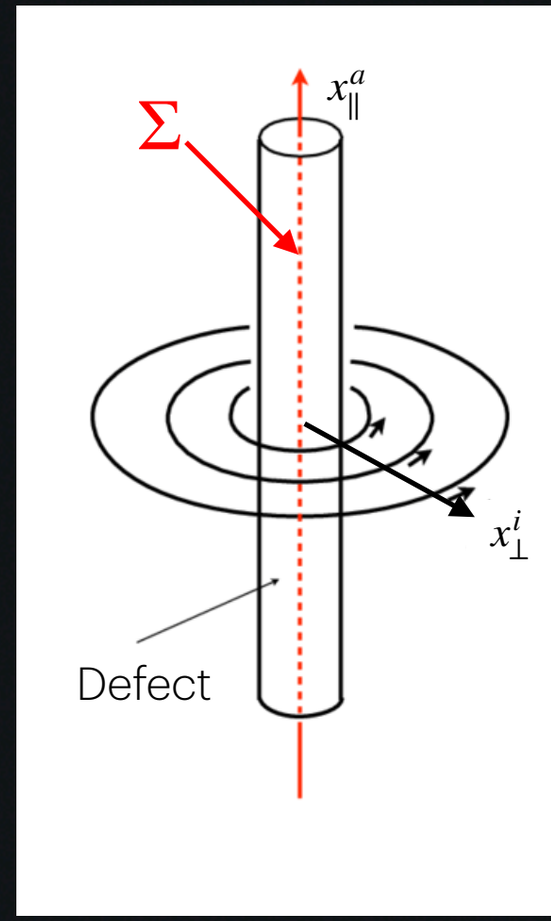


Why conical singularity?

- Generalized background with conical singularities:
  Why? R-symmetry!

- The Killing spinor equation in the is, roughly,

$$D_\mu \epsilon = \left(\partial_\mu + \frac{1}{4} \gamma^{ab} \omega_{ab\mu} - i A_\mu^{(R)} \right) \epsilon = 0$$

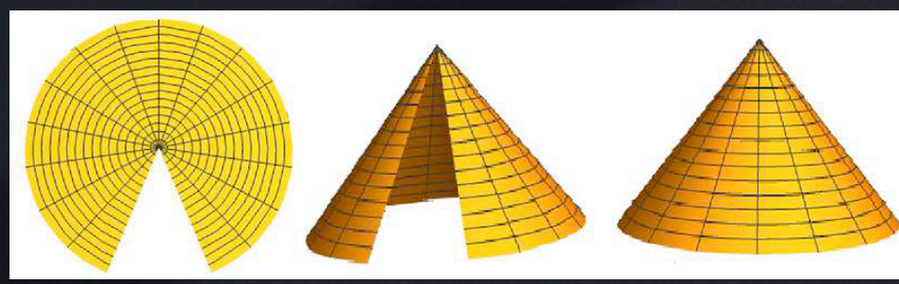


- Integrability: $[D_\mu, D_\nu]\epsilon = \left(\frac{1}{4} R_{ab} \gamma^{ab} + F^{(R)} \right) \epsilon = 0$

- If we have $F^{(R)} = \mu_R \, \delta^{(2)}(\Sigma) \, dx \wedge dy$, we can fix the chirality of the spinor, $\gamma^{12}\epsilon = \pm \epsilon$ and satisfy with

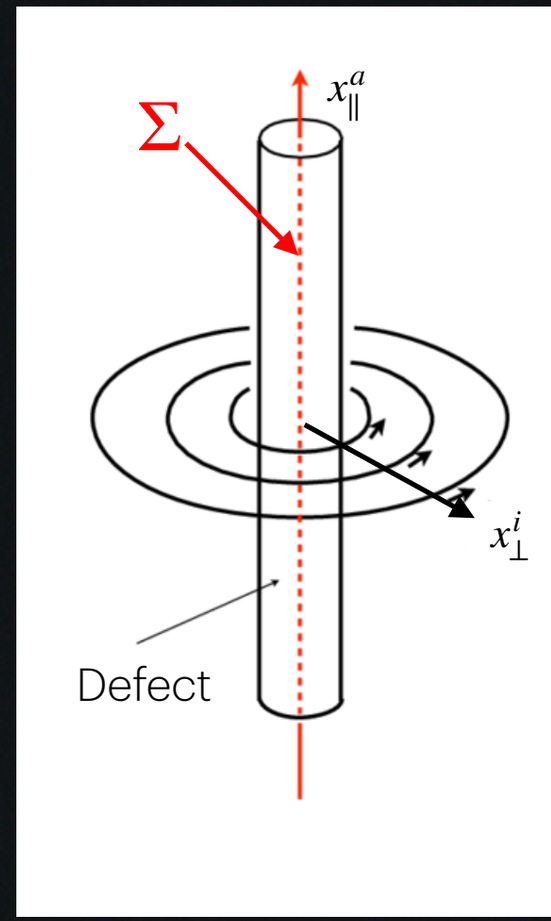
- $R_{12} \sim \pm \mu_R \delta^{(2)}(\Sigma) \, dx \wedge dy$:

  - conical singularity!



Magnetic defects

- Trace anomaly of 4d/2d system classified completely (ignoring parity odd possibilities ...)
- Standard ambient space terms, $\langle T^{\mu}{}_{\mu} \rangle \supset cW^2 - aE_4$
- A nontrivial defect gives three new "central charges",



- $\langle T^{\mu}{}_{\mu} \rangle \supset -\frac{1}{24\pi}\delta^{(2)}(\Sigma)\left(bR^{(\Sigma)} + d_1 \tilde{K}^{\mu}_{ab}\tilde{K}^{ab}_{\mu} - d_2(\phi * W)^{ab}{}_{ab} \right)$
- b does not depend on defect marginal couplings (type A)
- $d_{1,2}$ can (type B)
- There is a b-theorem for flows driven by defect couplings, with $b_{IR} < b_{UV}$

Magnetic defects

- $\langle T^{\mu}{}_{\mu} \rangle \supset -\frac{1}{24\pi}\left(bR^{(\Sigma)} + d_1 \tilde{K}^{\mu}_{ab}\tilde{K}^{ab}_{\mu} - d_2 (\phi * W)^{ab}{}_{ab} \right)$

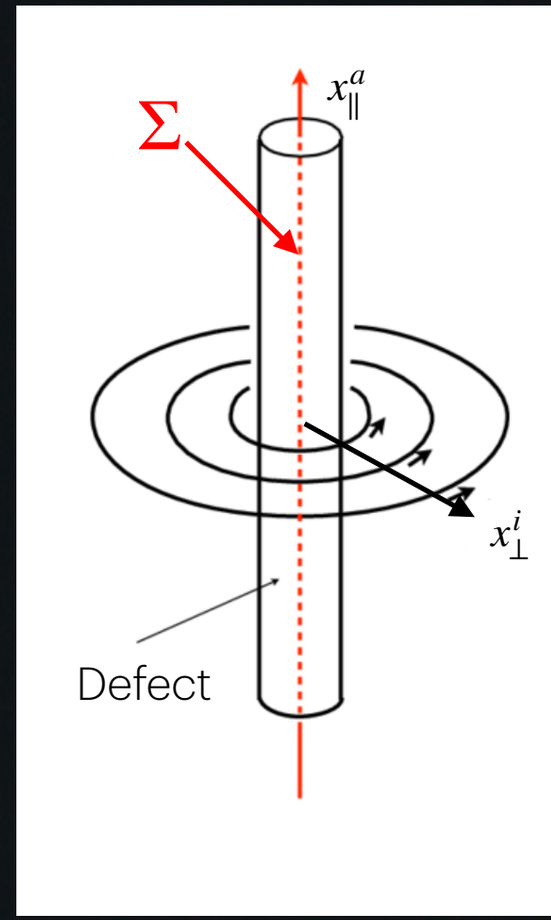
- For a flat defect, writing $x^i_\perp x^i_\perp = \rho^2$, recall

- $\langle T^{\mu}{}_{\nu} \rangle = -\frac{h_D}{2\pi\rho^4}\begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -3 \end{pmatrix}, \quad d_2 = 18\pi n h_D$

- With $(0,2)$ SUSY:

  - b indep. of some bulk marginal couplings, $d_1 = d_2$

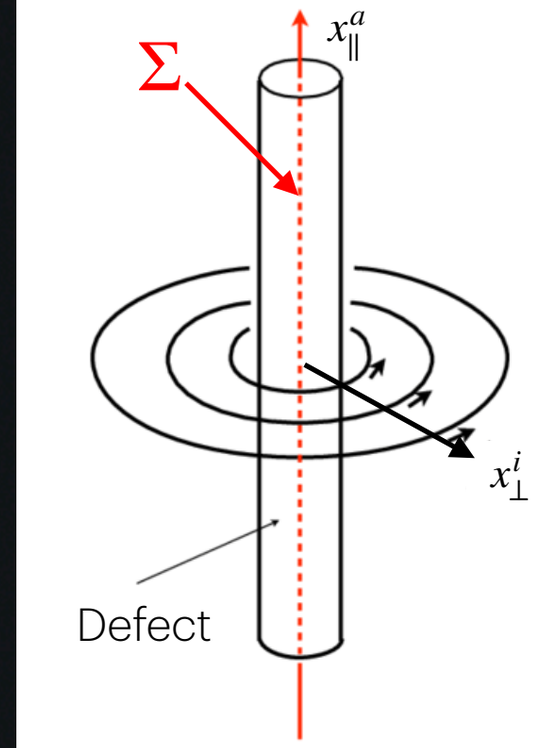
  - $\langle (J_R)_\theta \rangle = \frac{n h_D}{2\pi\rho^2}, \qquad C_R = \frac{n h_D}{2\pi} = \frac{d_2}{(6\pi)^2}$



Defect

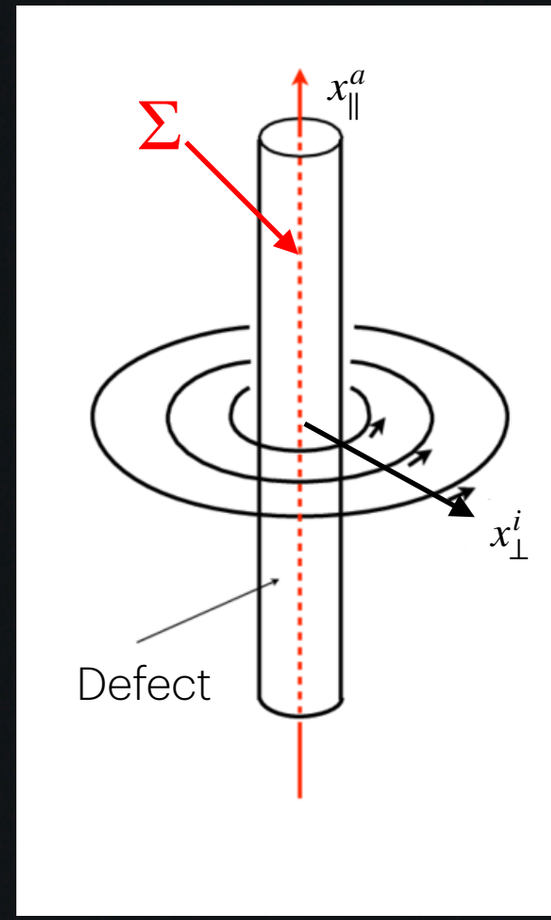
Magnetic defects

- For monodromy defects, for flavor currents
  - $\langle (J_I)_\theta \rangle = \frac{C_I(\mu)}{\rho^2}$,
- It was shown that $\frac{d}{d\mu_I} b(\mu) = \frac{1}{n} 12\pi^2 C_I(\mu)$
  - Calculation involves relating b to on-shell action in spherical Weyl frame
- To integrate b we need also $\partial_n b = -\frac{1}{n} d_2 + 12a$
  - Related to Weyl map which gives super-Renyi entropy
- And thus from calculating $\langle T \rangle$, $\langle J \rangle$ we can derive b



Holographic defects

- Magnetic defects have been studied in free theories
  - We would like to see what happens with interactions
  - Difficult to study perturbatively: No momentum space Feynman diagram techniques
    - Gnarly position-space propagators
- "Standard" tool of string theory: Holography



AdS/CFT

- Strongly coupled large *N* gauge theories are dual to classical gravity in higher dimensions (anti-deSitter space, aka hyperbolic space)

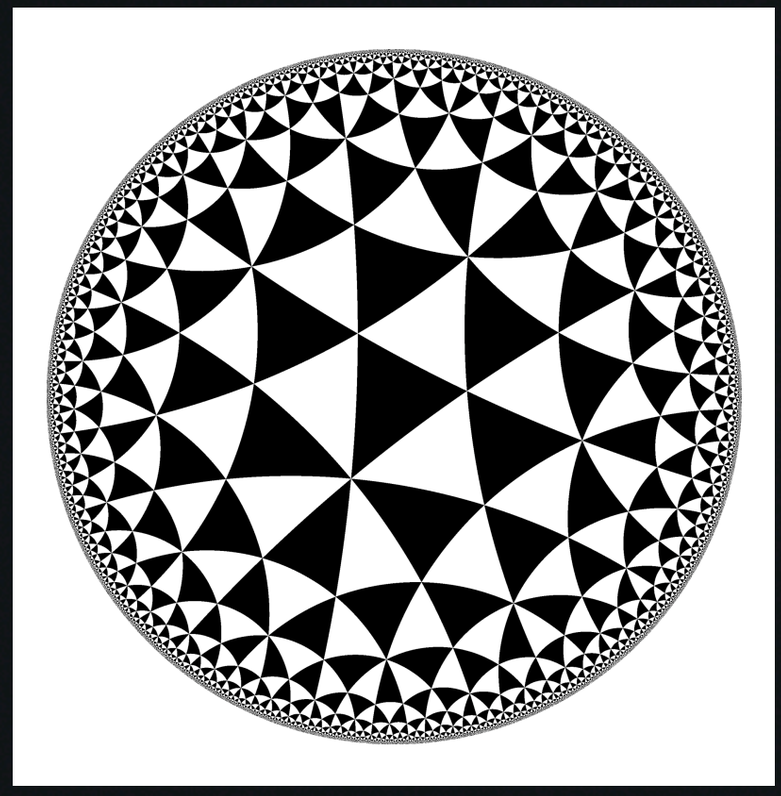
- AdS_{d+1} is not globally hyperbolic - impose boundary conditions to have well-posed Cauchy problem for e.g. Einstein's equations

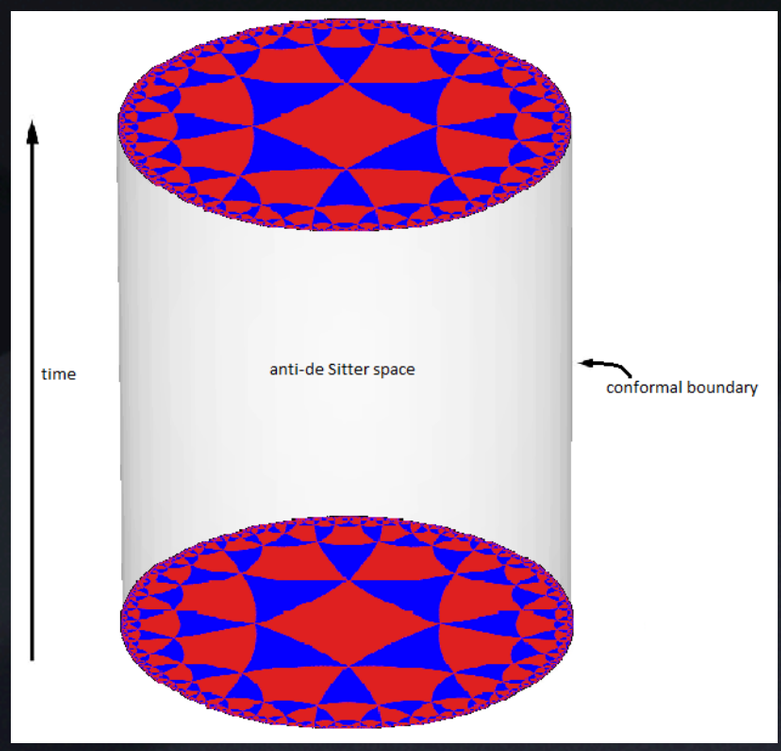
- Boundary conditions of fields correspond to sources

- $$Z_{CFT}[A_i, g^{(\text{bdy.})}] = \int \mathscr{D}\Phi e^{-S[\Phi]-A_i J^i[\Phi]} = e^{-S_{cl.}(g_{\mu\nu},\ldots)}$$

- $$ds^2 \sim dr^2 + e^{2r/L} g_{ij}^{(\text{bdy.})}(x^i)dx^i dx^j + \ldots, \; A \sim A_i(x^i)dx^i + \ldots$$

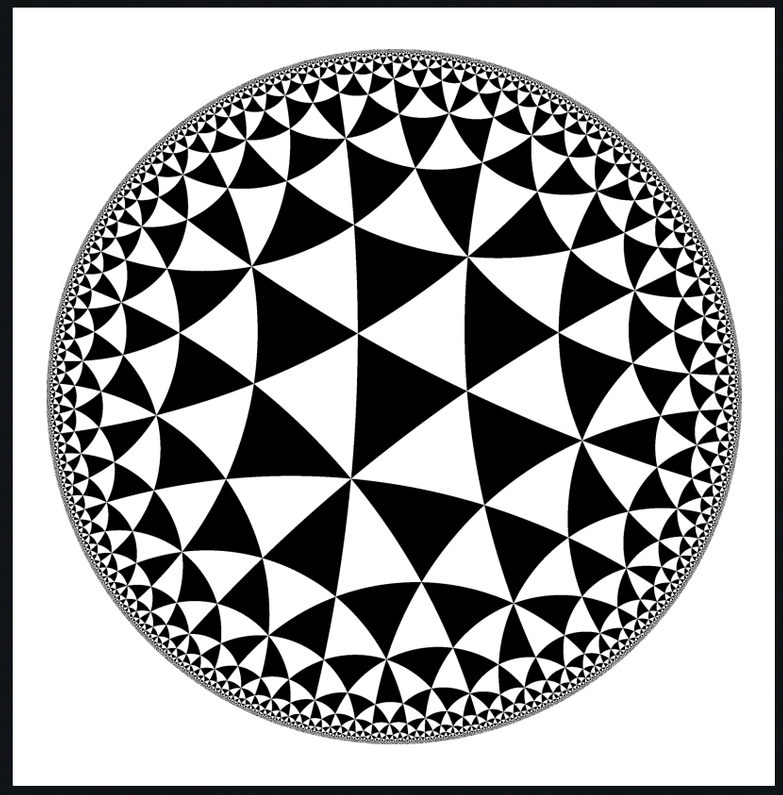
- $$\frac{\delta \ln Z}{\delta A_i} = \langle J^i \rangle, \qquad \frac{\delta^2 \ln Z}{\delta A_i \delta A_j} = \langle J^i J^j \rangle, \ldots$$

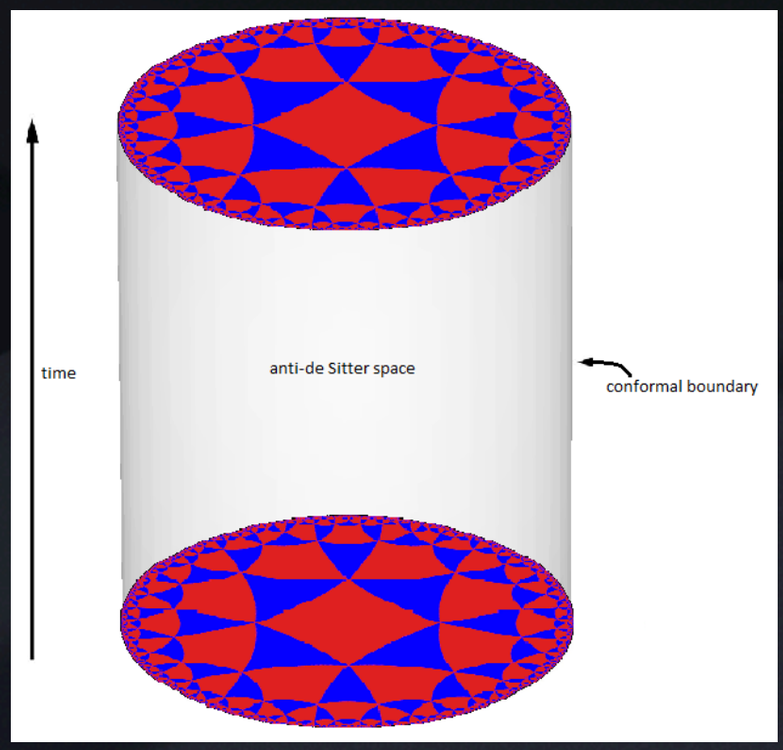




AdS/CFT

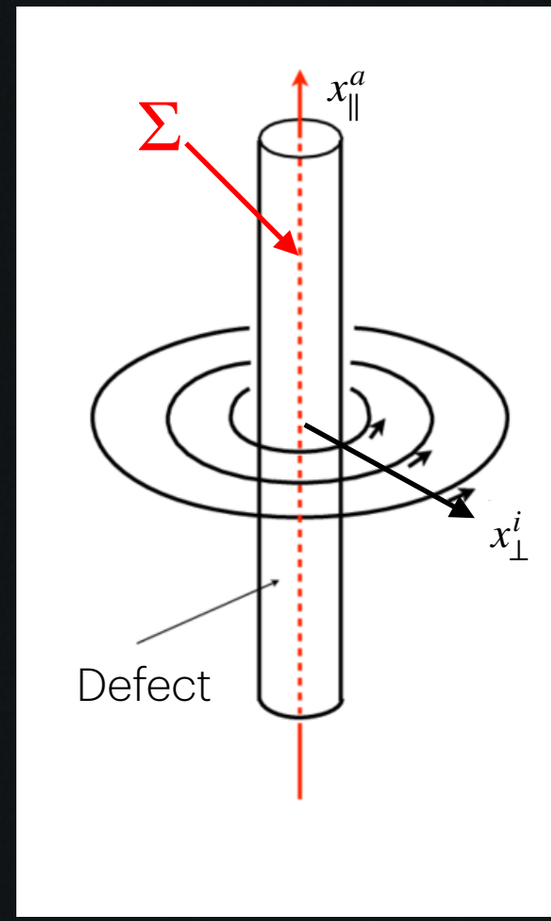
- Studying classical gravitation and geometry allows us to learn about strongly coupled QFTs
- Large N, λ means semiclassical gravity
- $g_{ij}^{(\mathrm{bdy})}$ is source for stress-energy tensor
- A_i is source for global symmetry currents
- Conformal symmetry of CFT_d is geometrized as isometry group of AdS_{d+1}
- To study system with reduced symmetry, consider backgrounds with less isometries





Holographic defects

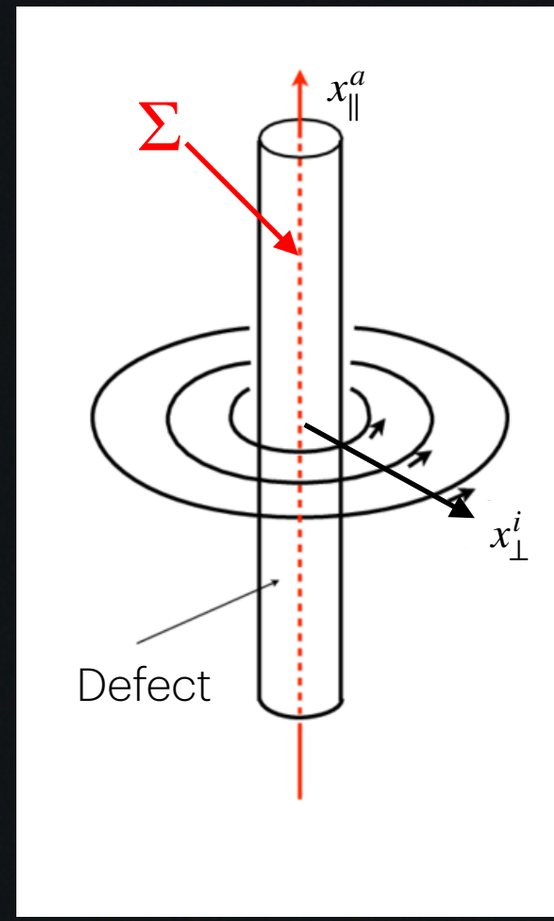
- Recall that the symmetry of the conformal defect is
  - $SO(2,2) \times SO(2)$
- Since we are studying a CFT, we can do calculations Weyl frame, $\hat{g}_{\mu\nu} = e^{2\Omega(x^\mu)} g_{\mu\nu}$

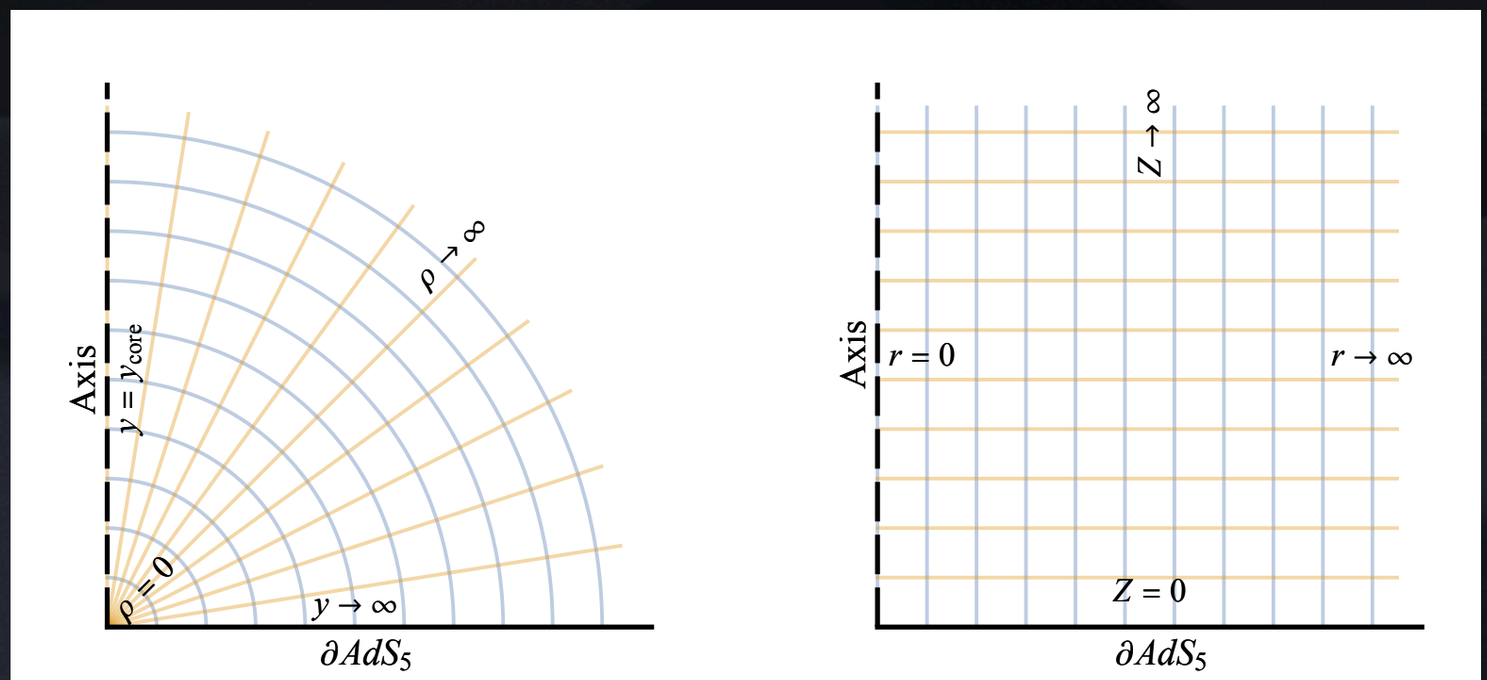


- We can take a more convenient Weyl frame,
  - $$ds^2 = -dt^2 + dx^2 + d\rho^2 + n^2\rho^2 d\theta^2 = \left(\frac{-dt^2 + dx^2 + d\rho^2}{\rho^2} + n^2 d\theta^2 \right) \times \rho^2$$
  - This is simply $AdS_3 \times S^1$, w. isometry group $SO(2,2) \times SO(2)$
  - In this frame, $A = \mu d\theta,\ dA = 0$, nontrivial monodromy on S^1

Holographic defects

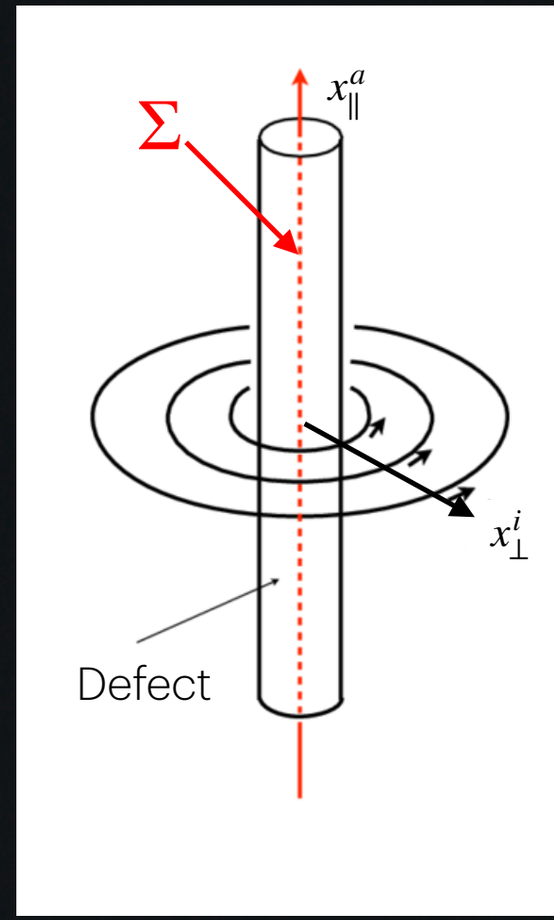
- In the vacuum (no magnetic flux), we can see this easily:
- $$ds^2 = \frac{-dt^2 + dx^2 + dr^2 + r^2 d\theta^2 + dZ^2}{Z^2}$$
- Change the radial coordinate, $Z = \rho \cos y,\ r = \rho \sin y$
- $$ds^2 = \frac{1}{\cos^2\theta}\left(\frac{-dt^2 + dx^2 + d\rho^2}{\rho^2}\right) + \tan^2 y d\theta^2 + \frac{dy^2}{\cos^2 y}$$



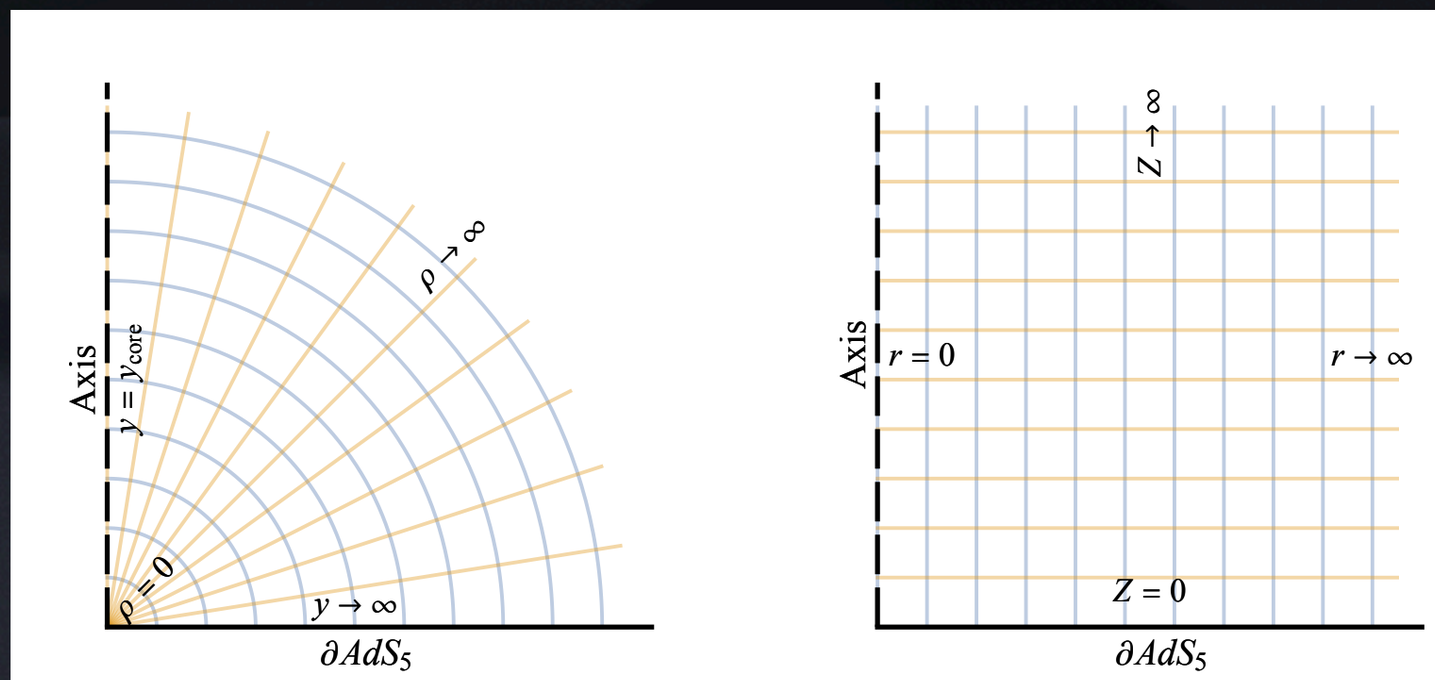


Holographic defects

- More generally, we will consider solutions of the form
- $ds^2 = e^{2V} d(AdS_3)^2 + h^2 d\theta^2 + f^2 dy^2, A^i = a^i(y) d\theta$
  - Plus some scalar fields (eg dilatons in the vector multiplet)
- $h^2 \sim (y - y_0)^2$, $h^2/f^2 = (y - y_0)^2 + \ldots, a_i(y_0) = 0$ for bulk regularity



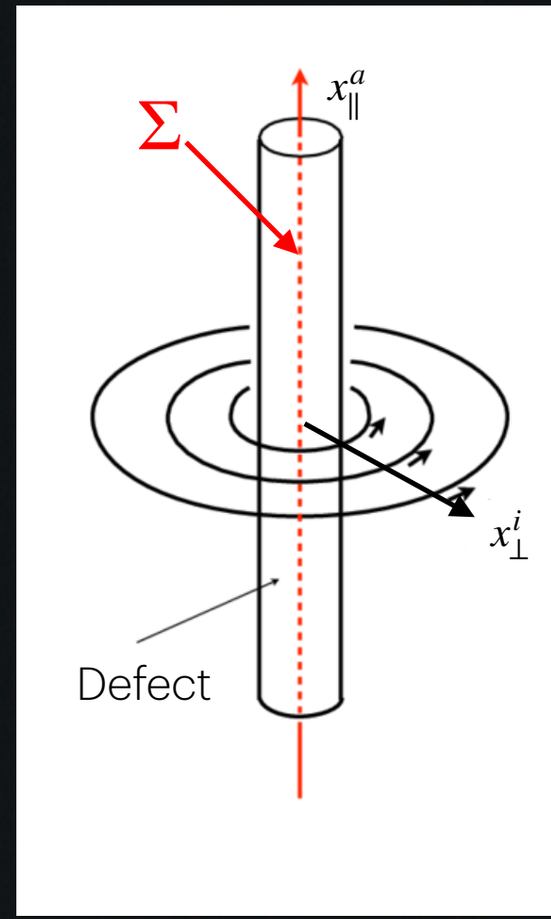
- Even for $\mu_R \neq 1$, there is no conical singularity in the bulk - just that the radius of AdS_3 and S^1 may be different!



Holographic defects

- We wish to study holographic duals of conformal magnetic defects - natural starting point is $\mathcal{N}=4$ SYM

- $AdS_3 \times S^1$ foliated solutions of 5d gauged supergravity, with nontrivial monodromy around the S^1



  - Preserving only $(0,2)$ SUSY, we can turn on monodromies in $(U(1)^2)_F \times U(1)_R \subset SU(4)_R$

- We can also consider the $\mathcal{N}=1$ LS theory

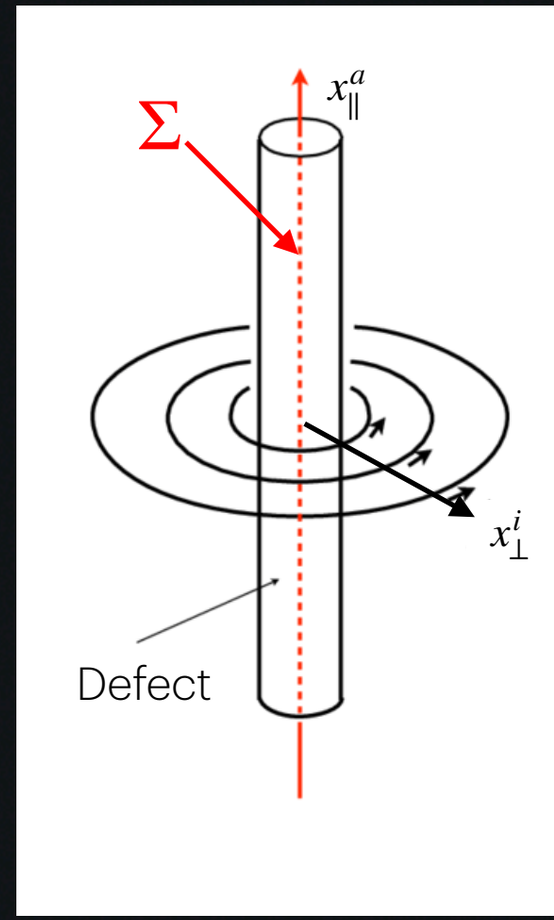
  - Turn on a mass for one of the three chiral multiplets in $\mathcal{N}=4$ SYM, flows to strongly coupled $\mathcal{N}=1$ theory

  - Monodromies in $U(1)_F \times U(1)_R \subset SU(2)_F \times U(1)_R$

Holographic defects

- $\mathcal{N} = 4$: Monodromies in $(U(1)^2)_F \times U(1)_R$
- $\mathcal{N} = 1$: Monodromies in $U(1)_F \times U(1)_R$

- If we satisfy an algebraic constraint on the monodromies, we can also turn on the SUSY preserving mass deformation to flow from $\mathcal{N} = 4$ to $\mathcal{N} = 1$
  - SUSY: $Dm_F = 0$, where $D = \partial + i(A^1 + A^2 - A^3)$
    - Holomorphic due to $(0,2)$ chirality
  - constant m_F then requires $\mu_1 + \mu_2 - \mu_3 = 0$
- In this restricted case, we can compare b at UV and IR

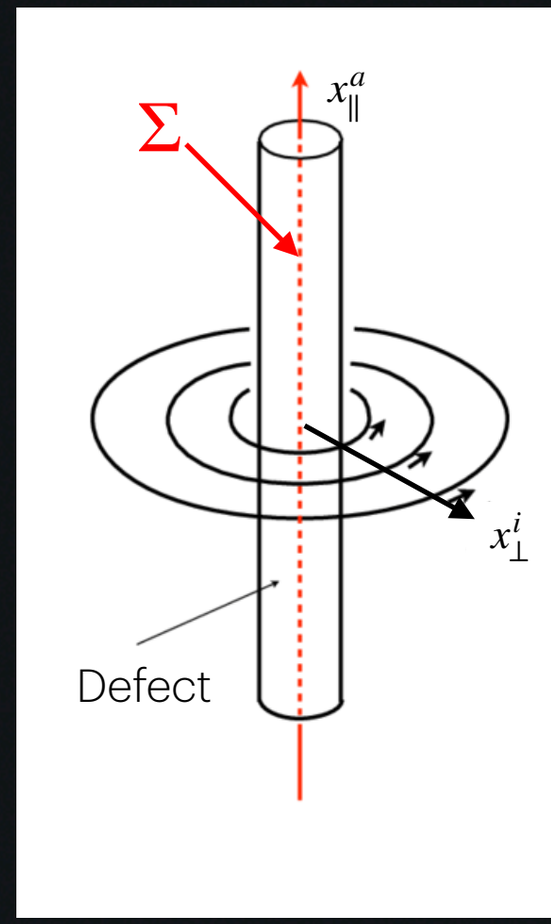


Integrating up b

- Must dial conical deficit n to be able to integrate b from $\langle J \rangle$, $\langle T \rangle$
- $$\frac{d}{d\mu_I} b(\mu, n) = \frac{1}{n} 12\pi^2 C_I(\mu, n),$$

$$\partial_n b(\mu, n) = -\frac{1}{n} d_2(\mu, n) + 12a$$

- From holographic perspective, no big change
  - Dialing asymptotic radii of AdS_3 and S^1 boundary parts



$\mathcal{N} = 4$ defects

- We have three monodromy parameters μ^i, and on the primary branch of solutions (adiabatically connected to the vacuum) we find
  - $\mu_R = \mu_1 + \mu_2 + \mu_3 = \pm (1 - n)$
    - where ± 1 is the SUSY chirality ($(0,2)$ vs $(2,0)$
      - Will fix $(0,2)$ for the remainder
- We will parameterize solutions by μ_i remembering implicitly that we are always satisfying the above SUSY constraint.

$\mathcal{N} = 4$ defects

- Where do solutions exist? Not for all monodromies

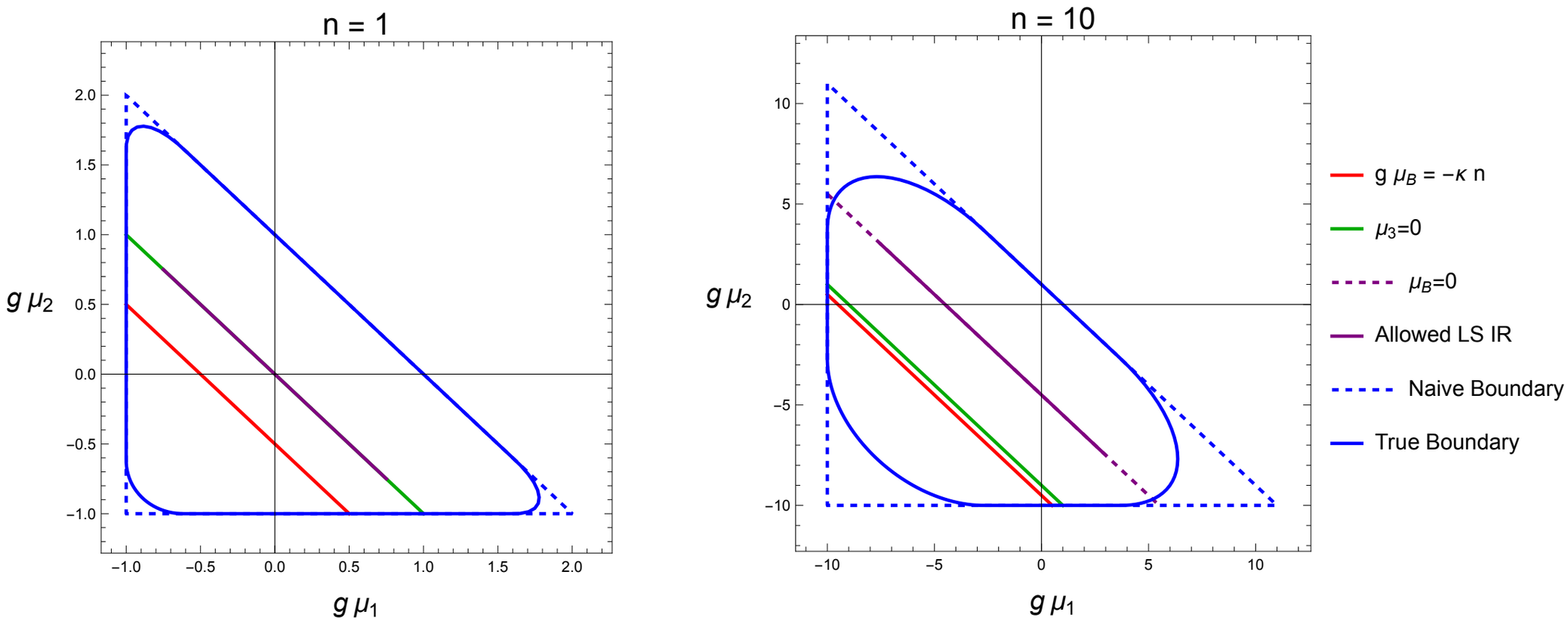


Figure 5: Solutions space of STU solutions as a function of two independent monodromy parameters $g\mu_1$, $g\mu_2$ for $n = 1$ (left plot) and $n = 10$ (right plot), for the main branch of solutions with $s = -\kappa/2$ (we have set $\kappa = +1$).

$\mathcal{N} = 4$ defects

- Currents:

$$\langle J_1 \rangle = \frac{N^2}{4\pi^2}\mu_1 \left(1+\frac{\mu_2}{n}\right)\left(1+\frac{\mu_3}{n}\right),$$

$$\langle J_2 \rangle = \frac{N^2}{4\pi^2}\mu_2 \left(1+\frac{\mu_3}{n}\right)\left(1+\frac{\mu_1}{n}\right),$$

$$\langle J_3 \rangle = \frac{N^2}{4\pi^2}\mu_3 \left(1+\frac{\mu_1}{n}\right)\left(1+\frac{\mu_2}{n}\right), \qquad \mu_1+\mu_2+\mu_3 = (1-n)$$

- Stress tensor:

$$h_D = -\frac{2\pi}{3n}\left(\langle J_1\rangle + \langle J_2\rangle + \langle J_3\rangle\right) = -\frac{2\pi}{n}\langle J_R^{\mathcal{N}=4}\rangle$$

  - (What about periodicity $\mu \sim \mu + 1$?)

$\mathcal{N} = 4$ defects

- To calculate b recall

$$db = 12\pi^2 \left(\frac{1}{n} \sum_i \langle J_i \rangle d\mu_i - 3 \frac{h_D}{2\pi} dn \right) + 12 a_{\mathcal{N}=4} dn$$

  - Where $a_{\mathcal{N}=4} = \frac{N(N-1)}{4} \approx \frac{N^2}{4}$ at large N

    - It is a nontrivial check that the above is actually closed given our results, subject to the constraint $\mu_R = \mu_1 + \mu_2 + \mu_3 = \kappa(1-n)$

- It can be integrated to yield

  - $b_{\mathcal{N}=4} = -12 a_{\mathcal{N}=4} n(F^{\mathcal{N}=4} - 1) = -3N^2 n(F^{\mathcal{N}=4} - 1)$

  - Where $F^{\mathcal{N}=4} = \left(1 + \frac{\mu_1}{n}\right)\left(1 + \frac{\mu_2}{n}\right)\left(1 + \frac{\mu_3}{n}\right)$

$\mathcal{N} = 4$ defects

- $b_{\mathcal{N}=4} = -12a_{\mathcal{N}=4}n(F^{\mathcal{N}=4} - 1) = -3N^2n(F^{\mathcal{N}=4} - 1)$
- Where $F^{\mathcal{N}=4} = \left(1 + \frac{\mu_1}{n}\right)\left(1 + \frac{\mu_2}{n}\right)\left(1 + \frac{\mu_3}{n}\right)$ and $\mu_1 + \mu_2 + \mu_3 = 1 - n$

- Remarkably, this agrees with free theory results (modulo assumptions about b.c. in free system)
  - When studying free field theory, there are boundary conditions at the core allowing L^2 integrable singular modes
  - We can identify these b.c. to match the above precisely!
  - Matching fixes $\xi_i = 0$ or 1 (natural b.c. from finite size solenoid)

$\mathcal{N} = 4$ defects

- We can also calculate the (appropriately renormalized) on-shell action, and we find

  - $$S = \frac{N^2}{2\pi}\text{vol}(AdS_3)[nF^{\mathcal{N}=4}]$$

- We can equivalently write this as

  - $$S = \text{vol}(AdS_3)\frac{1}{6\pi}\left(12a_{\mathcal{N}=4}n - b_{\mathcal{N}=4}\right)$$

$\mathcal{N} = 1$ LS defects

- The mass deformation flowing to $\mathcal{N} = 1$ LS requires
  - $\mu_B = \mu_1 + \mu_2 - \mu_3 = 0$
- Which leaves us with one flavor monodromy
  - $\mu_F = \mu_1 - \mu_2$
- And the $U(1)_R$ monodromy related to the conical deficit,
- $\mu_R = 1 - n$

$\mathcal{N} = 1$ LS defects

- Currents:
  - $\langle J_R^{LS} \rangle = \frac{N^2}{32\pi^2} \frac{n+1}{n^2} \left(1 - n^2 - \mu_F^2\right)$
  - $\langle J_F \rangle = \frac{N^2}{8\pi^2} \mu_F \left(1 + \frac{1}{2n}\right)$
- And as required by SUSY,
  - $h_D = -\frac{2\pi}{n} \langle J_R^{LS} \rangle$

$\mathcal{N} = 1$ LS defects

- To calculate b recall

$$db = 12\pi^2 \left(\frac{1}{n} \left(\langle J_R^{LS} \rangle d\mu_R + \langle J_F \rangle d\mu_F \right) - 3 \frac{h_D^{LS}}{2\pi} dn \right) + 12 a_{LS} dn$$

  - Where now $a_{LS} = \frac{27}{128} N^2$ at large N

    - Again a nontrivial check that the above is actually closed given our results, subject to the constraint $\mu_R = 1 - n$

- It can be integrated to yield

  - $b_{LS} = -12 a_{LS} n (F^{LS} - 1) = -\frac{81}{32} N^2 n (F^{LS} - 1)$

  - Where $F^{LS} = \left(1 + \frac{4\mu_1}{3n}\right)\left(1 + \frac{4\mu_2}{3n}\right)\left(1 + \frac{2\mu_3}{3n}\right)$

$\mathcal{N} = 1$ LS defects

- We can also calculate the (appropriately renormalized) on-shell action, and we find

  - $$S = \frac{N^2}{2\pi}\mathrm{vol}(AdS_3)[nF^{LS}]$$

- We can equivalently write this as

  - $$S = \mathrm{vol}(AdS_3)\frac{1}{6\pi}\left(12a_{LS}n - b_{LS}\right)$$

$\mathcal{N} = 4$/LS bulk flows

- We have calculated properties of defects for both $\mathcal{N} = 4$ and LS theories - let's compare them (when mass def. allowed)

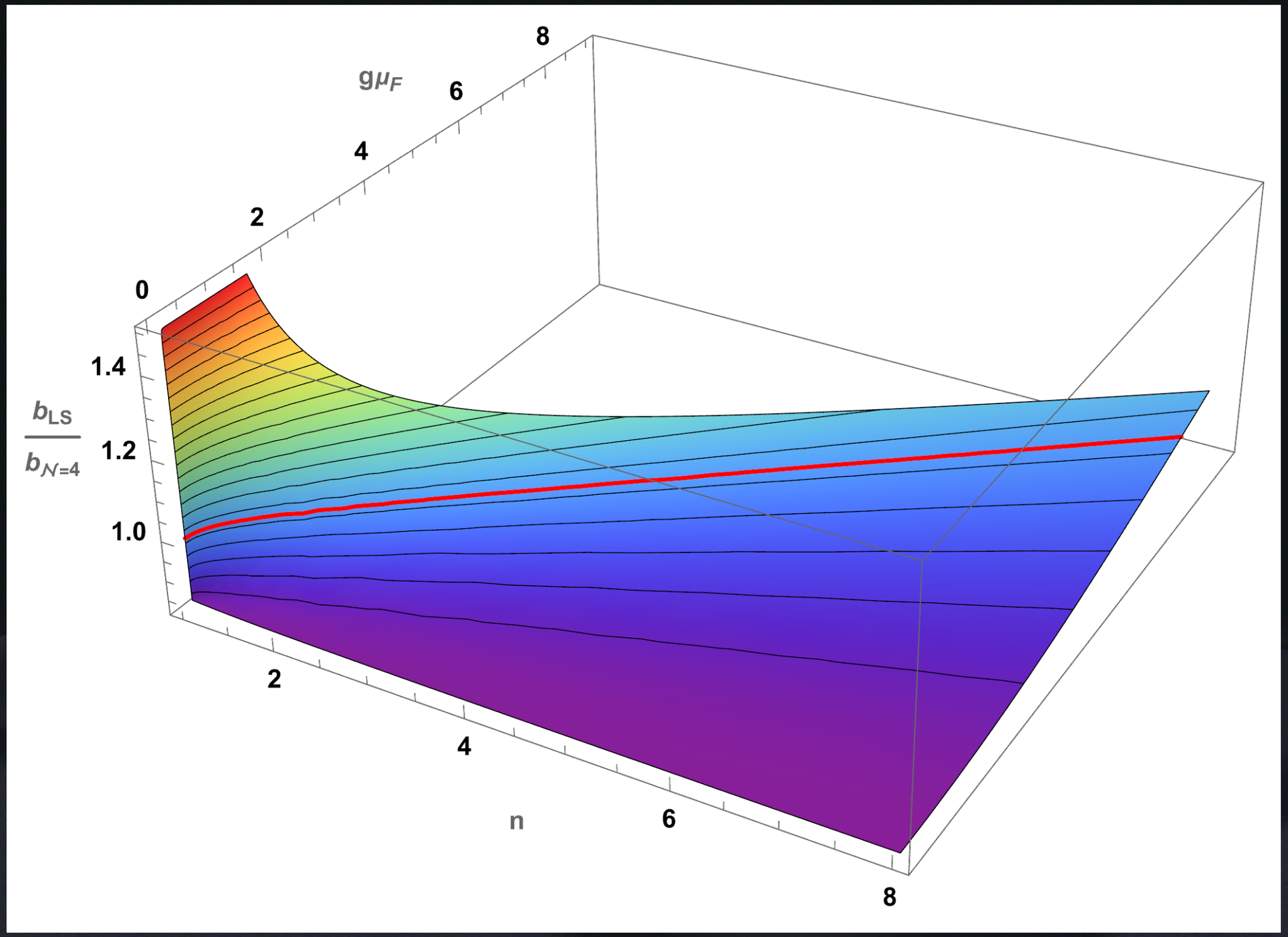
  - $b_{\mathcal{N}=4}|_{\mu_B=0} = \frac{3N^2}{32n^2}\left((n-1)(1+8n+23n^2)+4(1+n)\mu_F^2\right)$

  - $b_{LS} = \frac{3N^2}{32n^2}\left((n-1)(1+7n+19n^2)+4(1+2n)\mu_F^2\right)$

- First, note that for $n = 1$, $b_{LS} = \frac{3}{2} b_{\mathcal{N}=4}$, it increases!

- For $n \neq 1$, it can either increase or decrease!

$\mathcal{N}=4$/LS bulk flows - b_{IR}/b_{UV}



$\mathcal{N} = 4$/LS bulk flows

- We can similarly compare the on-shell actions, which depend only on $a,\ b$ central charges.

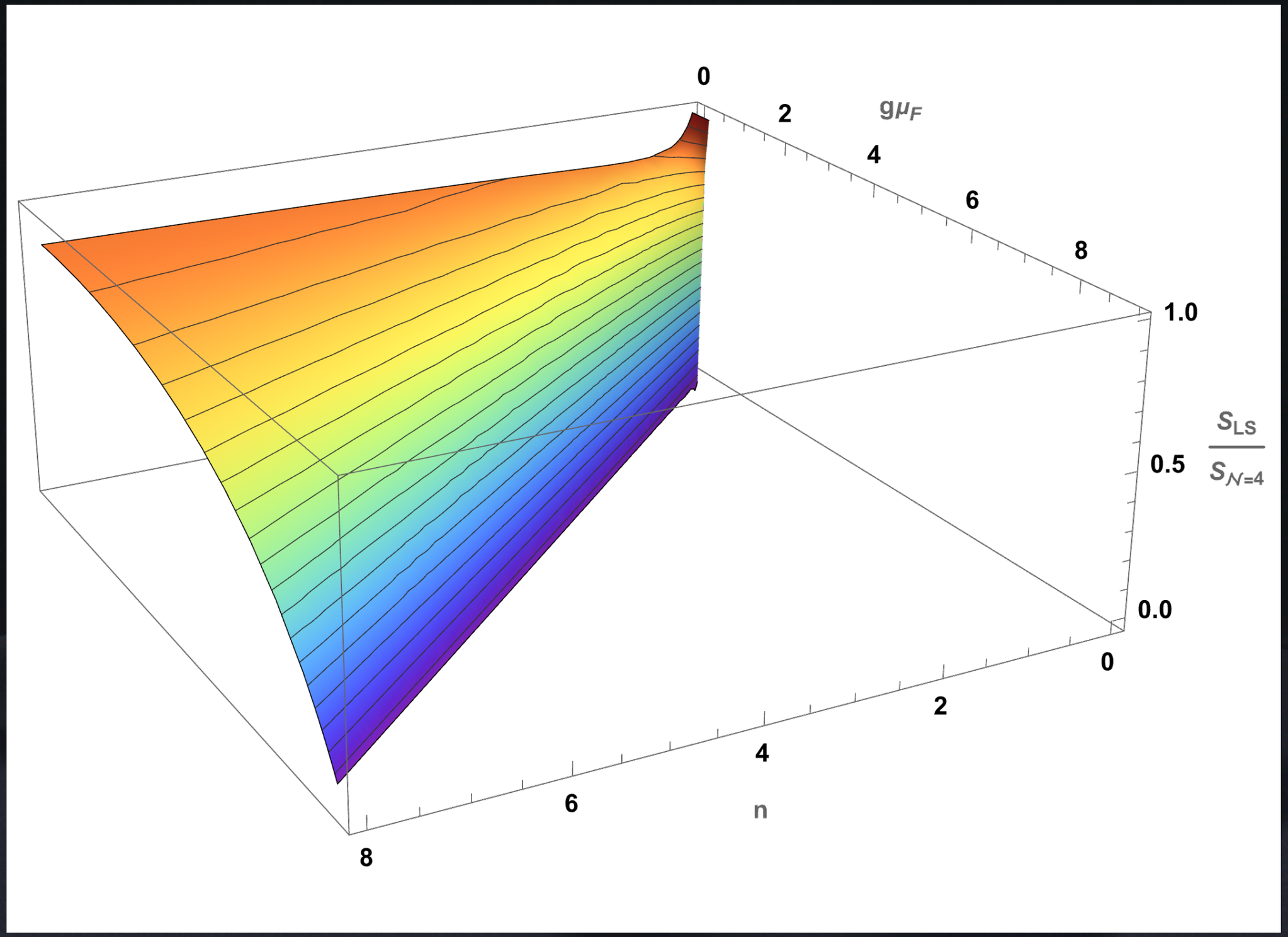
  . $S = \mathrm{vol}(AdS_3)\frac{1}{6\pi}\left(12an - b\right)$

- One can use our previous results to explicitly demonstrate that $S_{LS}/S_{\mathcal{N}=4} < 1$

  - (Explicit expressions ugly, in the paper)

- Strongly indicative that $12an - b$ is a good RG monotone

$\mathcal{N} = 4$/LS bulk flows - S_{IR}/S_{UV}

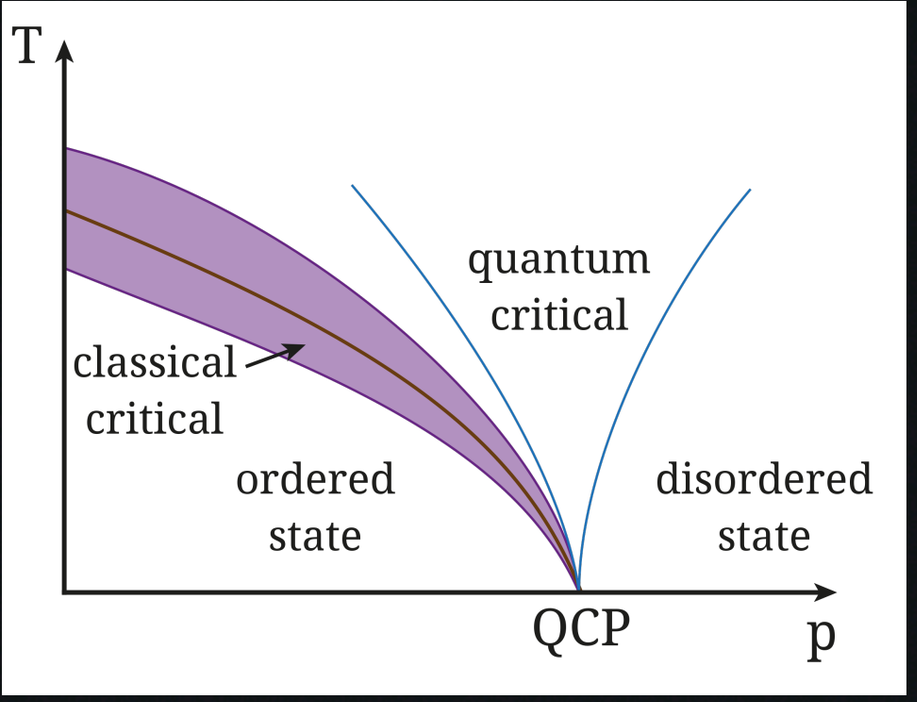


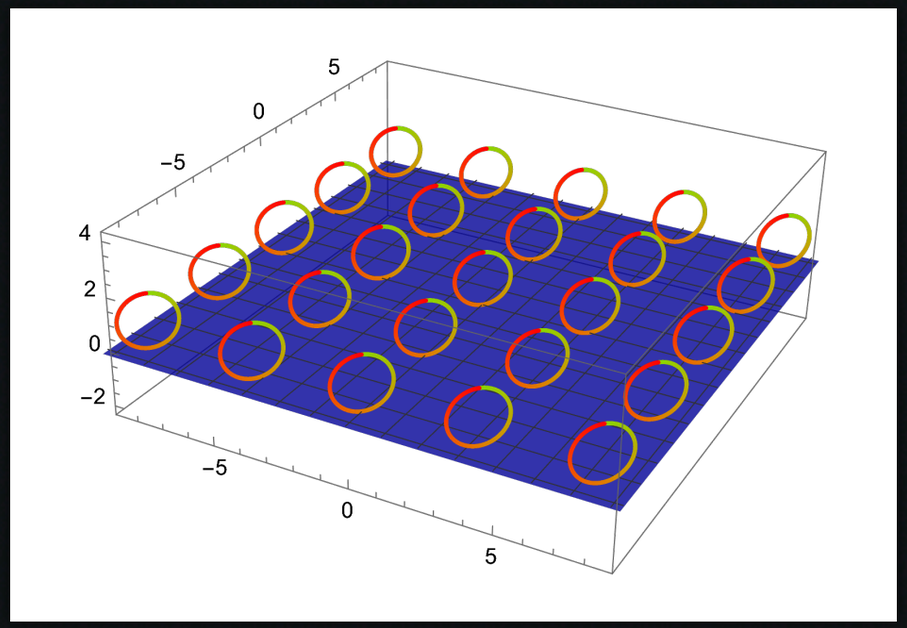
Topics I don't have time for

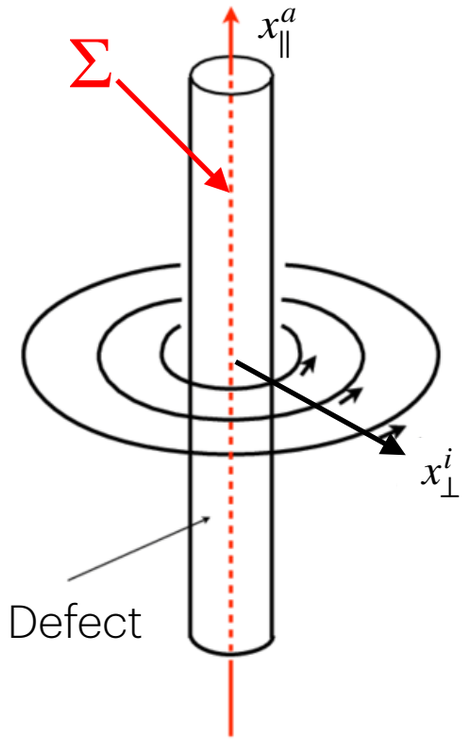
- Matching to free results
- Other branches of solutions (not adiabatically connected to no flux)
- Supersymmetric Renyi entropy (via the conical singularity)
- "Baroque" conformal defects with $m_F = \frac{\lambda}{\rho}$
- Details of how we extracted all these VEVs without analytic solutions
- Periodicity of monodromies / boundary conditions for regular solutions
- 3d conjectured monotones (on-shell action)

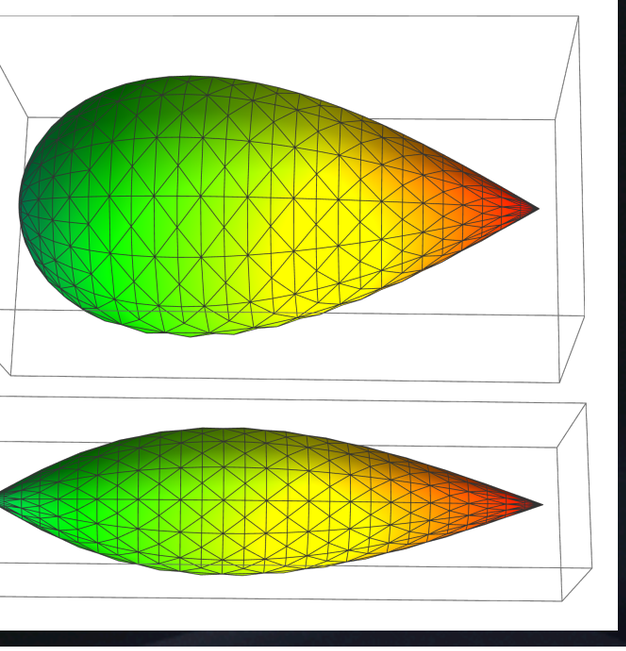
Conclusions

- New extensive study of $(0,2)$ 2d/4d magnetic defects
  - Explicit expressions for new central charges
- **New conjectured monotonic quantity**!
- To do:
  - Scattering (two-point functions), entanglement entropy, ...
  - Detailed analysis of defect core holographic renormalization

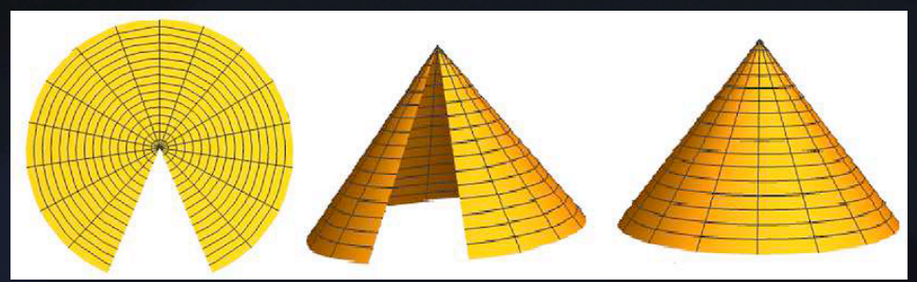

T
quantum
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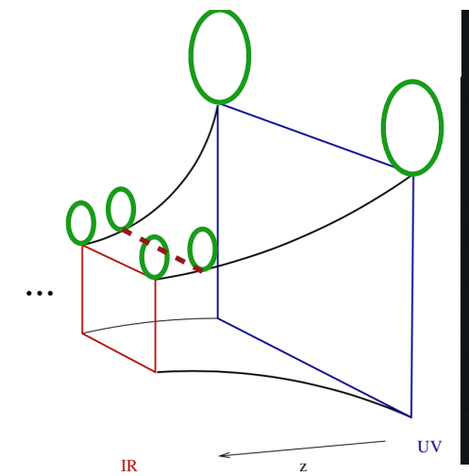


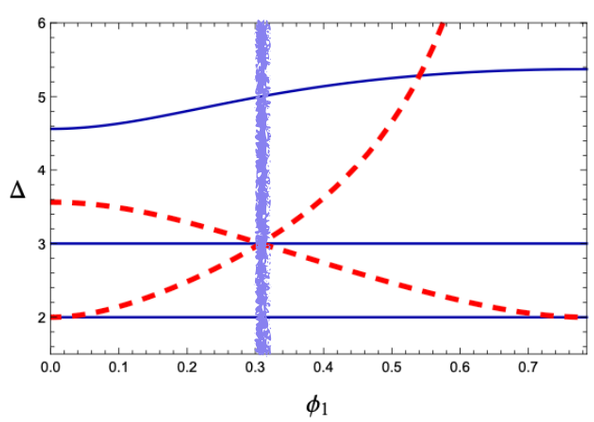

$x^a_{\parallel}$
Σ
$x^i_{\perp}$
Defect

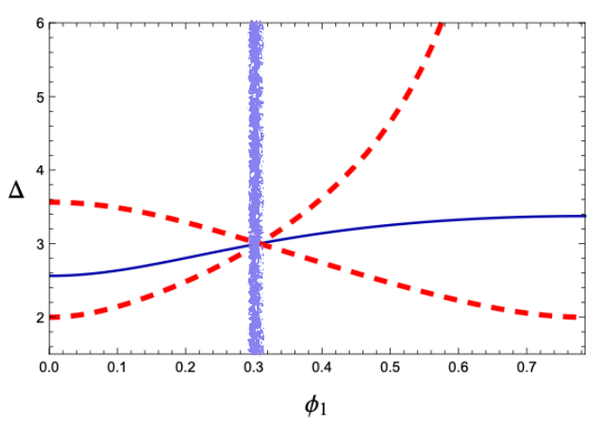


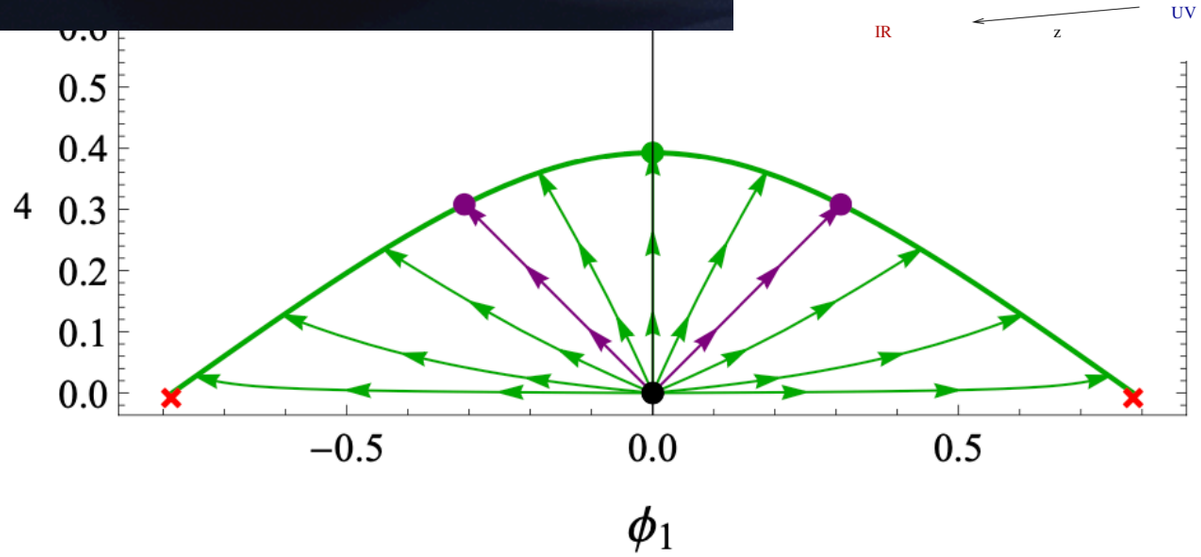
Thank you!




...
IR
z
UV


Δ
ϕ_1
U(1)² invt. source
other source

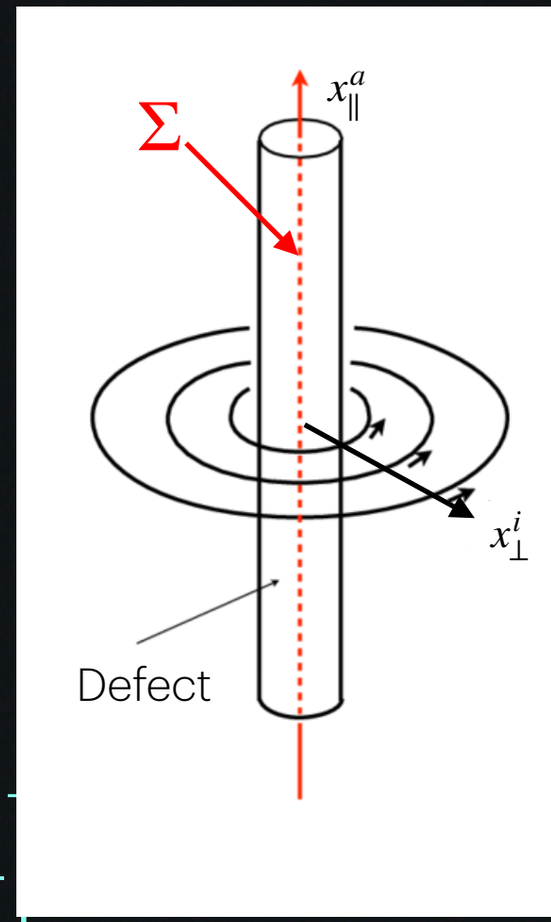

Δ
ϕ_1
U(1)² invt. vev
other vev


ϕ_1

SUSY and bulk solutions

Holographic defects

- In holography we would look for solutions to gauged SUGRA, a classical theory of Einstein gravity coupled to finitely many matter fields, a KK reduction of 10d IIB SUGRA



- $S = \frac{1}{16\pi G_5}\int\sqrt{-g}\left[R - V - \sum_i \frac{(\partial\varphi^i)^2}{2} - \frac{1}{4}\sum_I (X^I)^{-2}(F^I)^2 - \frac{(\partial\rho)^2 + \sinh^2\rho(D\theta)^2}{2}\right.$

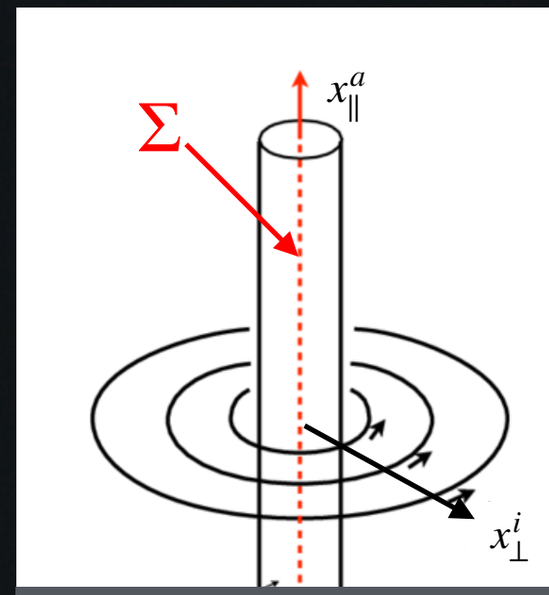
- $X^I = e^{a_i^I\varphi^i}, \qquad X^1X^2X^3 = 1, \qquad D\theta = d\theta - \zeta_I A^I$

- $V = 2\sum_i\left(\frac{\delta W}{\delta\varphi^i}\right)^2 + 2\frac{\delta W}{\delta\rho} - \frac{4}{3}W^2$

- $R_{\mu\nu} - \frac{R}{2}g_{\mu\nu} = \ldots, \qquad \nabla^2\varphi^i = \ldots, \qquad \nabla_\mu(\ldots F^{I\mu\nu}) = \ldots$

Holographic defects

- We are looking for $\mathcal{N} = (0,2)$ solutions to 5d SUGRA
- We solve 1st order equations: gravity, vector and hyper



$$\delta_\epsilon \psi_\mu = \left[\nabla_\mu - \frac{i}{2} Q_\mu + \frac{W \Gamma_\mu}{6} + \frac{i}{24} \sum_I (X^I)^{-1} F^I_{\nu\rho} (\Gamma_\mu{}^{\nu\rho} - 4\delta_\mu^\nu \Gamma^\rho) \right] \epsilon = 0$$

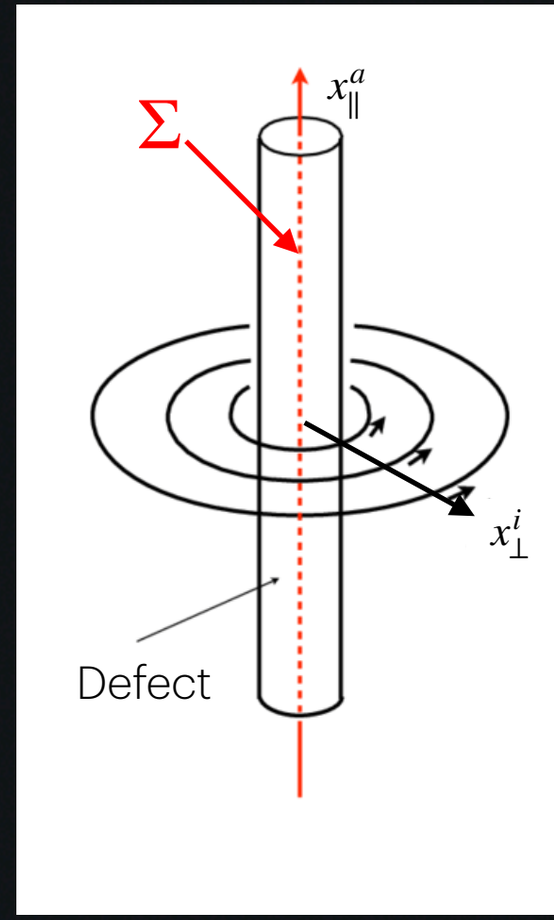
$$\delta_\epsilon \lambda = \left[\Gamma^\mu \partial_\mu \varphi^i - 2 \frac{\delta W}{\delta \varphi^i} + \frac{i}{2} \sum_I \frac{\delta (X^I)^{-1}}{\delta \varphi^i} F^I_{\mu\nu} \Gamma^{\mu\nu} \right] \epsilon$$

$$\delta_\epsilon \chi = \left[\Gamma^\mu \partial_\mu \rho - 2 \frac{\delta W}{\delta \rho} + 2i \frac{\delta Q_\mu}{\delta \rho} \Gamma^\mu \right] \epsilon = 0$$

- A solution consists of bosonic fields and a Killing spinor ϵ, and corresponds to a supersymmetric state in the (defect) SCFT

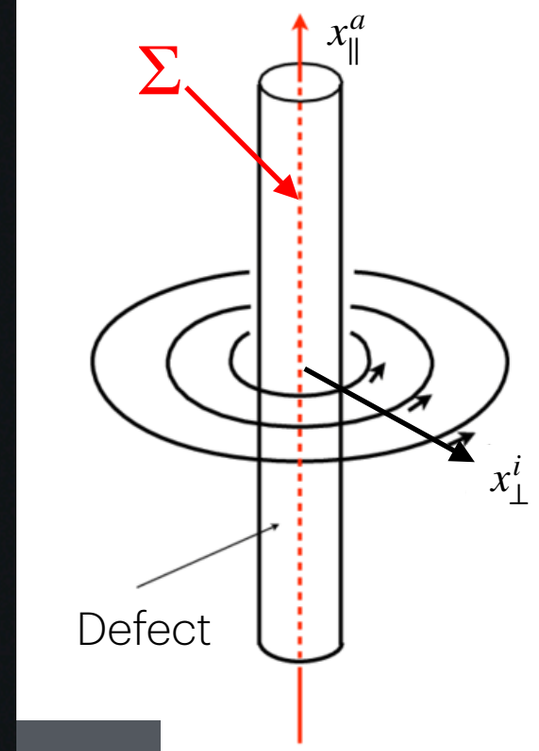
Holographic defects

- Product manifold $ds^2 = e^{2V} d(AdS_3)^2 + h^2 d\theta^2 + f^2 dy^2$
- $\epsilon^{(5)} = \psi^{(3)} \otimes \chi^{(2)}$, where $\psi^{(3)}$ is left (or right) invariant conformal Killing spinor on AdS_3
- With this assumption, we can reduce the fermion variations into a linear equation for $\chi^{(2)}$ and first order equations for metric and matter fields
- Fixing $\psi^{(3)}$ left-handed, we impose one projection condition of the form $\left[\cos \xi \gamma^{34} + \sin \xi \gamma^{3}\right] \epsilon = + i\epsilon$
- Integrate the spinor ODE as $\epsilon = e^{\frac{\xi}{2}\gamma^4} e^{V/2} e^{isz} \eta_0$, $\gamma^3 \eta_0 = + i\gamma^4 \eta_0$, and are left with first order equations for matter fields and phase ξ (split even vs odd rank)



Holographic defects

- BPS equations depend on chirality choice, and a general solution will only satisfy $\delta_\epsilon \psi = 0$ for fixed chirality. Only one η_0
  - Preserves $(0,2)$ SUSY
- A consequence of SUSY and Maxwell is the existence of conserved quantities of the solution
- Maxwell: $(e^{3V} e^{a_i \varphi^i} F^{(i)}_{34})' = 0 \rightarrow e^{3V} e^{a_i \varphi^i} F^{(i)}_{34} = E^{(i)}$
- Gaugino: $e^{a_i \varphi^i} F^{(i)}_{34} \sim -(1 + \partial_{\varphi_i}) W \cos\xi - e^{-V}$
- This gives constants of motion for BPS solutions
- $E^i = e^{2V}\left[2e^V \cos\xi - 2(1 + \partial_{\varphi_i})W\right]$
- We can use these constants of motion to extract asymptotics without actually solving the full BPS equations analytically!



Holographic defects

$$
\begin{aligned}
f^{-1}\xi' &= gW\cos\xi + 2\kappa e^{-V}\,,\\
f^{-1}V' &= \frac{g}{3}W\sin\xi\,,\\
f^{-1}\alpha' &= -\frac{g}{12}\partial_\alpha W\sin\xi\,,\\
f^{-1}\beta' &= -\frac{g}{4}\partial_\beta W\sin\xi\,,\\
f^{-1}\varphi' &= -\frac{g}{2}\frac{\partial_\varphi W}{\sin\xi}\,,\\
f^{-1}\frac{h'}{h} &= \frac{1}{\sin\xi}\Big(2\kappa e^{-V}\cos\xi + \frac{gW}{3}(1+2\cos^2\xi)\Big)\,,
\end{aligned}
\tag{A.3}
$$

along with the two constraint equations

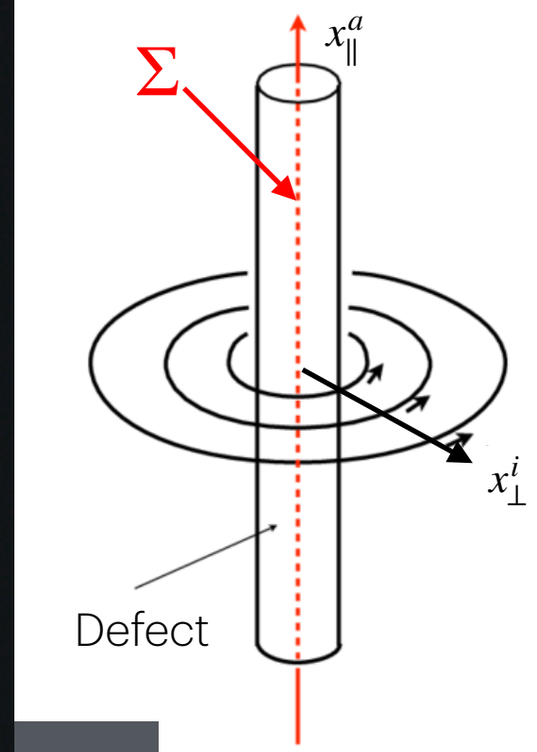
$$
\begin{aligned}
(s-Q_z)\sin\xi &= -\frac{1}{2}gWh\cos\xi - \kappa h e^{-V}\,,\\
\frac{g}{2}\partial_\varphi W\cos\xi &= \partial_\varphi Q_z\sin\xi h^{-1}\,.
\end{aligned}
\tag{A.4}
$$

The frame components of the field strengths are given by

$$
F^{(i)}_{34} = f^{-1}h^{-1}(a^i)'\,,
\tag{A.5}
$$

and the derivation of the BPS equations leads to

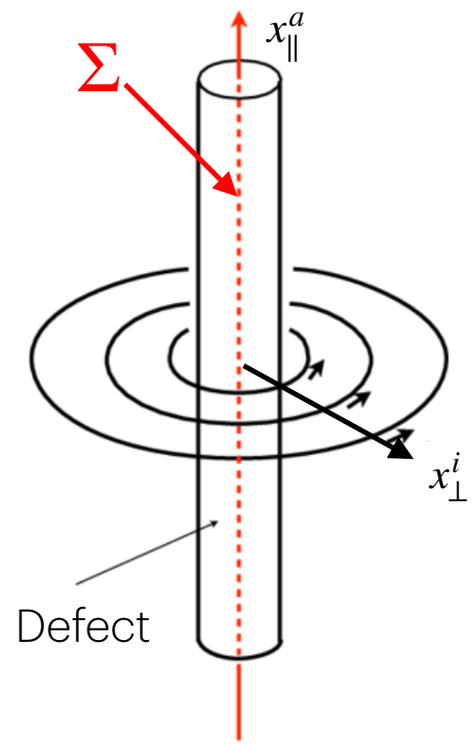
$$
\begin{aligned}
e^{2\alpha-2\beta}F^1_{34} &= -\frac{g}{12}[4W-\partial_\alpha W+3\partial_\beta W]\cos\xi - \kappa e^{-V}\,,\\
e^{2\alpha+2\beta}F^2_{34} &= -\frac{g}{12}[4W-\partial_\alpha W-3\partial_\beta W]\cos\xi - \kappa e^{-V}\,,\\
e^{-4\alpha}F^3_{34} &= -\frac{g}{6}[2W+\partial_\alpha W]\cos\xi - \kappa e^{-V}\,.
\end{aligned}
\tag{A.6}
$$



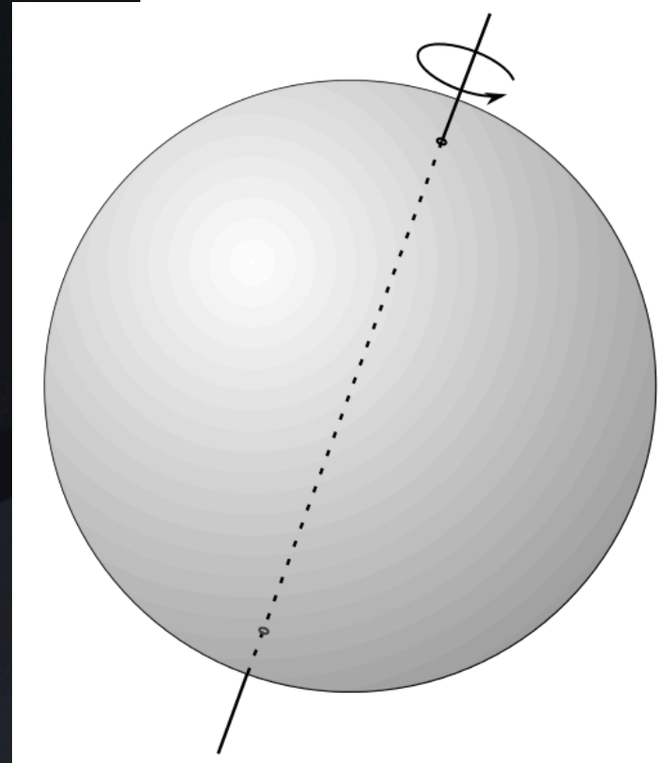
Entanglement entropy and b

Magnetic defects

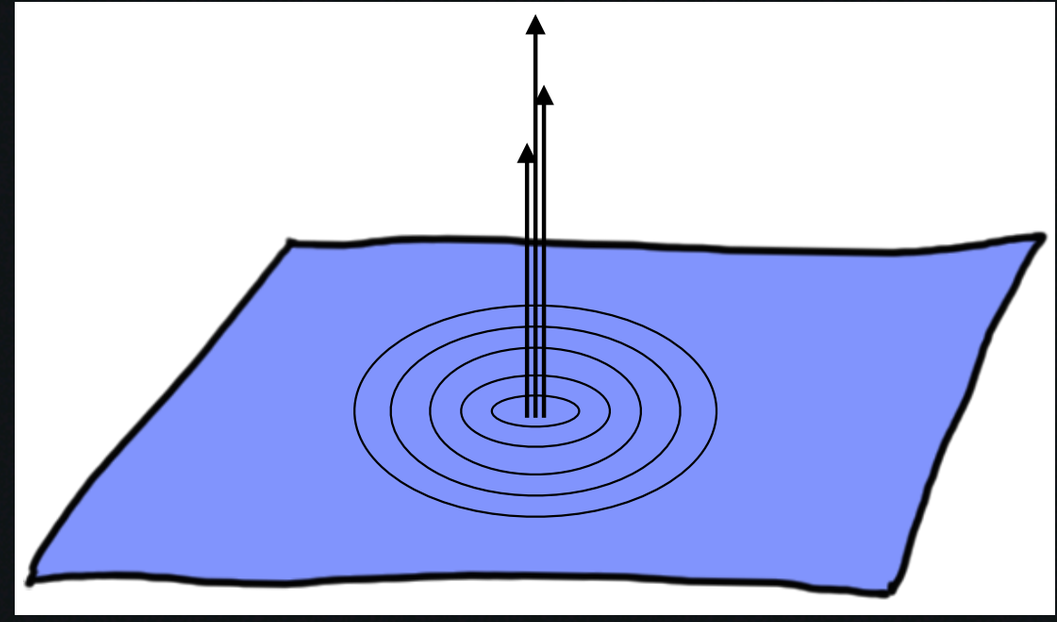
- We could also extract b from entanglement entropy: for $n = 1$
  - $S_{EE} = \gamma_1 \frac{\ell^2}{\epsilon^2} - 4a \ln(\ell/\epsilon) + \frac{1}{3}(b - d_2/3)\ln(\ell/\epsilon)$



- This is a more complicated calculation we leave to future work (hints how to proceed in literature…)
- Note that the "universal" part of the entanglement is no longer monotonic
- Unclear precisely how to generalize to general n



3d/1d Defects



- Imagine a sheet slicing through the solenoid, with d.o.f. confined there

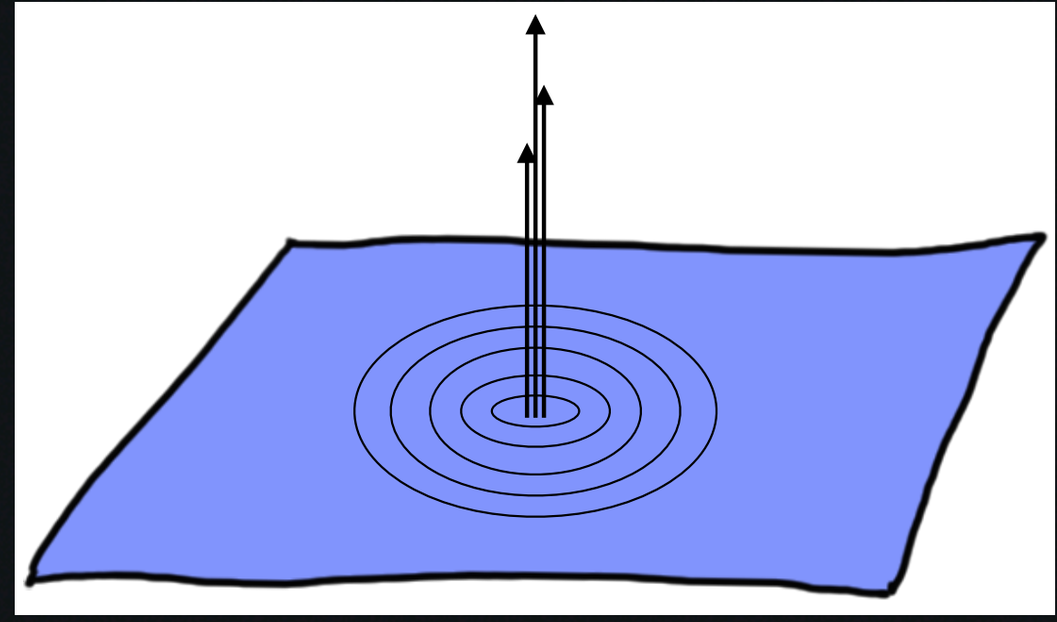
- We will again expect to find circulating currents and stress tensor

- $\langle J_\theta \rangle = \dfrac{C(\mu)}{\rho}, \qquad \langle J_\rho \rangle = \langle J_{x_\parallel} \rangle = 0$

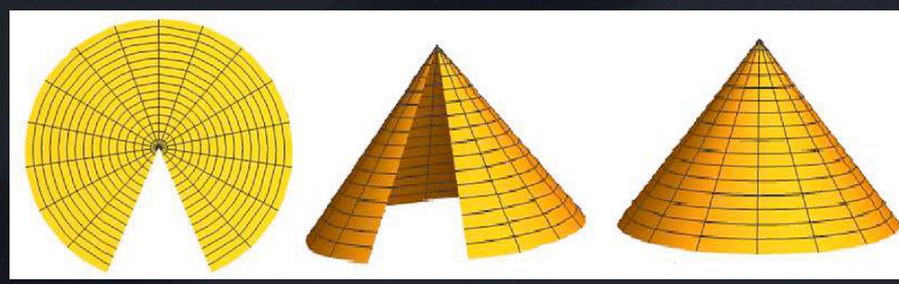
- $\langle T^\mu{}_\nu \rangle = -\dfrac{h_D}{2\pi\rho^3}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix} \qquad x^\mu = (t, \rho, \theta)$

- In 3d/1d system, there are no trace anomalies, so no analog of $a,\ c,\ b\ , d_1,\ d_2$

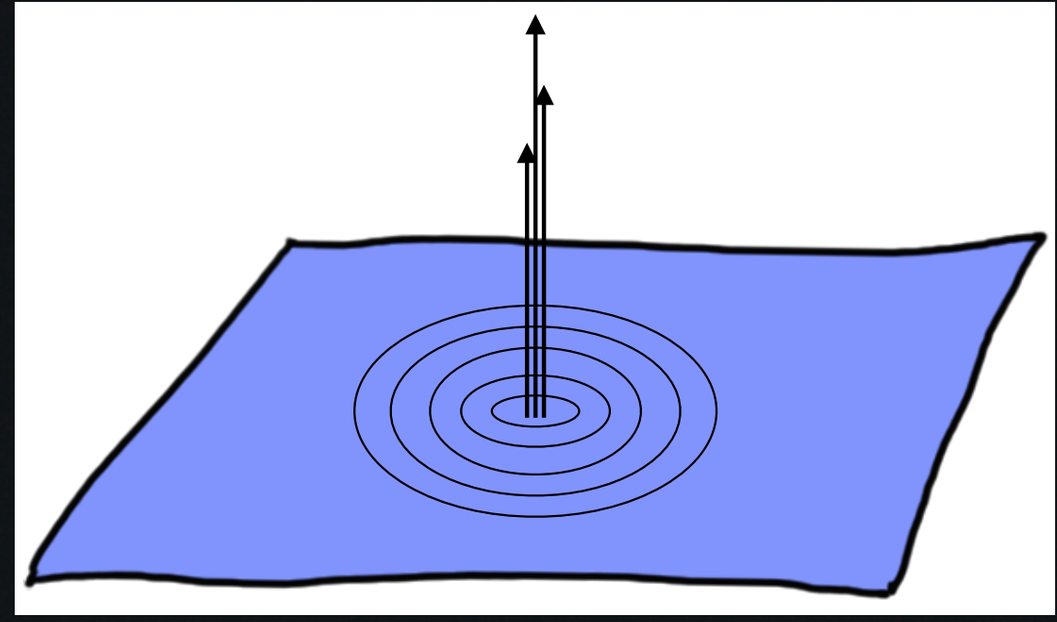
3d/1d Defects



- Maximally SUSY 3d theory, $\mathcal{N} = 8$ ABJM
  - Global symmetry is $SO(8) \supset U(1)^4$
  - We can tune four independent fluxes, moduli SUSY constraint
  - As before, we may add conical singularity on defect
  - $\mu_R = \mu_0 + \mu_1 + \mu_2 + \mu_3 = \kappa(1 - n)$
  - Using same techniques as before, we calculate $\langle J_\mu \rangle$, $\langle T_{\mu\nu} \rangle$



3d/1d Defects



- $\langle J^{\alpha}_{\theta} \rangle = \frac{1}{\rho} \frac{N^{3/2}}{3\sqrt{2}\pi} \frac{\mu^{\alpha} F_{ABJM}}{1 + \frac{2\mu^{\alpha}}{n}}, \ \alpha = 0, \ldots 3$

- $F_{ABJM} = \sqrt{\left(1 + \frac{2\mu^{0}}{n}\right)\left(1 + \frac{2\mu^{1}}{n}\right)\left(1 + \frac{2\mu^{2}}{n}\right)\left(1 + \frac{2\mu^{3}}{n}\right)}$

- Structurally similar to 4d result

- $h_D = \frac{\pi}{2n} \sum \rho \langle J^{\alpha} \rangle = -\frac{N^{3/2}}{6\sqrt{2}} \sum \frac{\mu^{\alpha} F_{ABJM}}{1 + 2\mu^{\alpha}}$

- No central charges, so no immediate monotonicites